Part-time Employment Traps and Childcare Policy

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Abstract

We model educational investment, wages and employment status (full-time, part-time or non-participation) in a frictional world in which heterogeneous workers have different productivities, both at home and in the workplace. We investigate the degree to which there might be under-employment and distortions in human capital investment, and we then show how childcare policy can be used not only to correct the ex post under-participation problem but also to provide efficient incentives to invest optimally ex ante in education.

Keywords: Part-time, full-time, education, market failure, childcare policy.

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1 Introduction

Childcare and work-family reconciliation policies have attracted much recent interest in developed countries. Reasons advanced to justify such policy intervention are that it will increase female participation, assist child development, and encourage greater investment by women in human capital, thereby improving their labor market opportunities. This discussion is premised on the notion that the labor market, left on its own, will produce a less than optimal outcome.[1] The purpose of our paper is to investigate the impact of childcare policy in this context.

It is certainly the case that countries with supportive policies towards childcare have high rates of female participation in the market sector. Panel A of Table 1 reports the estimates of a simple regression of female participation on the OECD index of supportive childcare policies, controlling for the proportion of the female population with tertiary education. The index ranges between a low of -3.4 for Greece to a high of 2.9 for Denmark. The estimates provide clear evidence of a positive and statistically significant correlation between childcare policies and female participation. This is illustrated in Figure 1.

Table 1 and Figure 1 near here.

A second stylised fact, not captured by the cross-country regressions, is the considerable variation in hours spent in market and home production across women within a given country. Since childcare policies typically recognize the fact that childcare involves a large time input, whether through home production or through purchase of market-provided services, it is instructive to consider the actual amount of time spent in market and home production across employment states. Panel B of Table 1 shows that, on average, women with children living at home typically work long hours in home production. The observed variation in means across employment states is likely to reflect differences in tastes as well as heterogeneity in workplace productivity. But it is also likely to reflect heterogeneity in home productivity, and differences across individuals in relative productivities in market and home production, which may affect participation.[2]

[1] For examples of discussion of childcare policy issues, see the websites of various US federal and state governments, and in the UK see http://www.number-10.gov.uk/output/page1430.asp

[2] In labor supply theory, differences in participation rates arise owing to differences in preferences and differing budget constraints (reflecting exogenously given market-wage rates and non-labor income). Differences in home

The goal of this paper is to consider optimal labor market policy within an equilibrium model of general human capital investment, wage formation and participation in a non-competitive labor market. A central feature is that workers are heterogeneous, having different productivities both in the home and in the workplace.[3] Importantly, labor market distortions affect such workers differently. For example, the talented home-maker who has high home productivity but low workplace productivity is unaffected by imperfect competition in the labor market. Such homemakers earn their marginal product in the household and are largely unaffected by market failures in the workplace. In contrast, the incompetent parent with high workplace productivity is most affected by imperfect competition in the labor market - that worker participates in the labor market with probability one and may suffer a significantly reduced pay packet through imperfect wage competition. However the corresponding deadweight loss is small because the worker still chooses to participate in the labor market, which is the socially optimal outcome. The efficiency losses are instead greatest for those who are relatively talented in both dimensions. For these workers, a non-competitive labor market may lead to large substitution effects to home production.

Those deadweight losses are magnified by ex-ante education decisions. In the first phase of their lives, youngsters can increase their future workplace ability by investing in general skills that affect workplace productivity. Such investments may not necessarily improve future home productivity. For example, they might invest in a mathematics course or a qualification in information technology, imbuing them with expertise that is invaluable in the workplace but is unlikely to increase their skills at home-decorating or their patience at child-rearing. Of course if the labor market were perfectly competitive, so that workers were paid their full market value, each worker would invest in general human capital at the socially optimal level. But if a worker expects to receive less than the full marginal return to human capital investment, the worker

productivity are typically ignored. Important contributions to the literature considering the allocation of time to home production - albeit in a different context to ours - include Becker (1965, 1981), Gronau (1977), Sandmo (1990), and Apps and Rees (1997).

[3] Some may have very high levels of workplace productivity and low levels of domestic productivity - such as the talented physicist who cannot imagine that the Big Bang could be something happening to the kids in the kitchen. Others may be highly proficient in both workplace and home production: the so-called superwomen, such as Ma Baker and Sandra Day O'Connor in the US, and Cherie Blair in the UK. Others again might be characterized by rudimentary literacy and numeracy abilities, rendering them of low workplace productivity, but with a high level of competence at home-making. Other less fortunate individuals might be untalented in both dimensions.

not only underparticipates in the labor market ex-post but also underinvests in human capital ex-ante.

A central insight of the paper is that the ex-post participation decision of workers endogenously generates increasing marginal returns to education. It is shown that this non-convexity generates a part-time employment trap. For youngsters in that trap, the social planner's optimum implies the youngster should choose a high level of education and ex-post enjoy a high participation probability in full time employment. But as workers are not paid their full value in the labor market, these youngsters substitute to home production. They make low skills investments ex-ante, and participate with low probability in the labor market ex-post - and only in part-time employment if they do choose to participate. The corresponding deadweight losses are potentially large.

So what are the reasons for such increasing returns? First, an increase in general human capital leads to better wage offers and the worker is more likely to participate in the workplace. Ex-ante, a higher expected participation rate increases the expected marginal return to human capital investment. Second, an increase in general human capital can make it worthwhile for the worker to switch from part-time to full-time employment (assuming diminishing marginal returns to home production). Rather than only spend a proportion of time $T < 1$ in the workplace, the switch to full time work increases the expected return to human capital investment by a factor of $1/T$.

Even with a competitive labor market, there are increasing marginal returns to education. But the assumed market imperfection (described below) generates a third increasing return to education. Wages in the model are determined using a Hotelling-type pricing structure.[4] That structure implies that wage competition becomes more intense as the worker's value of employment increases; i.e., wages rise more quickly with productivity as productivity increases, and rises one for one with productivity once the worker (endogenously) participates with probability one.

