Firms Agglomeration and Unions

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Abstract

This paper develops a model in which the interaction between product market imperfections, transportation costs, unions and workers immobility across regions creates a tendency for agglomeration of firms when transportation costs are low. The model fits quite well the European experience. It is able to explain the emergence of a center-periphery pattern with equally populated regions. In the center, most people work in the unionized industry and earn large wages. In contrast, workers in the periphery are employed at a low wage in a constant return to scale industry.

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1 Introduction

In the last few years, there has been a revival of interest among economists in understanding the location of firms across countries. In particular the modeling of product market imperfections has allowed the construction of formal models that explain the agglomeration of firms in a single region (see e.g. Ottaviano and Puga (1998), Fujita, Krugman and Venables (1999) and Fujita and Thisse (forthcoming)). In a seminal paper, Krugman (1991a) shows how a home market effect creates agglomeration. Firms settle in regions where the demand for their product is large. Workers prefer to live in regions that have good access to the products that they consume. Since the demand that is addressed to firms comes from workers, firms locate where workers are. Moreover workers go where firms are located. Thus, if, for whatever reason, a region has a larger concentration of firms, it attracts workers from the other regions. The increase in demand that follows the migration of workers in that region attracts more firms. Krugman show that one can end up with all firms being agglomerated in one region.

This story is relevant when the labor force is highly mobile across regions, as for example in the United States (see also Puga (1999) and Ottaviano (2001) for the importance of labor mobility in similar models). However, in a European context, language, cultural and institutional differences hinder worker mobility. Decressin and Fatás (1995), Bentolila (1997) and Faini, Galli, Gennari and Rossi (1997) show that migrations across European countries or within each European country are quite low. As Krugman (1991b) recognizes, “in spite of considerable migration from South to North in the 1960s and early 1970s, there has been no wholesale concentration of population and employment in the areas of early industrialization. The reason is obvious: Europe has historically been far less integrated, both in terms of factor mobility and in terms of trade, than the United States” (Krugman, 1991b, p.93).

Still, disparities across the regions of the European Union are much wider than across the Untited States (see Puga (1999)). Krugman adds “... Europe is characterized by a very strong center-periphery pattern when one considers not population but purchasing power. Interregional income differentials within Europe are much larger than within the United States, and they are closely associated with geographical position." (Krugman, 1991b, p.93-94). The center-periphery pattern in Europe is difficult to explain on the basis of Krugman's (1991a) model alone. Rather than focusing on interregional migration, Krugman and Venables (1995) and Venables (1996) use an input-output structure to explain agglomeration of firms. When industries are imperfectly competitive and when it is costly to ship goods from one region to the other, upstream firms prefer to settle in regions where the demand for their product is large. Thus, they locate near their customers, that is, near the downstream firms. More upstream firms in one region imply a lower price for the intermediate good and attracts downstream firms. The mechanism is similar to that in Krugman's (1991a) model: if, for whatever reason, a region has a concentration of upstream firms, it attracts downstream firms from the other regions. The increase in demand that follows the inflow of downstream firms attracts more upstream firms in that region. As a result, firms may cluster in a single region. Venables (1996) demonstrates this result when workers are immobile whereas Puga (1999) extends the model to account for labor migration.

In this paper we propose an alternative explanation for the emergence of a center-periphery pattern in Europe. As noted before, European center-periphery pattern is related more to purchasing power than to population. We focus on two features of European countries: worker mobility is low and unions are influential. This last feature is at the heart of the model of this paper. Standard wage bargaining models predict that wages are rigid if the demand for labor is iso-elastic. Then, changes in productivity entirely fall on employment. This result is repeatedly emphasized in the literature, see e.g. McDonald and Solow (1981), Oswald (1985) and Blanchard and Fischer (1989). In this paper, we show that within the Dixit and Stiglitz (1977) framework, changes in the firms location also leave wages unaffected and strongly influence employment. Moreover, wages in the unionized industry are larger than in the non-unionized and constant return to scale (agricultural) sector. Workers who do not have the chance to find a job in the unionized sector are employed in the agricultural sector at a lower wage. Thus, the total earnings of workers in one region increase with the number of unionized firms that settle in that region. The preference of firms is to settle in a region where the demand for their product is large. Hence, they locate near their customers, that is, in the region with the highest proportion of unionized workers. Thus, if, for whatever reason, a region has a larger concentration of firms, it also has a larger income. The larger income attracts more firms, which increases the income further.

In this paper, we prove that according to this process, a center-periphery pattern may emerge. A large proportion of people living in the central region work in the unionized firms and earn high wages, whereas in the periphery, workers are employed in the low-wage agricultural sector. However, both regions remain equally populated. This accords quite well with the above observation by Krugman that Europe is characterized by a very strong center-periphery pattern when one considers not population but purchasing power.

The remainder of the paper is organized as follows. Section 2 presents the model with the behavior of consumers, workers, firms and unions. Section 3 solves for the location equilibria. Welfare is analyzed in Section 4. This is followed by the conclusions.