Increasing marginal returns to education implies workers specialize. Those with a com-

[4] Bhaskar, Manning and To (2002) also note that transport costs can usefully summarize the variety of reasons for imperfect competition in the labour market - such as imperfect information about alternative jobs, mobility costs, and heterogeneous preferences over the non-wage characteristics of a job.

parative advantage in workplace production invest ex-ante in high levels of human capital and participate ex-post in full-time employment with probability close to one. Those with a comparative advantage in home production choose little education ex-ante and focus on home production ex-post, though possibly taking part-time employment (assuming decreasing returns to home production). Workers in the part-time employment trap have the most distorted labor supply decision and so generate the largest deadweight losses.

The policy section establishes that an employment subsidy, paid to the worker, not only corrects the ex-post under-participation problem, but also corrects the ex-ante under-education problem. Furthermore as the title of the paper suggests, the policy can be appropriately targeted by noting who, statistically, might be caught in the part-time employment trap; i.e. what sectors of society are characterised by low participation rates/part-time employment and relatively low education levels.

Table 2 describes male and female participation rates, and type of employment contract, by demographic group. The data source is the British Household Panel Survey (BHPS). Columns [1] and [3] report male and female participation rates, while Columns [2] and [4] report the proportions of the total male and female working populations respectively that are employed part-time. Notice that women have similar participation rates to men in each demographic group except for those who are married and, more significantly, those who have kids between 0-16 years. Furthermore, within this latter group, over 50% of women who participate take part-time employment, while 98% of men who participate take full time employment. As female part-time wage rates are only 70% of female full time wage rates in the U.K. [OECD (1999)] it may not be surprising that the participation rate of women with kids is so low.

Table 2 near here.

Table 2 demonstrates that, on average, women take the brunt of child care, which in the context of this paper might be interpreted as relatively high home productivity. Such women are over-represented in part-time employment and we know that part-time employment traps occur for such workers. An obvious employment subsidy which targets precisely this group is a state-subsidised childcare scheme. Such an employment subsidy may generate large welfare gains as it not only corrects the ex-post underparticipation distortion, but also encourages women to invest more in education when young. Indeed, Table 3 demonstrates that women with kids in the part-time employment sector have surprisingly low education levels.

Table 3 here.

Table 3 summarizes the ex-ante education decisions, and ex-post participation rates, of working-age men and women who have kids up to 16 years old living at home. First consider the full-time employment row. For each sex and conditional on having kids up to 16 years old, the full time employment row describes the proportion of workers who have a particular level of education. It is striking that the composition of education is very similar between the two sexes in full-time work. This is consistent with the theory developed below which argues that skills levels are not distorted for the very highest workplace ability types (who typically work full time). Presumably many of these high ability types pay for private child care.

The distortion becomes evident as we consider the part-time employment and non-participation rows in Table 3. These establish that, for women with kids who choose to work part-time, 62% have no qualification higher than that taken at the end of compulsory schooling (the 'GCSE'), while the corresponding figure for men is 37%. Similarly, of women with kids who choose not participate, 65% have no qualification higher than GCSE, while the corresponding figure for men is 53%.

For simplicity, the paper establishes the existence of part-time employment traps assuming the labor market is imperfectly competitive. It should be noted, however, that distortionary taxation, where government revenue is raised by taxing labor income while home production is not taxed, generates the same market failure. By extracting workplace rents, the government induces workers to substitute into home production.[5] As argued above the largest substitution effects, and hence greatest deadweight losses, arise for workers who are relatively productive both in the workplace and in home production. Furthermore as there are increasing marginal returns to education (even in a competitive labor market), the same part-time employment trap occurs, except it is the government which is extracting rents from employees. We shall return to this insight in the Conclusion.

The next section describes the model and determines equilibrium remuneration and partici-

[5] This possibility was also noted by Sandmo (1990).

pation rates of workers by productivity type. Section 3 examines a worker's optimal investment in (workplace) human capital given non-competitive wage formation in the workplace. It shows how undertraining and under-participation is closely associated with part-time employment. Section 4 develops the implications for optimal childcare policies and draws some conclusions.

2 The Model

Each worker is productive both at home and in the workplace. A representative worker is born in the first period with ability a and has expectations of future home productivity b. In the first period, the worker at cost $\phi(k)$ can invest in k units of general skills, whereupon the worker's second period productivity in the workplace is denoted $\alpha = \alpha(a, k)$ where α is strictly increasing in both arguments and differentiable. The worker's home productivity b is also determined in the second period, and could be considered as a random draw from some underlying population distribution. For simplicity, however, we shall assume the worker knows b in the first period.

Given (α, b) in the second period, the worker has a unit time endowment which is allocated between production at home and in the workplace. There are diminishing marginal returns to home production, but constant returns in the workplace. For simplicity, if the worker allocates time h to home production, assume the value of home output, $x(h; b)$, is given by

$$\begin{aligned} x(h; .) &= bh \text{ if } h \leq 1 - T \\ &= b(1 - T) \text{ if } h > 1 - T. \end{aligned}$$

where $T \in (0, 1)$ is fixed. This production technology assumes that, for all workers, home production provides a zero marginal return for $h > 1 - T$, but workers with different home productivities b have different marginal returns for $h < 1 - T$.

Given the worker supplies l units of labor to the workplace, the value of the resulting output is αl, where the time constraint requires $l + h = 1$. Potentially workers may have some disutility to work in the workplace, where the worker obtains an additional utility payoff $-ql$ for time l spent at work. As this is not disimilar to assuming net productivity $(\alpha - q)$ in the workplace, we simplify by assuming $q = 0$.

The labor market is imperfectly competitive and has a Hotelling-type structure. If the worker accepts employment at firm i, then the worker has an additional utility (or transport) cost $c_i \geq 0$ to working there, where c_i is considered as a random draw from some underlying c.d.f. F with finite support $[0, \overline{c}]$. Assume F is twice differentiable and concave over that support (i.e. the density exists and is non-increasing).

Wages are determined in the second period by Bertrand competition between two firms, $i = 1, 2$. The worker's productivities (α, b) are common knowledge, but the idosyncratic utility costs (c_1, c_2) are private information to the worker. Given the observed productivities (α, b), each firm $i = 1, 2$ simultaneously makes a contract offer (y^i, l^i), where y^i is the amount paid to the worker in return for providing l^i units of labor time. Given those contract offers, the worker either accepts one, say at firm i, and so obtains period 2 utility $U_2 = x(1 - l^i; b) + y^i - c_i$, or rejects both and so obtains period 2 utility $U_2 = b[1 - T]$ through home production. Should the worker accept firm i's contract offer, firm i makes profit $\alpha l^i - y^i$, while the other firm obtains zero profit.

Throughout we shall only consider symmetric pure strategy equilibria. In the second period and given (b, α), each firm $i = 1, 2$ chooses (y^i, l^i) to maximise expected profit, given the acceptance strategy of the worker. The corresponding symmetric Nash equilibrium implies contract offers $(y^i, l^i) = (y^*(b, \alpha), l^*(b, \alpha))$. Given those equilibrium contract offers (y^*, l^*), we can then compute the worker's expected second period utility, denoted $U_2^*(\alpha, b)$. This problem is considered in Section 3.

Given expected utility $U_2^*(\alpha, b)$ in the second period, we investigate in Section 4 the worker's optimal investment decision in the first period. In particular, given ability a and expectations of future home productivity b, the worker chooses skills k to maximize total expected utility:

$$\max_k \left[-\phi(k) + U_2^*(\alpha(k, a), b) \right].$$

In anticipation of the results below, it is useful to define the following.

Definition: Given (α, b), the value of workplace employment, $V = V(\alpha, b)$, is defined as

$$V(\alpha, b) = \max[\alpha T, \alpha - b(1-T)]. \tag{1}$$

$\alpha > b$ implies a full time employment contract is more efficient than a part-time one. The above definition with $\alpha > b$ implies the value of workplace employment $V = \alpha - b(1-T)$; assuming a full time employment contract, V is total workplace output, α, less foregone home production. Alternatively $\alpha \leq b$ implies a part-time employment contract is more efficient. The above definition with $\alpha \leq b$ implies $V = \alpha T$; assuming a part time employment contract in this case, V is the value of workplace output with no foregone home production. Note that $V(.)$ is continuous, strictly increasing in α and (weakly) decreasing in b.

3 Equilibrium Wages

This section considers equilibrium in the second period, where (α, b) are taken as given. Given contract offers (y^i, l^i) and idiosyncratic utility costs c_i, the worker's second period payoff is

$$U_2 = \max\left[x(1-l^1;b) + y^1 - c_1, x(1-l^2;b) + y^2 - c_2, b(1-T)\right]$$

where the worker either accepts firm 1's offer, accepts firm 2's offer or rejects both. This section characterizes the (symmetric, pure strategy) Nash equilibrium where, given (α, b), the firms simultaneously make contract offers (y^i, l^i) to maximise expected profit, given the job acceptance strategy of the worker.

To start, consider parameters satisfying $b > \alpha$ (we consider the case $b < \alpha$ below). $b > \alpha$ implies marginal home productivity is greater than workplace productivity for hours $h < 1-T$. The jointly efficient contract is therefore a part-time contract and so, as productivities are common knowledge, this describes a firm´s optimal contract offer. Hence both firms offer $l^i = l^* = T$.

Suppose that firm i offers compensation y^i for part-time employment. Given contract offers $(y^i, l^* = T)$, the worker can either

(i) reject both and so obtain payoff $b[1-T]$ through home production, or

(ii) accept firm $i's$ part-time employment offer and so obtain payoff $b[1-T]+y^i-c_i$.

The worker's optimal job acceptance strategy is to accept employment at firm 1 if $y^1-c_1>\max[0,y^2-c_2]$.

Given this job acceptance strategy, consider now firms 1's choice of compensation y^1. If firm 2 offers compensation y^2, then firm 1's expected profit, denoted π_1, by offering y^1 is

$$\pi_1(y^1,y^2;\alpha,b)=P(y^1-c_1\geq\max[0,y^2-c_2])[\alpha T-y^1],$$

where $P(.)$ is the probability that the worker accepts firm 1's job offer,[6] whereupon the firm makes profit $\alpha T-y^1$.

To compute this probability, note that for each c_1 satisfying $y^1-c_1\geq 0$; i.e. for $c_1\leq y^1$, the worker prefers employment at firm 1 rather than pure home production. Further for such c_1, the worker also prefers firm 1's employment offer to firm 2's as long as $y^2-c_2\leq y^1-c_1$; i.e. for $c_2\geq y^2-y^1+c_1$, which occurs with probability $1-F(y^2-y^1+c_1)$. Hence integrating over such c_1, the probability the worker accepts firm 1's contract offer is

$$\int_0^{y^1}[1-F(y^2-y^1+c_1)]f(c_1)dc_1.$$

Hence firm 1's expected profit is

$$\pi_1(y^1,y^2;.)=[V-y^1]\int_0^{y^1}[1-F(y^2-y^1+c_1)]f(c_1)dc_1 \tag{2}$$

where $b>\alpha$ implies $V=\alpha T$. It will become clear later why we have substituted in V here.

Given y^2, firm 1's best response satisfies the necessary condition for a maximum:

$$\begin{aligned}\frac{\partial\pi_1}{\partial y^1} &= -\int_0^{y^1}[1-F(y^2-y^1+c_1)]f(c_1)dc_1 \\ &\quad +[V-y^1]\left[[1-F(y^2)]f(y^1)+\int_0^{y^1}f(y^2-y^1+c_1)]f(c_1)dc_1\right]=0.\end{aligned} \tag{3}$$

[6]Note that there are no mass points in F by assumption.