2 The Model

2.1 Consumption

We consider an economy with two identical regions $K \in \{A, B\}$. There are two kinds of goods in this economy: agriculture (denoted by subscript 0) and manufactures (denoted by subscript M). All individuals h share the same preferences for the two kinds of goods:

$$U_h = C_{0h}^{1-\mu} C_{Mh}^{\mu}$$

where

$$C_{Mh} = \left(\int_0^N (C_{ih})^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} .$$

The consumption of the agricultural good is denoted by C_{0h} whereas C_{Mh} is the index of consumption of manufactured goods. Given the Cobb-Douglas form of U_h, the share of revenue spent on agricultural goods is $1-\mu$ and the share of revenue spent on manufactured goods is μ. There is monopolistic competition among the N manufactures. The consumption of manufactured good i is C_{ih}. The elasticity of substitution among manufactured products is $\sigma > 1$.

In each region K, individual h faces the following budget constraint:

$$C_{0h} + \int_0^N p_i^K C_{ih} di \leq W_h^K,$$

where p_i^K is the price paid to consume one unit of good i in region K, W_h^K is the income of the individual h belonging to region K. We take the price of the agricultural good as the numeraire. The individual h maximizes his utility subject to his budget constraint. His optimal consumption of good i is

$$C_{ih}^K = \left(\frac{p_i^K}{P^K}\right)^{-\sigma} \left(\frac{\mu W_h^K}{P^K}\right) \text{ where } P^K = \left(\int_0^N \left(p_i^K\right)^{-(\sigma-1)} di\right)^{\frac{-1}{\sigma-1}} \tag{1}$$

P^K is the price index of the manufactured goods in region K. Consumption of good i by individual h decreases with the relative price p_i^K/P^K and increases with the real income spent on manufactured goods $\mu W_h^K/P^K$. Note that the prices p_i^K and the index P^K are computed in the region K where individual h consumes. Since preferences are homogenous, the utility increases with real income W_h^K/P_G^K where P_G^K is the general price index of both agriculture and manufactures consumed in region K.

2.2 Labor

It is a stylized fact of European labor markets that workers are rather immobile between European countries and within each country (see e.g. Decressin and Fatás (1995), Bentolila (1997) and Faini, Galli, Gennari and Rossi (1997)). To fit this European fact, we assume that workers cannot move across regions. This contrasts with Krugman (1991a) who mainly focuses on the long-run behavior of workers in the United States.

In our model, each region is populated by the same number of individuals, $\bar{L}$, who can work either in the agricultural or in the manufactured sector. In contrast to Krugman (1991a), but following Krugman and Venables (1995), we do not assume that workers are specific to one sector. Thus, we assume that workers are able to work in both sectors. If L_M^K persons are employed in the manufacturing industry, the remaining $\bar{L} - L_M^K$ workers must be employed in the agricultural sector.

The agricultural good is produced with constant returns to scale. One unit of agricultural good requires one unit of labor. Since the agricultural good is the numeraire, wages are equal to one in the agriculture. Note that we use the label 'agriculture' to follow the terminology used in most papers on this topic. However, the label need not be interpreted literally. For our purpose, the important characteristic of this sector is that returns to scale are constant. In region $K \in \{A, B\}$, the total earnings of farmers are made of their wages and their share in the total profits of both regions Π:

$$W_0^K = \left(\bar{L} - L_M^K\right)\left(1 + \frac{\Pi}{2\bar{L}}\right)$$

In this expression, it is obviously assumed that all individuals receive an equal share of the firms' profits. The total earnings of industrial workers in region $K \in \{A, B\}$ are

$$W_M^K = L_M^K\left(w^K + \frac{\Pi}{2\bar{L}}\right)$$

where w^K is the mean wage in the industry of region K. Total earnings in region K are given by $W^K = W_0^K + W_M^K$.

2.3 Firms

In the manufactured sector, each firm produces a differentiated good i and is located in one region only. The production of one unit of good i requires β workers. Suppose that firm i is located in region K. It chooses the mill price q_i^K that maximizes its profits

$$\pi_i^K = q_i^K z_i^K - w_i^K \beta z_i^K - f \tag{2}$$

where z_i^K denotes firm i's sales and where w_i^K is the wage in firm i. Superscript K indicates that firm i is located in region K. The firm incurs a fixed cost f. For simplicity this fixed cost is paid in terms of agricultural goods. Thus, fN goods are needed for the fixed costs in the manufacturing firms and the number of farmers should therefore be such that $2\overline{L} > fN$. We prove in the Appendix that this condition will always be fulfilled.

The agricultural good is traded at no cost whereas transportation costs are incurred for the manufactured goods. We assume that these costs take the Samuelson's iceberg form in which only a fraction $\tau < 1$ of each unit shipped from one region arrives to the other region. Therefore, if good i is consumed where it is produced, then the consumer price is equal to the mill price: $p_i^K = q_i^K$. If good i is consumed in the other region, the consumer price is larger than the mill price: $p_i^L = q_i^K/\tau > q_i^K$, with $L \neq K$. Hence, by (1), if firm i is located in region K, the demand for good i is

$$z_i^K = \left(\frac{q_i^K}{P^K}\right)^{-\sigma}\left(\frac{\mu W^K}{P^K}\right) + \frac{1}{\tau}\left(\frac{q_i^K}{\tau P^L}\right)^{-\sigma}\left(\frac{\mu W^L}{P^L}\right), \tag{3}$$

where $L \neq K$ and $K \in \{A, B\}$. Equivalently, one can write:

$$z_i^K = \left(\frac{q_i^K}{P^K}\right)^{-\sigma}\eta^K \text{ where } \eta^K = \mu\left[\left(\frac{W^K}{P^K}\right) + \tau^{\sigma-1}\left(\frac{P^L}{P^K}\right)^{\sigma}\left(\frac{W^L}{P^L}\right)\right] \tag{4}$$