This expression is the standard monopsony rule: the firm trades a lower wage offer (the first term describes expected employment) against a lower acceptance probability (the second term). As a pure strategy, symmetric equilibrium implies $y^1 = y^2 = y^*$, the above condition implies equilibrium y^* satisfies

$$\int_0^{y^*} [1 - F(c_1)] f(c_1) dc_1 = [V - y^*] \left[[1 - F(y^*)] f(y^*) + \int_0^{y^*} [f(c_1)]^2 dc_1 \right].$$

As the left hand side is integrable, this simplifies to:

$$\frac{1}{2} \left[1 - [1 - F(y^*)]^2 \right] = [V - y^*] \left[[1 - F(y^*)] f(y^*) + \int_0^{y^*} [f(c_1)]^2 dc_1 \right], \tag{4}$$

which is an implicit function for $y^* = y^*(V)$. (4) therefore describes the equilibrium compensation offer y^* given the value of workplace production $V = \alpha T$ in a part time employment contract (which is optimal when $b > \alpha$). To interpret this condition, note that $1 - [1 - F(y^*)]^2$ is the probability a worker accepts an employment offer [i.e. at least one c_i is below y^*]. Symmetry implies the left hand side of (4) describes expected employment at firm 1. For a small increase in y^1 above y^*, the left hand side describes the marginal loss in firm 1's profit by paying workers more, while the right hand side describes the marginal increase in profit by attracting more workers, where $f(y^1 = y^*)[1 - F(y^*)]$ is the measure of workers who are marginally attracted from non-participation (workers whose $c_1 = y^*$ and $c_2 > y^*$), and $\int_0^{y^*} f(c)^2 dc$ is the measure of workers who are marginally attracted from firm 2 (workers whose $c_1 = c_2 \leq y^*$). Optimality requires that these two margins are equal.

As y^* describes equilibrium compensation, we briefly describe its mathematical properties.

Claim 1. $y^*(V)$ is continuously differentiable and satisfies $y^* = 0$ at $V = 0$ and
(i) for $V \in (0, \overline{c} + d)$, $y^*$is strictly increasing with $dy^*/dV < 1$ and $y^*(V) < \overline{c}$;
(ii) for $V \geq \overline{c} + d$, $y^*(V) = V - d \geq \overline{c}$;
where $d > 0$ is defined by:

$$d = \frac{1}{2 \int_0^{\overline{c}} [f(c)]^2 dc}. \tag{5}$$

Proof. (4) immediately implies $y^*(0) = 0$. Differentiating (4) w.r.t. V and rearranging implies

:

$$\frac{dy^*}{dV} = \frac{[1-F(y^*)]f(y^*) + \int_0^{y^*}[f(c_1)]^2 dc_1}{2[1-F(y^*)]f(y^*) + \int_0^{y^*}[f(c)]^2 dc + [V-y^*][1-F(y^*)][-F''(y^*)]} \tag{6}$$

Noting $V > 0$ implies $y^* < V$ [a firm never offers $y \geq \alpha T$ as it implies a negative profit] then F concave over its support implies $0 < \frac{dy^*}{dV} < 1$ while $0 < y^* < \overline{c}$. As F is twice differentiable, dy^*/dV is continuous for $y^* < \overline{c}$ and note $y^* \to \overline{c}^-$ implies $dy^*/dV \to 1$. Putting $y^* = \overline{c}$ in (4) implies $V = \overline{c} + d$ where d is defined in the Claim. Finally (4) implies $y^* = V - d$ for $y^* \geq \overline{c}$, and so $dy^*/dV = 1$ for such y^*. This completes the proof of the Claim.

Recall that $V = \alpha T$ in this case. Claim 1 establishes that given part-time contract offers, equilibrium compensation, y^*, is zero if $\alpha = 0$, and is strictly increasing in workplace productivity α. For large enough α, so that $\alpha T > \overline{c} + d$, the worker is offered the value of output, αT, less a lump sum deduction d.We discuss further this property of equilibrium below. Before doing that we consider the case $b < \alpha$.

$b < \alpha$ implies the optimal contract offer is a full time employment contract; i.e. $l^* = 1$. Suppose that each firm i also offers compensation $y^i = b(1-T) + \widehat{y}^i$; i.e. the wage compensates for foregone home production, plus some additional surplus $\widehat{y}^i$. It follows that the worker will accept firm 1's contract offer if $\widehat{y}^1 - c_1 > \max[0, \widehat{y}^2 - c_2]$. Computing this probability as before, firm 1's expected profit is

$$\begin{aligned}\pi_1(\widehat{y}^1, \widehat{y}^2) &= [\alpha - y^1] \int_0^{\widehat{y}^1} [1 - F(\widehat{y}^2 - \widehat{y}^1 + c_1)] f(c_1) dc_1 \\ &= [V - \widehat{y}^1] \int_0^{\widehat{y}^1} [1 - F(\widehat{y}^2 - \widehat{y}^1 + c_1)] f(c_1) dc_1\end{aligned}$$

as $V = \alpha - b(1-T)$ for $b < \alpha$. Note that the functional form for π_1 is identical to (2). Hence the same results apply: a pure strategy, symmetric equilibrium implies $\widehat{y}^i = y^*$ as described in Claim 1 and each firm offers compensation $b(1-T) + y^*(V)$. The insights are therefore the same, the only difference being that the firms compensate workers for their foregone home production. We have therefore established the following Theorem.

Theorem 1. Equilibrium Contract Offers.

Given (α, b) and corresponding $V = V(\alpha, b) > 0$, a pure strategy, symmetric wage contracting equilibrium implies:

Case (A) for $V < \overline{c} + d$, the worker is offered:

(i) a part-time contract ($l^* = T$) if $\alpha < b$, and compensation y^*;

(ii) a full-time contract ($l^* = 1$) if $\alpha \geq b$, and compensation $b(1-T) + y^*$.

where y^* is defined by (4).

Case B if $V \geq \overline{c} + d$, the worker is offered:

(i) a part-time contract, $l^* = T$ if $\alpha < b$, and compensation $\alpha T - d$

(ii) a full-time contract, $l^* = 1$ if $\alpha \geq b$, and compensation $\alpha - d$.

Proof. Case A has already been established. Consider Case B with $V \geq \overline{c} + d$. Claim 1 implies $y^* = V - d$. If $\alpha < b$, the contract offer is a part time employment contract with total compensation $y^* = V - d$ and, as $V = \alpha T$ for this case, this reduce to $\alpha T - d$. If $\alpha \geq b$, the contract offer is a full time employment contract with total compensation $b(1-T) + y^*$. Claim 1 then implies the Theorem, noting that $V = \alpha - b(1-T)$ for this case. This completes the proof of the Theorem.

The Technical Appendix describes conditions which guarantee the existence of a symmetric pure strategy equilibrium. Theorem 1 establishes that, in such an equilibrium, the type of contract offered, part-time or full-time, depends on whether home productivity b exceeds workplace productivity α or not. This is not surprising as it is jointly efficient that the worker should only work part-time if $b > \alpha$, and should work full-time otherwise.

An important element of Theorem 1 is that the value of workplace employment, V, is not only a sufficient statistic describing the surplus offered by firms, y^*, it also determines the worker's participation probability. In particular, given $V = V(\alpha, b)$ and offer $y^* = y^*(V)$, the worker accepts a job offer as long as one of the c_i is less than y^*; i.e. the worker's participation probability is

$$P(y^*) = 1 - [1 - F(y^*)]^2$$

and Claim 1 implies that participation rates increase with V. Figure 2 now describes how remuneration, y^*, and participation rates $P(y^*)$ vary with the underlying productivities α, b.

Figure 2 here.

The region below the 45^0 line is the part-time employment region (where $b > \alpha$). In that case $V = \alpha T$ and so the iso-V contours correspond to horizonal lines. Such contours also describe iso-y^* contours, and iso-participation contours. Increased workplace productivity α implies higher V, more generous offers (higher y^*) and higher participation rates.

Above the 45^0 line, $V = \alpha - b(1-T)$ and so iso-V contours correspond to straight lines with slope $1 - T$. Again, higher α implies a higher V, y^* and participation probability P. Although the 45^0 line implies a switch from full time to part-time contracts, the participation contours are continuous across this switch.

$\overline{\alpha}(b)$, as drawn in Figure 2, is defined where $V(\overline{\alpha}, b) = \overline{c} + d$. Claim 1(i) shows that workers with productivity $\alpha < \overline{\alpha}(b)$ receive relatively low wage offers; i.e. receive $y^* < \overline{c}$. As workers do not participate if both utility costs c_i exceed y^*, then $y^* < \overline{c}$ implies these workers participate in the labor market with probability less than one. More importantly, the non-competitive wage offer implies they underparticipate relative to the social optimum; $y^* < V$ implies $P(y^*) < P(V)$. This not only implies their participation rates are ex-post inefficient but, as we shall see in the next section, the expected return to training is consequently too low and ex-ante investments are also too low.

In contrast, workers with productivity $\alpha \geq \overline{\alpha}(b)$ have high V (i.e. $V \geq \overline{c} + d$) and Claim 1 implies they receive offers $y^* \geq c$. These workers participate in the workplace with probability one. This region is efficient - although $y^* < V$, workers participate with probability one and so $P(y^*) = P(V) = 1$. More surprisingly, the next section shows that workers in this region also invest ex-ante at the socially optimal level.

Efficiency therefore depends critically on whether $V \lesseqgtr \overline{c} + d$. To understand why this threshold occurs, note that the worker's participation strategy implies there are two oligopsony margins: a poaching margin and a participation margin. By offering higher wages, a firm might not only attract an employee from a competing firm - the poaching margin - but also attract a non-participant into the market sector.

The participation margin does not bind for workers with sufficiently high V that, in equilibrium, they accept a job offer with probability one. A useful analogy is the Hotelling pricing literature where we might interpret c_i as the worker's transport cost to work at firm i. The case "V sufficiently high that an offer is always accepted" is typically referred to as a "covered market". The equilibrium is that both firms offer a wage equal to the worker's value of output less 'price' $d > 0$. Equilibrium d reflects the marginal probability that a small increase in the offered wage will poach the worker away from the competing firm and, in a symmetric equilibrium, d depends only on the assumed distribution of transport costs.

In contrast, the participation margin binds for workers with V less than $\overline{c} + d$. Such workers include low workplace-productivity workers and intermediate productivity workers with high home productivities. An important property of the Hotelling pricing structure is that, as the value of employment increases, wage competition at the margin becomes more intense. In particular, Claim 1 implies $dy^*/dV = 0$ at $V = 0$, $dy^*/dV < 1$ for $V < \overline{c} + d$ and $dy^*/dV \to 1$ as $V \to \overline{c} + d$. Hence wages rise more quickly with productivity as the participation margin peters out, where $dy^*/dV = 1$ for all $V \geq \overline{c} + d$

Given wages as described by Theorem 1, the next section examines the worker's optimal training decision in the first period. To do that, we initially compute expected second period utility conditional on productivities α, b which we denote as $U_2^*(\alpha, b)$. The results of Theorem 1 imply

$$\begin{aligned} U_2^*(\alpha, b) &= E_{c_1,c_2} \max[b(1-T), b(1-T) + y^* - c_1, b(1-T) + y^* - c_2] \\ &= b(1-T) + E_{c_1,c_2} \max[0, y^* - c_1, y^* - c_2]. \end{aligned}$$

Let $c = \min[c_1, c_2]$ and note this random variable has c.d.f. $1-(1-F)^2$. Its density is therefore $2(1-F)f$, and so

$$U_2^*(\alpha, b) = b(1-T) + \int_0^{y^*} [y^* - c] 2(1 - F(c)) f(c) dc.$$

Integration by parts implies

$$U_2^*(\alpha, b) = b(1-T) + \int_0^{y^*} \left[1 - (1 - F(c))^2\right] dc. \tag{7}$$

The first term is the value of home production, the second is the expected surplus from employment, which depends on V and labor market imperfections as $y^* = y^*(V)$.