We assume that there is a large number of manufacturing firms so that each firm considers the index P^K and the revenues W^K as constants. Given the iso-elastic product demand z_i^K, the firm sets the product mill price to

$$q_i^K = \frac{\sigma}{\sigma - 1}\beta w_i^K \tag{5}$$

By (4), the demand for labor of firm i producing in region K is thus iso-elastic:

$$l_i^K = \beta z_i^K = \beta\left(\frac{\sigma\beta w_i^K}{(\sigma - 1)P^K}\right)^{-\sigma}\eta^K \tag{6}$$

By (2), (4) and (5), profits can be written as

$$\pi_i^K = w_i^{1-\sigma}\frac{\beta}{\sigma - 1}\left(\frac{\sigma}{\sigma - 1}\frac{\beta}{P^K}\right)^{-\sigma}\eta^K - f \tag{7}$$

2.4 Wage Bargaining

In the manufactured sector, we assume a decentralized wage setting with one independent union per firm. First, the union and the firm bargain over wages. Then the firm chooses employment given wages (the firm has the *right to manage*). According to the Nash solution, wages maximize the following product

$$N \equiv \left[V_i^K - \overline{V}^K\right]^{\phi} \left[\pi_i^K - \bar{\pi}^K\right]^{1-\phi}$$

where ϕ is the union bargaining power, V_i^K is the union utility, π_i^K is the firm's profits, $\overline{V}^K$ and $\bar{\pi}^K$ are the fall-back utilities.

We assume that the objective of each union is to maximize the real wages of workers in the firm: $\left(w_i^K/P_G^K\right) l_i^K$. In case of persistent disagreement with the firm, workers receive the real wage of the agricultural sector: $\left(1/P_G^K\right)$. Hence, the union contribution to the Nash product is:

$$V_i^K - \overline{V}^K = \left(\frac{w_i^K}{P_G^K} - \frac{1}{P_G^K}\right) l_i^K.$$

The firm maximizes profits (7). In case of persistent disagreement with the union, the firm still incurs the fixed cost. Thus, the firm's contribution to the Nash product is:

$$\pi_i^K - \bar{\pi}^K = w_i^{1-\sigma} \frac{\beta}{\sigma - 1} \left(\frac{\sigma}{\sigma - 1} \frac{\beta}{P^K}\right)^{-\sigma} \eta^K.$$

There is large number of firms and unions. Therefore, firms and unions consider the indices P_G^K and P^K, the revenues W^K and thus the variables η^K as constants. Taking into account the relationship between employment and wages (6), the maximization of the Nash product with respect to the wage gives

$$w_i^K = w \equiv 1 + \frac{\phi}{\sigma - 1}, \; K \in \{A, B\} \tag{8}$$

This wage is a fixed mark-up over the agricultural wage and is identical for every firm in any region.

3 Location Equilibrium

In this section, we first analyze the general equilibrium when the total number of firms $N = N_A + N_B$ is fixed. Before proceeding to the characterization of the location equilibrium, it is convenient to take advantage of the symmetry of the problem by setting $\Delta N \equiv N_A - N_B$. We then study the equilibrium under free entry of firms. A location equilibrium is defined as follows:

Definition 1 *A* location equilibrium *is such that no locational deviation by a single firm is profitable.*

Hence, there must be no incentives for firms to relocate. If a region offers higher profits than the other, firms move to that location until the *profit differential* $\Delta\pi(\Delta N) = \pi_i^A - \pi_i^B$ between the regions falls to zero or until all firms are located in that region. A *location equilibrium* arises at an interior point $\Delta N \in (-N, N)$ when $\Delta\pi(\Delta N) = 0$, or at corners points $N_A = 0$ when $\Delta\pi(-N) \leq 0$, and $N_A = N$ when $\Delta\pi(N) \geq 0$. In the first case, we have two clusters whereas in the last two cases, we have a single cluster.

Definition 2 *A location equilibrium ΔN^* is* stable *if, in the neighborhood of ΔN^*, no locational deviation by a group of firms (non zero mass) is profitable.*

Continuity of the profit functions implies that corner solutions are always stable. For interior solutions ΔN^* where $\Delta\pi(\Delta N^*) = 0$, stability implies that $\pi_i^A - \pi_i^B$ decreases (resp. increases) if a group of firms moves from B to A (resp. A to B). That is, the slope of $\Delta\pi(\Delta N)$ must be negative in the neighborhood of the equilibrium.

In order to evaluate $\Delta\pi(\Delta N)$, we need to compute prices and output. Since wages are constant (see (8)), the mill prices (5) are identical across regions:

$$q_i^K = q \equiv \left(\frac{\sigma}{\sigma - 1}\right)\beta w, \;\; K \in \{A, B\}.$$

In region K, the consumption of one unit of good produced in that region costs $p_i^K = q$ whereas the consumption of one unit of good produced in the other region costs $p_i^L = q/\tau$. Therefore, by (1) the regional price indices of manufactured goods can be written as

$$P^K = q\left(N_K + N_L\tau^{\sigma-1}\right)^{\frac{-1}{\sigma-1}} \tag{9}$$

with $L \neq K$, $K, L \in \{A, B\}$. Using (9), the expression of η^K can be simplified to

$$\eta^K = \frac{\mu}{P^K}\left(W^K + W^L\tau^{\sigma-1}\frac{N_K + N_L\tau^{\sigma-1}}{N_L + N_K\tau^{\sigma-1}}\right) \tag{10}$$

One can easily check that the production of manufacturing firms located in the same region $K \in \{A, B\}$ are identical: $z_i^K = z^K \ \forall i \in K$.