4 The Worker's Optimal Education Decision

Given ability a and expected home productivity b, the worker in the first period chooses general human capital investment or training k to maximise $-\phi(k) + U_2^*(\alpha, b)$ where workplace productivity $\alpha = \alpha(a, k)$.[7] For simplicity, assume $\alpha(.) = a + k$, and that the training cost technology $\phi(.)$ is increasing and strictly convex with $\phi(0) = \phi'(0) = 0$. The worker's optimisation problem can then be reformulated as:

$$\max_{\alpha} U_2^*(\alpha, b) - \phi(\alpha - a)$$

where the worker chooses second period productivity $\alpha \geq a$ at personal cost $\phi(\alpha - a)$. The necessary condition for a maximum is

$$\partial U_2^* / \partial \alpha = \phi'(\alpha - a),$$

where (7) implies

$$\begin{aligned} \frac{\partial U_2^*}{\partial \alpha} &= [1 - (1 - F(y^*))^2] \frac{dy^*}{dV} \frac{\partial V}{\partial \alpha} \\ &= P(y^*) \frac{dy^*}{dV} \frac{\partial V}{\partial \alpha}. \end{aligned} \tag{8}$$

$\partial U_2^* / \partial \alpha$ describes the marginal expected return to education and depends on three components: $P(y^*)$ is the probability the worker participates in the labor market; dy^*/dV is the rate at which compensation y^* increases with V; and $\partial V / \partial \alpha$ describes how V increases with skills α. Figure 3a plots $\partial U_2^* / \partial \alpha$ for fixed b, satisfying $b < (\overline{c} + d)/T$, and this is denoted MR. Most importantly for what follows, note that there are increasing marginal returns. This occurs for three reasons:

(i) Participation effects: an increase in productivity implies firms offer higher wages which increases the worker's participation probability (see Claim 1). The higher participation probability increases directly the marginal return to education.

[7] We sometimes use the term 'training' to denote human capital investments made by individuals ex ante. However this is education or training received 'before the job' rather than on the job.

(ii) The full-time/part-time switch: an increase in productivity at $\alpha = b$ implies the worker switches from part-time to full-time employment. The marginal return to education increases at this point by a factor of $1/T$, where (1) implies $\partial V/\partial\alpha = T$ for $\alpha < b$ and $\partial V/\partial\alpha = 1$ for $\alpha > b$.

(iii) Increasing wage competiveness: as the value of employment V increases, firms at the margin bid more competitively for the worker's services. In particular, $dy^*/dV = 0$ at $V = 0$, while $dy^*/dV \to 1$ as $V \to \overline{c} + d$ (see Claim 1).

It is straightforward to show that $b < (\overline{c} + d)/T$ implies $\overline{\alpha}(b) > b$ (recall, $\overline{\alpha}$ is defined where $V(\overline{\alpha}, b) = \overline{c} + d$). For such b, we then have (i) $MR = 1$ for all $\alpha \geq \overline{\alpha} > b$ [as $P(y^*) = 1$, $dy^*/dV = 1$ and $\partial V/\partial\alpha = 1$ for such α], (ii) $MR < 1$ for all $\alpha < \overline{\alpha}$ [as $P(y^*) < 1$, $dy^*/dV < 1, \partial V/\partial\alpha \leq 1$] and (iii) $MR = 0$ at $\alpha = 0$ [as $P(y^*) = 0$]. Also notice that MR is not continuous at $\alpha = b$; at this point the worker switches from part-time to full-time employment.

Figure 3a here

$\phi'(\alpha - a)$ is the marginal cost to skill accumulation and is denoted MC_a in Figure 3a. The assumptions on ϕ imply $MC_a = 0$ at $\alpha = a$ and is strictly increasing in α. The optimal skills investment decision of a worker with ability a occurs where MC_a crosses MR, though this may not be a sufficient condition for a maximum as there are increasing marginal returns.

There are two marginal cost curves drawn in Figure 3a corresponding to medium ability a_M and high ability a_H. Note that an increase in ability implies a rightward shift in MC_a. This automatically implies that, ceteris paribus, workers with higher ex-ante ability invest in education to a higher ex-post skill level α.

Now consider workers with high ability, $a = a_H$, as drawn in Figure 3a. These workers invest so that $\alpha = \overline{\alpha}$. The previous paragraph implies that workers with ability $a > a_H$ invest so that $\alpha > \overline{\alpha}$. These workers are interesting as, at such investment levels, the private marginal return to education equals the social marginal return, and so these workers invest to the socially optimal level (even though ex-post they earn less than their value; i.e. $y^* < V$). To see this, note that with constant returns to workplace production, the competitive outcome implies workers should be offered their full value $y^* = V$. In that case, the marginal social return to education, denoted SR, is

$$SR = P(V)\frac{\partial V}{\partial \alpha},$$

as $P(V)$ is the probability that the worker participates in the competitive labor market, and $\partial V/\partial\alpha$ is the corresponding increase in output. Although $\alpha \geq \overline{\alpha}$ implies $y^* = V - d < V$, Claim 1 still gives $P(y^*) = P(V) = 1$ and $dy^*/dV = 1$. Hence $SR = MR = 1$ for $\alpha \geq \overline{\alpha}$ and so workers with $a \geq a_H$, who choose $\alpha \geq \overline{\alpha}$, train to the socially optimal level and participate with probability one.

In contrast, Claim 1 implies $SR > MR$ for $\alpha < \overline{\alpha}$ [as $P(y^*) < P(V)$ and $dy^*/dV < 1$]. This implies that workers with $a < a_H$ undertrain - their private marginal return to education is strictly less than the marginal social return. They also underparticipate ex-post.

Now consider an intermediate ability type with $a = a_M$ as drawn in Figure 3a. Such an individual is interesting as, given the two shaded areas are equal, this person is indifferent to investing to $\alpha = \alpha_2 \gg b$ or investing to $\alpha = \alpha_1 \ll b$. The discontinuity in the marginal return to education at $\alpha = b$ implies discontinuous investment behaviour. Workers with ability $a > a_M$ train so that $\alpha > \alpha_2 \gg b$; such workers have high V ex-post, work full time and have relatively high participation probabilities. In contrast, those with ability $a < a_M$ invest so that $\alpha < \alpha_1 \ll b$. Such workers have low V ex-post, work part-time and have relatively low participation probabilities.

It is worth contrasting this market outcome with the competitive case. As there are increasing social marginal returns ($P(V)$ is increasing in V), the competitive outcome also implies some critical threshold $a^c(b)$, where workers with ability $a \geq a^c(b)$ choose $\alpha \gg b$ (and work full time with a high participation probability) while lower ability workers invest $\alpha \ll b$ (and work part time with a low particiaption probability). But as $SR > MR$ for $\alpha < \overline{\alpha}$, it follows that $a^c < a_M$; in the competitive market, worker a_M would strictly prefer to invest to some $\alpha > \alpha_2$. The oligopsonistic behaviour of firms therefore implies a *part-time employment trap;* workers with ability $a \in [a^c, a_M)$ severely underinvest. The social planner's optimum implies training $\alpha \gg b$, full time employment with a high participation rate, but imperfect wage competition implies that these workers instead substitute to home production, consider only part-time employment and underparticipate. As $MR < SR$ for $\alpha \in [\alpha_1, \alpha_2]$, the large switch in investment implies a large deadweight loss.

Figure 3a describes a_M and a_H for a particular value of home productivity b. More generally for any b, let (a_M, b) denote the worker who is indifferent to investing to high α and working full-time, or investing low α and working part-time with a low probability. As home productivity changes the value of employment, a_M varies with b. The following characterises $a_M = a_M(b)$. Similarly, let (a_H, b) denote a worker who invests to $\alpha = \overline{\alpha}(b)$ and so describes the efficiency frontier; i.e. all those with $a \geq a_H(b)$ invest to the socially optimal level and participate with probability one. Figure 3b establishes how these margins change with b (for $b < (\overline{c} + d)/T$).

Figure 3b here

An increase in b shifts the MR curve where

$$MR = P(y^*) \frac{dy^*}{dV} \frac{\partial V}{\partial \alpha}$$

The effect of an increase in home productivity b depends on whether b exceeds α or not. First fix an α and b satisfying $\alpha > b$ and consider a (small) increase in b. This implies V strictly decreases and so implies a rightward shift in MR (a higher α is then necessary to hold V constant). Now fix an α and b satisfying $\alpha < b$ and consider a (small) increase in b. This time an increase in b does not affect V and so MR is unaffected. Figure 3b illustrates this comparative static for the case $\overline{\alpha} > b$ (which holds for $b < (\overline{c} + d)/T$).

An increase in b does not affect the MC curve. The cost curve denoted MC in Figure 3b corresponds to the MC curve drawn in Figure 3a; it is the MC curve of type a_M who, with productivity b, is indifferent to choosing high or low α. Now consider an increase in b, say to $b' > b$, which implies a right shift of the MR curve. Given home productivity b', our original worker with workplace ability $a = a_M(b)$ now strictly prefers to choose low α. Instead consider the cost curve denoted MC'. Assuming the two shaded areas are equal, then given home productivity b', MC' describes the worker who is indifferent to choosing high or low education; i.e. the rightward shift in MR implies $a_M(b') > a_M(b)$.

Similarly for $a_H(b)$. Recall that $\overline{\alpha}(b)$ is the lowest value of α where investment is socially efficient. The case $\overline{\alpha} > b$ implies this occurs where $MR = 1$. The right shift in MR now implies $\overline{\alpha}$ increases with b. As $a_H(b)$ is identified by the MC curve which passes through $\overline{\alpha}(b)$, an increase in b implies $a_H(.)$ must also increase. Figure 3b therefore establishes the following:

(i) The optimal training decision of workers with ability $a < a_M(b)$ does not change with b - these workers intend only to work part-time and increased b does not affect their training margin.

(ii) The part-time employment trap, $a_M(b)$, increases with b. At the old a_M an increase in b reduces the marginal return to training (MR shifts to the right) and the worker then strictly prefers $\alpha = \alpha_1 < b$.

(iii) The efficiency threshold a_H increases and so workers with higher b are more likely to underparticipate and to underinvest in education.

Similar insights go through for $\overline{\alpha} < b$, which occurs for large b. The above information is incorporated in Figure 4 which depicts the training and ex-post participation contours based on ex-ante abilities (a, b).

Figure 4 here.

For any level of home productivity b, workers with ability $a \geq a_H(b)$ participate with probability one (either full time or part-time depending on b) and train to the socially optimal level. The insight is that, although these high ability types have amount d deducted from their wages by firms (see equation 5), this lump sum does not affect their marginal training decision nor their participation probabilities.

Workers with $a < a_H(b)$ participate with probability less than one and undertrain. For those in the full time region [where $a > a_M(b)$], they choose $\alpha \gg b$ and their training and participation contours are upward sloping. In this region, an increase in home productivity reduces the marginal return to training and so workers train marginally less and ex-post have lower participation probabilities. The lower participation probability, of course, reinforces the decision to train less.

Those workers with $a < a_M(b)$ anticipate part-time employment and choose $\alpha \ll b$. Conditional on being in this region, the iso-participation and iso-training contours are flat. However, notice that participation and training contours are not continuous across $a_M(b)$, the large drop in skills investment across this frontier implies a large drop in participation rates. Those in the part-time employment trap, i.e. those with ability $a_c < a < a_M$, would in a competitive labor market choose much higher skills and participate in full time employment with high probability.

5 Policy and Discussion

The previous section shows that endogenous participation behaviour generates increasing returns to education. Even with a competitive labor market, this non-convexity implies that workers tend to specialize either in home production (choosing low ex-ante skills investment and low ex-post participation probabilities) or workplace production (choosing high ex-ante investment and high ex-post participation rates). A non-competitive labor market, however, generates a part-time employment trap. For workers in that trap, the social planner's optimum implies the worker should make greater investments in human capital and have a high participation probability in full time employment. Instead, these workers substitute to home production – they make low skills investments and participate with low probability in part-time employment. We also demonstrated that this trap is increasing in home productivity: thus workers with intermediate ability are more likely to be caught in this trap if their home productivity is relatively high.

To correct the under-participation problem, optimal policy requires increasing the return to participation in the labor market relative to non-participation. The obvious approach is either to (i) tax non-participants with a home production tax, or (ii) subsidise participation with an employment subsidy. The first approach, a tax on non-participation, is unlikely to be politically feasible and so we focus on the latter.

Suppose for the moment that the government observes the worker's ex-post productivity parameters α, b. The government can achieve the first best allocation by offering an employment subsidy $s = S(V)$ to workers who participate in the labor market, where $V = V(\alpha, b)$ as defined before. Repeating the analysis as before, it is straightforward to show that, in a pure strategy symmetric equilibrium, the equilibrium compensation offered by firms is $y^*(V + S) - S$. In other words, the firms extract the employment subsidy from the worker (the $-S$ term), but their equilibrium offers then reflect that the value of workplace employment is $V + S$. Given such offers, workers obtain an offer with net value $y^*(V+S)$.