Finally, the profits of any firm i located in a same region $K \in \{A, B\}$ are also identical. Indeed

$$\pi_i^K = \pi^K = (q - w\beta)\, z^K - f \tag{11}$$

The profit differential between regions A and B is therefore

$$\Delta\pi = \pi^A - \pi^B = (q - w\beta)\left(z^A - z^B\right)$$

Interior equilibria occur when the firm's production is identical across regions, $z^A = z^B$. In order to get expressions for these production levels, we first write the total earnings of individuals belonging to region $K \in \{A, B\}$ as

$$W^K = W_0^K + W_M^K = \left(\bar{L} - L_M^K\right) + L_M^K w + \frac{\Pi}{2}$$

where $\Pi = N_A\pi^A + N_B\pi^B$. In each region total profits are equal to the value of production minus labor and fixed costs:

$$N_K\pi^K = qz^K N_K - wL_M^K - fN_K \tag{12}$$

Using this expression, the fact that $L_M^K = \beta N_K z^K$ and the expression for W^K, we find

$$W^K = \left(\bar{L} - \frac{1}{2}fN\right) + \left(\beta\,(w-1) + \frac{q - w\beta}{2}\right)N_K z^K + \frac{q - \beta w}{2}N_L z^L \tag{13}$$

for all $K \neq L$ and $K, L \in \{A, B\}$. Using (4), (9) and (10), we compute that

$$z^K = \frac{\mu}{q}\left(\frac{W^K}{N_K + N_L\tau^{\sigma-1}} + \frac{W^L\tau^{\sigma-1}}{N_L + N_K\tau^{\sigma-1}}\right) \tag{14}$$

Solving expressions (13) and (14) allows to get z^K (see Appendix). Plugging this result in the profit differential and using the optimal values of w and q yields (see Appendix)

$$\Delta\pi = \Delta N\left(\tau^{\sigma-1} - \frac{\sigma^2 - \sigma\,(1 - \phi + \mu\phi) + \mu\phi}{\sigma^2 - \sigma\,(1 - \phi - \mu\phi) - \mu\phi}\right)\Psi(\Delta N) \tag{15}$$

where it is shown in the Appendix A that $\Psi(\Delta N) > 0$. This result allows to fully characterize the stable equilibria. Let

$$\widehat{\tau} \equiv \left(\frac{\sigma^2 - \sigma\left(1 - \phi + \mu\phi\right) + \mu\phi}{\sigma^2 - \sigma\left(1 - \phi - \mu\phi\right) - \mu\phi} \right)^{\frac{1}{\sigma - 1}} < 1.$$

Proposition 3 *Symmetric location* ($\Delta N = 0$) *is the unique stable locational equilibrium if* $\tau < \widehat{\tau}$. *Agglomeration in one region* ($\Delta N = \pm N$) *is a stable locational equilibrium if* $\tau > \widehat{\tau}$. *Any location* ($\Delta N \in [0, N]$) *is a stable equilibrium if* $\tau = \widehat{\tau}$.

Proof. Stable equilibria are given by the behavior of $\Delta\pi(\Delta N)$. Since $\Psi(\Delta N) > 0$, the behavior of $\Delta\pi(\Delta N)$ is similar to that of $(N_A - N_B)\left(\tau^{\sigma-1} - \widehat{\tau}^{\sigma-1}\right)$. First, there exists an interior equilibrium when $N_A = N_B$. This equilibrium is stable when the slope of this function is negative, that is, when $\tau < \widehat{\tau}$. Second, there exist corner equilibria when $\pm\Delta\pi\left(\Delta N\right) > 0$ at $\Delta N = \pm N$, that is, when $\tau > \widehat{\tau}$. Finally, when $\tau = \widehat{\tau}$, the function takes zero values for any value of ΔN. Hence, all locations are stable equilibria. ■

Globalized economies are characterized by diminishing transportation costs (larger τ). The location pattern results from the trade-off between two forces: product market competition and home market effect. For large transport costs (low τ), the effect of product market competition dominates. Exports are negligible and firms produce for their domestic market. If domestic markets have equal sizes, firms locate where competition is the lowest, that is, in the region with the smallest number of firms. Hence, the location equilibrium is symmetric. However, when the transportation cost drops, the home market effect plays a larger role. The relocation of firms to a region indeed increases the number of high paid workers ($w > 1$) and also decreases the price index in that region. Hence, workers' real earnings rise, which raises the product demand. Since profits go up with product demand, firms tend to cluster in a single region. When the home market effect is larger than the competition effect ($\tau > \widehat{\tau}$), agglomeration is the location equilibrium. Globalization thus leads to increased agglomeration in regions with immobile workers and wage rigidities.

The transportation cost threshold $\widehat{\tau}$ decreases in μ and ϕ whereas it increases in σ (see the Appendix). Hence, a larger share of the manufactures in the individual's spending (larger μ) implies a larger home market effect. Therefore, the symmetric location equilibrium is less likely (smaller $\widehat{\tau}$). In the same spirit, larger union bargaining power (ϕ) increases nominal wages and strengthens the home market effect. Finally, more elastic product demand (σ) decreases the profitability of firms and pushes wages down. The home market effect is weakened and symmetric location equilibrium becomes more likely. Globalization thus induces firms to agglomerate sooner in a single cluster when the manufacturing industry and the union bargaining power are large or when the demand elasticity of manufactures is low.

It is important to note that the threshold $\widehat{\tau}$ *does not depend* on fixed costs, f. Hence, when the number of firms (and varieties) is fixed, increasing returns to scale are not a necessary condition for agglomeration: the home market effect may dominate the effect of market competition even under constant returns to scale. This contrasts with the large strand of the literature, which is based on the assumption of increasing returns to scale (Krugman (1991a)). This assumption is associated to the hypothesis of free entry. Under free entry, profits must fall to zero, which requires strictly positive fixed costs. While free entry refers to the appealing concept of long run equilibrium, it also allows to avoid the issue of profit distribution. We now follow the literature by considering the relationship between fixed costs and free entry.

Let N^* denote the equilibrium number of firms set by free entry. At N^*, Proposition 3 applies and only two kinds of equilibria can be envisaged: symmetric location and agglomeration. Since the threshold value $\hat{\tau}$ that separates both equilibria does not depend on the total number of firms (N), it is also the threshold that separates the two equilibria under free entry. In the following, we determine the equilibrium values of the number of firms in both kinds of equilibria.

Under free entry, profits in both regions fall to zero when firms locate symmetrically. Using (11), we can write the output level under free entry as

$$\overline{z}^K = \overline{z} \equiv \frac{f}{q - w\beta} = \frac{f}{\beta}\frac{(\sigma-1)^2}{(\sigma-1+\phi)} \text{ for all } K \in \{A, B\}. \tag{16}$$

Moreover by (13-14), the equilibrium production at $N_K = N/2$ is also (see the Appendix)

$$z_K^* = z^* \equiv \frac{\mu\left(\sigma-1\right)^2\left(2\bar{L}-fN\right)}{\beta N\left(\sigma^2-\sigma\left(1+\mu\right)+\sigma\phi\left(1-\mu\right)+\mu\right)}$$

Entry occurs until $z_K^* = \overline{z}^K$, that is, until the number of firms is equal to

$$N^* = \frac{2\mu\left(\sigma-1+\phi\right)\bar{L}}{f\left(\sigma^2-\sigma+\phi\left(\sigma-\mu\sigma+\mu\right)\right)} \tag{17}$$

Let us now examine the solution with agglomeration. Suppose agglomeration occurs in region K: $N_K = N$ and $N_L = 0$. In the Appendix, we compute the production z_K^* of firms located in region K. In this model, the production in case of agglomeration takes the same value as the production in the symmetric case: $z_K^* = z^*$. Under free entry, the profits of firms in the region where firms agglomerate fall to zero whereas firms make losses in the other region. Profits are driven to zero if $z_K^* = \overline{z}^K$. Thus, under free entry, the number of firms is equal to N^*, as in the symmetric case.

Proposition 4 *Under free entry, the number of firms (and thus varieties) is independent of the transportation cost and the location pattern of the manufacturing industry. In addition, the number of firms increases with the union power.*

The expression (17) also confirms standard results on firms' entry. The number of firms increases with the share of manufactures in consumption and with the size of the labor force. It decreases with the fixed costs and with the elasticity of the demand for manufactures.

The positive correlation between union bargaining power and the number of firms is worth noticing. An increase in union power raises wages, which has two effects. For a given number of firms (N), it first reduces profits by increasing the costs. Second, since the increase in wages affects all workers, earnings go up in both regions and the demand for each variety rises, which fosters profits. This general equilibrium effect has a Keynesian flavor and is closely related to the home market effect according to which profits are larger where the demand is higher. In this model, the second effect dominates the first. Hence, the increase in union power raises profits. New firms are attracted and new varieties are created.

This positive effect of union power on variety must be balanced with its effect on production. Indeed, it is straightforward to check from (16) that the supply of each variety, $\overline{z}$, decreases with union power. From equations (16) and (17), one can also see that the total supply of all varieties, $\overline{z}N^*$ decreases with union power. By the same token an increase in union power reduces total employment in manufactures, $\beta\overline{z}N^*$.

4 Welfare: Agglomeration versus Dispersion

In the previous section, we have shown that firms choose either to agglomerate in a single region or to spread evenly across regions. Such changes in location equilibria have strong implications on the welfare of the individuals belonging to each region. It is natural to question the impact of the market equilibrium on welfare when the economic activity shifts from symmetry to agglomeration. To this aim, we first analyze the distribution of gainers and losers in this transition. We show that losers cannot be compensated by gainers. Indeed, if gainers compensated losers, agglomeration would increase the waste in transportation while it would not alter production possibilities. Second, we show that the location equilibrium corresponds to the location chosen by a planner who would minimize waste in transportation. Finally, we focus on a utilitarian planner. We show that, for some intermediate values of transportation costs, she may prefer agglomeration whereas the market selects the dispersion of economic activity.