To identify the optimal subsidy note that, with constant returns to scale, the competitive outcome implies workers are paid full value (and the resulting allocation is efficient). Hence the optimal subsidy, S^*, satisfies

$$y^*(V+S^*) = V,$$

which is an implicit function for S^*, where $y^*(.)$ is defined by (4).

Claim 2. The optimal employment subsidy $S^*(.)$ satisfies $S^* = 0$ at $V = 0$,

(i) for $V \in (0, \bar{c})$, $S^*$is strictly increasing with $S^*(\bar{c}) = d$, and

(ii) for $V \geq \bar{c}$, $S^* = d$.

Proof: follows from Claim 1 and the equation for S^*.

Claim 2 establishes that, in order to guarantee efficient participation **and** efficient training, the government needs to compensate for the oligopsony rents extracted by firms. Of course for many types the welfare gains through such a scheme may be small (and is zero for types with $a \geq a_H(b)$ who participate ex-post with probability one). In contrast, workers with the most distorted participation rates are those found in the part-time employment trap. These workers are characterized by low participation rates and, conditional on being employed, take part-time employment.

As the subsidy scheme can be targeted by restricting eligibility to workers with certain values of b, it is worth identifying types who are likely to fall into the part-time employment trap. Table 2 in the Introduction clearly establishes that women have similar participation rates to men in each demographic group except for those who are married and, more significantly, those who have kids between 0-16 years. Furthermore, within this latter group, over 50% of women who participate take part-time employment, while 98% of men who participate take full time employment. Table 3 demonstrates that such women also have unusually low education levels. It is striking that the composition of education is very similar between the two sexes in full-time work. This is consistent with the theory, which argues that skills levels are not distorted for the very highest workplace ability types (who typically work full time). The distortion becomes evident as we consider the part-time employment and non-participation rows in Table 3. These establish that, for women with kids who choose to work part-time, 62% have no qualification higher than GCSE, while the corresponding figure for men is 37%. Similarly, of women with kids who choose not participate, 65% have no qualification higher than GCSE, while the corresponding figure for men is 53%. Together these facts suggest that women with kids are the most likely to be caught in a part-time employment trap. An obvious employment subsidy that targets precisely this group is a state-subsidised childcare scheme.

6 Conclusion

As noted in the Introduction, the underlying rationale for the renewed interest of OECD governments in childcare policies seems to be based on three related points: (i) Childcare policies enable women to get back to work, which might be especially vital in areas where there are skill shortages (under-participation). (ii) Early learning in good quality childcare can assist child development (low domestic productivity can harm child development). (iii) Childcare might provide a way out of poverty for women, especially lone mothers with low skills who cannot afford to work or train (under-participation and under-investment in skills).[8]

Our analysis indicates that an employment subsidy may generate large welfare gains in a non-competitive labor market. This is not only because it will correct the ex-post underparticipation distortion, but it will also encourage women to train more when young (as they are more likely to participate in the labor force in the future). An employment subsidy that targets precisely this group is a state-subsidised childcare scheme.

Manning (2003) provides extensive evidence of oligopsonistic wage setting for the two countries he analyses, the US and the UK.. However even if one were to insist that the labor market is competitive, the same part-time employment trap arises if the government taxes labor income but does not tax home production (as is typically the case). Workers then substitute to home production and increasing marginal returns to education generate the part-time employment trap. In fact we know from the optimal taxation literature that tax rates should be highest on those goods that are traded inelastically. An income tax on the incompetent parent who has high workplace ability is not going to distort that worker's participation probability. But

[8]Political economy factors - that we have not investigated in this paper - might also be at work: governments might respond to the preferences of working women in order to be re-elected.

it will distort the labor market decision of the intermediate ability worker with relatively high home productivity. The government's optimal tax rate therefore depends not only on workplace productivity but on home productivity (as that determines the magnitude of the substitution effect). As Table 2 suggests women with kids have the largest substitution effects, an effective way to counteract the worst participation distortions implied by a general level of income tax, is to target an employment subsidy through child care support.

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7 Technical Appendix

To address the existence issue, suppose firm 2 announces $y^*$and suppose firm 1 deviates by announcing y. Let

$$L(y, y^*) = \int_0^y [1 - F(y^* - y + c_1)] f(c_1) dc_1$$

which is the probability the worker accepts firm 1's job offer. Hence

$$\pi_1 = L(y, y^*)[V - y].$$

Note that $\pi_1 \equiv 0$ for $y \leq y^* - \bar{c}$ (as $L = 0$) and $\pi_1 \leq 0$ for $y \geq V$. Hence define $\Gamma(y^*) = (\max[0, y^* - \bar{c}], V) \subseteq [0, V]$ and note that Claim 1 implies this set is non-empty. Without loss of generality we can restrict attention to $y \in \Gamma(y^*)$ - all other offers yield negative profit. As π_1 is not concave in y over this domain, we need instead that π_1 is single peaked. This requires that at any $y \in \Gamma(y^*)$ with $\partial \pi_1 / \partial y = 0$, then $\partial^2 \pi_1 / \partial y^2 < 0$. Using the above definition of π_1, this requires

$$L \frac{\partial^2 L}{\partial y^2} - 2 \frac{\partial L}{\partial y}^2 < 0 \text{ for all } y^* > 0, \ y \in \Gamma(y^*). \tag{9}$$

Given the definition of L, (9) describes a restriction on F which guarantees existence of a symmetric, pure strategy Nash equilibrium: Claim 1 implies y^* always exists and given $y^2 = y^*$, then $y^1 = y^*$ is optimal for firm 1.

Unfortunately this condition does not reduce to anything simple. Although assuming F concave over its support guarantees non-paradoxical comparative statics; i.e. ensures that wage offers increase with productivity α, we have been unable to show it is sufficient to guarantee (9).

It is well known in the Hotelling framework with linear transport costs that a pure strategy equilibrium may not exist. The problem there is that demand is discontinuous - a small price cut can imply a jump in demand. Such demand discontinuities do not arise here - idiosyncratic match values implies demand $L(.)$ is continuous in y . The text implicitly assumes F satisfies (9) and Theorem 1 then describes the (unique) symmetric pure strategy equilibrium.