In this section, we consider free entry of firms, which ensures that profits fall to zero. Each individual's utility is measured by his/her real earnings. The general price index of both agriculture and manufactures consumed in region K is defined as

$$P_G^K = \left(\frac{P^K}{\mu}\right)^{\mu}\left(\frac{1}{1-\mu}\right)^{(1-\mu)} \equiv \left(P^K\right)^{\mu}\frac{1}{\mu_0},$$

where, by (9),

$$P^K = P^L = P_s \equiv q \ \left(N^*\right)^{\frac{-1}{\sigma-1}}\left(\frac{1+\tau^{\sigma-1}}{2}\right)^{\frac{-1}{\sigma-1}} \tag{18}$$

under symmetric location and where

$$P^K = P_a \equiv q \ \left(N^*\right)^{\frac{-1}{\sigma-1}} \text{ and } P^L = \frac{P_a}{\tau} \tag{19}$$

when firms agglomerate in region K. It is easy to show that

$$\frac{P_a}{\tau} \geq P_s \geq P_a$$

The cost of living is lower in the region where firms agglomerate than under symmetry; it is higher in the deserted region.

4.1 Gainers, Losers and Compensation

We consider the transition from symmetry to agglomeration in region K. All individuals in the deserted region L lose. Indeed, since the nominal earnings of farmers are equal to 1 and since the cost of living rises in the deserted region, the welfare of farmers falls in that region. Also, workers who were employed in the manufacturing sector lose their jobs. Therefore, in addition to the rise in the cost of living, these individuals face a drop in their nominal wages. In contrast, all individuals gain in the region where firms agglomerate. Whereas they keep the same nominal wage, farmers and workers in the manufacturing sector benefit from a better cost of living. Moreover, some farmers are now hired in the high paid manufacturing sector and benefit from an increase in nominal earnings.

This analysis sheds light on the preference of a Rawlsian planner who seeks to improve the welfare of individuals in the worst situation. It is obvious that if this planner can only select the location pattern of firms, she prefers symmetric location. Moreover, if she can also transfer earnings among individuals and across regions, she will prefer symmetric location. Indeed, in this model, gainers cannot compensate losers.

The reason why compensation between gainers and losers is not possible is that there is no efficiency gain in production when the economy switches from symmetry to agglomeration. As shown in the previous section, the production of each variety and the number of varieties do not depend on firms' location. Therefore, agglomeration does not bring any production surplus. However, the amount of exports varies according to firms' location. Larger exports imply a larger waste in transportation and thus smaller aggregate consumption. In this model, the waste in transportation is smaller when firms locate symmetrically than when they agglomerate in a single region and when the gainers compensate the losers. To show this, let us note that, the exports of variety i produced in region K are given by the last term of expression (3). Exports from region K are

$$\frac{1}{\tau}\left(\frac{q}{\tau P^L}\right)^{-\sigma}\left(\frac{\mu W^L}{P^L}\right)N_K.$$

Under symmetric location, total exports from both regions K and L are

$$\frac{1}{\tau}\left(\frac{q}{\tau P_s}\right)^{-\sigma}\left(\frac{\mu W^s}{P_s}\right)N^*, \tag{20}$$

where W^s denotes the nominal earnings of each region under symmetric location. Under agglomeration in region K, total exports are

$$\frac{1}{\tau}\left(\frac{q}{\tau\frac{P_a}{\tau}}\right)^{-\sigma}\left(\frac{\mu W^L}{\frac{P_a}{\tau}}\right)N^*. \tag{21}$$

Loosers will be indifferent between symmetric location and agglomeration if they get the same real earnings. Compensation then requires

$$\frac{W^L}{\frac{P_a}{\tau}}=\frac{W^s}{P_s}.$$

Since $P_a/\tau \geq P_s$, one can check from (20) and (21) that total exports under agglomeration and compensation are larger than under symmetric location: more goods are wasted in transportation. Since the total production is not altered, the aggregate consumption is smaller under agglomeration and compensation. In particular, the consumption in the region where firms agglomerate must be smaller than under symmetric location. Hence, compensation to losers eliminates all the gains to individuals living in the region where firms agglomerate.

We summarize these results in the following proposition.

Proposition 5 *Farmers and workers prefer agglomeration in their own region to symmetric location. They prefer symmetric location to agglomeration in the other region. Individuals who gain from economic agglomeration cannot compensate those who lose.*

4.2 Waste in Transportation

Although agglomeration generates losers who cannot be compensated, it can improve welfare by increasing the amount of goods available for consumption. Suppose again that the planner can only select the location pattern of firms (no compensation) and that she minimizes the waste in transportation. That is, she selects the location pattern of firms that minimizes the total exports of manufactures. By (20) and (21), total exports under symmetry are larger than under agglomeration if and only if

$$\frac{W^s}{W^L} \geq \left(\frac{\tau P_s}{P_a}\right)^{1-\sigma}$$

Earnings in the deserted region are $W^L = \bar{L}$ whereas under symmetric location, earnings are equal to

$$W^s = \bar{L} + (qz^* - \beta z^* - f)\, N^*/2. \tag{22}$$

Using (18) and (19), the condition becomes

$$Y \geq \frac{1+\tau^{\sigma-1}}{2\tau^{\sigma-1}}$$

where

$$Y \equiv \frac{W^s}{W^L} = \frac{\bar{L} + (qz^* - \beta z^* - f)\, N^*/2}{\bar{L}} = \frac{\sigma\,(\sigma - 1 + \phi)}{\sigma\,(\sigma - 1 + \phi) - \mu\,(\sigma - 1)\,\phi} > 1.$$

Whereas Y represents the ratio of regional earnings under agglomeration, it can also be interpreted as the average value of manufactures and agriculture in the economy. Finally, one can check that $\left(1 + \widehat{\tau}^{\sigma-1}\right) / \left(2\widehat{\tau}^{\sigma-1}\right) = Y$ where $\widehat{\tau}$ is the transportation cost at which the market switches from symmetry to agglomeration. As a result, agglomeration minimizes the waste in transportation if and only if $\tau \geq \widehat{\tau}$. This gives the following proposition:

Proposition 6 *The location equilibrium minimizes the waste in transportation of manufactures.*

Therefore, no planner is needed to minimize the transportation of manufactures and agglomeration yields the largest amount of manufactured goods when firms agglomerate.

4.3 Utilitarian Planner

The larger amount of consumables under agglomeration obviously benefit the consumers in the region where agglomeration takes place at the expense of the consumers in the deserted region. It is natural to question how the welfare of the aggregate population varies with firms location. To this aim, we assume that the planner is utilitarian and measures welfare as $V = U_K + U_L$ where $U_K = \int_{h \in K} U_h dh$. Since preferences are linearly homogenous, welfare in region K is equal to $U_K = \int_{h \in K} \left(W_h^K / P_G^K \right) dh = W^K / P_G^K$ where P_G^K is the general price index in region K.

Under symmetric locations, the total welfare in both regions can be derived from the definition of P_G^K, (18) and (22):

$$\begin{aligned} V_s &\equiv U_K + U_L \\ &= \frac{\mu_0 \left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^\mu} \left(\frac{1+\tau^{\sigma-1}}{2} \right)^{\frac{\mu}{\sigma-1}} \left(2\bar{L} + qz^* N^* - \beta z^* N^* - f N^* \right) \\ &= \frac{\mu_0 \left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^\mu} \left(\frac{1+\tau^{\sigma-1}}{2} \right)^{\frac{\mu}{\sigma-1}} 2Y\bar{L}. \end{aligned}$$

In case of agglomeration in region K, we have $W^L = \bar{L}$ in the deserted region, whereas $W^K = \bar{L} + \left(qz^* - \beta z^* - f\right) N^*$ in the region where firms agglomerate. Hence, from (19), the total welfare in both regions is equal to

$$\begin{aligned} V_a &\equiv U_K + U_L = \frac{\mu_0 \left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^\mu} \left[\bar{L} \left(1 + \tau^\mu\right) + qz^* N^* - \beta z^* N^* - f N^* \right] \\ &= \frac{\mu_0 \left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^\mu} \left[2Y - \left(1 - \tau^\mu\right) \right] \bar{L}. \end{aligned}$$

We seek to know when agglomeration yields a welfare that is larger than under symmetric location of firms. For this purpose, we analyze the difference $V_a - V_s$:

$$V_a - V_s = \frac{\mu_0 \bar{L} \left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^\mu} \left\{ 2Y \left[1 - \left(\frac{1+\tau^{\sigma-1}}{2} \right)^{\frac{\mu}{\sigma-1}} \right] - \left(1 - \tau^\mu\right) \right\}.$$

It is straightforward to check that in the absence of transportation costs ($\tau = 1$), total welfare is identical under symmetric location and under agglomeration ($V_a - V_s = 0$). Moreover, one can check that the derivative of $V_a - V_s$ with respect to τ is negative at $\tau = 1$. Hence, for τ close to one, agglomeration is welfare improving. Furthermore, we prove in the Appendix that there is at most one value of the transportation cost, $\tau = \bar{\tau}$, for which $V_a = V_s$. Hence, for τ lower than the threshold $\bar{\tau}$, the planner prefers the symmetric location of firms, whereas she prefers agglomeration for $\tau \in [\bar{\tau}, 1]$. Moreover, we prove in the Appendix that $\bar{\tau}$ is lower than $\hat{\tau}$, the transportation cost that induces firms to agglomerate. Therefore, the location decisions of competitive firms are congruent with the preference of the planner in two situations: either when they both prefer agglomeration, i.e., for $\tau \in [\hat{\tau}, 1]$, or when they both prefer symmetric locations, i.e., for $\tau \in [0, \bar{\tau}]$. In contrast, firms decisions are at odds with the preference of the planner for $\tau \in (\bar{\tau}, \hat{\tau})$. Firms locate symmetrically whereas the planner prefers agglomeration. Finally, it is easy to check that the average value of manufactures and agriculture, Y, increases with union power. Thus, $V_a - V_s$ takes negative values for low ϕ and positive values for high ϕ. The symmetric equilibrium is less likely to be chosen by the planner when unions are powerful. We summarize these three points in the following proposition.

Proposition 7 *(i) A utilitarian planner prefers agglomeration to symmetric locations if and only if $\tau \in [\bar{\tau}, 1]$. (ii) The threshold $\bar{\tau}$ is unique and lower than $\hat{\tau}$. For $\tau \in (\bar{\tau}, \hat{\tau})$, the planner prefers agglomeration whereas the market selects the dispersion of economic activity. For all other values of τ, the market selects the outcome preferred by the planner. (iii) Symmetric equilibrium is less likely to be chosen by the planner when unions are powerful.*

Proof. See Appendix. ■

The present analysis is instructive about the possible conflicts between the market outcomes and the social preferences in the context of globalization. As transportation costs decrease in globalizing economies, the economy moves from a symmetric equilibrium to agglomeration in one region. In this section, we have shown that this move may occur too late from the viewpoint of a utilitarian planner. Market forces here foster too much dispersion of the economic activity. A utilitarian planner would improve welfare by inducing firms to agglomerate sooner.

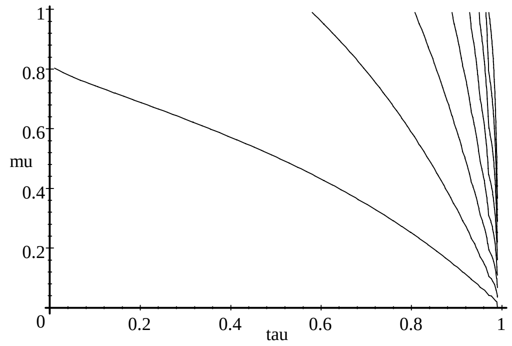

Figure 1: Agglomeration versus symmetric location: Welfare and competitive equilibrium (value of $\overline{\tau}(\mu, \theta)$ for $\phi = 0.5$ and $\theta = \{2, 3, ..., 9\}$).

It is unfortunately difficult to get more analytical results from the expression $V_a - V_s$. Still, one can see that its sign depends on four parameters: μ, σ, τ and ϕ. The proposition fully characterizes the effect of union power ϕ on welfare. To examine the impact of the other parameters, we fix, without loss of generality, the union power to the arbitrary value of $\phi = 0.5$ (firms and unions have the same bargaining power). Figure 1 depicts the values of $\tau = \bar{\tau}(\mu, \sigma)$ for which $V_a - V_s = 0$. Each curve corresponds to a specific value of σ. The lower curve corresponds to $\bar{\tau}(\mu, \sigma)$ at $\sigma = 2$ and the curves move upwards as σ increases from $\sigma = 2$ to $\sigma = 9$.

As predicted by the proposition, we observe in this figure that for each μ and σ, there exists at most one value of transportation cost $\bar{\tau}(\mu, \sigma)$ for which $V_a = V_s$. The figure also reveals that the zone in which symmetry is welfare improving increases with σ and decreases with μ. Thus, a large size of the manufacture industry (high μ) and a strong imperfection (low σ) induce the utilitarian planner to favor agglomeration. These last results seem general and have been confirmed in simulations with other values of σ, μ and ϕ.

5 Conclusions

The emergence of a center-periphery pattern has been the subject of recent research among economists. In particular, Krugman (1991a) shows that the interaction of labor migration across regions with product market imperfections and transportation costs induces firms and workers to cluster together as regions integrate. However, his model does not apply to regions in which mobility of workers is very limited. Thus, it does not apply to European countries in which language, cultural and institutional differences hinder worker mobility. However, another difference between European countries and the United States is the presence of influential unions.

It is well known that unions push wages up and also reduce the wage response to changes in the economic environment. Using the Dixit and Stiglitz (1977) model of imperfect competition, we show that wages in unionized firms consists of a markup over the alternative wage in the non unionized agricultural sector. The markup increases with the union bargaining power but is independent of firms location. Thus, if unions have the same bargaining power, wages are set at exactly the same levels across regions, whatever the number of firms. A relocation of firms from one region to another raises the number of highly paid workers in the latter and reduces it in the former. Income increases in the region where firms relocate, which raises the demand for goods. In contrast, income and demand fall in the deserted region. Since large demands attract firms, it is possible that they cluster in a single region. In that region, many workers are employed in the unionized high paid industry, whereas in the other region, people are working in the low paid agricultural sector. Thus, considering purchasing power rather than population, a center-periphery pattern emerges.

We prove that firms locate in symmetric locations when transportations costs are large. In contrast, they agglomerate in a single cluster for low transportation costs. We demonstrate that the tendency to agglomerate increases with the union bargaining power. We also show that larger union power reduces the total supply of manufactures, but raises product variety. Of course, adding labor mobility in the model would increase the likelihood of agglomeration since workers would move towards the region with the highest expected earnings, that is, towards the region where firms are clustered.

We finally analyze the welfare performance of the different equilibria. We first show that the gainers from one region can never compensate the losers located in a deserted region. Thus, agglomeration always creates losers even when compensation schemes are available instruments. Second, the location equilibrium minimizes the waste in transportation. Finally, the market selects the outcome preferred by a utilitarian planner when the transportation cost is either low or high. In contrast, for intermediate transportation costs, this planner prefers agglomeration whereas the market selects the dispersion of economic activity.

There obviously exists many interesting ways in which this model could be extended. We focus here on three directions for further research. First, research could put more geography into the model. By examining the location of firms among more than two regions it could be possible to study whether economic activity concentrates into a single region or into a set of regions. The price rigidities that are generated in union models, such as this one, show some remarkable properties that could be exploited in multi-region extension. Second, we have assumed that union power were equal in both regions. As a consequence, wages are equal in both regions and firms do not re-locate for cost differential reasons. Allowing for different union power across regions would make the model more complex but would add a new force by inducing firms to relocate towards regions with the lowest costs. Finally, we have focussed on one kind of labor market imperfections to conclude that larger union power induces firms to agglomerate in a single region. Other kinds of labor market imperfections also explain wage rigidities that are present in Europe. Thus it would be worthwhile to check whether these imperfections yield a similar result. However, the dynamics inherent in most of these models (e.g. search models, efficiency wage models, ...) are difficult to incorporate into the spatial analysis of the Krugman's (1991a) model.

6 Appendix

Derivation of the profit differential (15). We find (15) by solving the four linear equations (13-14) with four unknowns z^K and W^K, $K \in \{A, B\}$.

$$
\begin{aligned}
z^A &= \frac{\mu}{q}\left(\frac{W^A}{N_A+N_B\tau^{\sigma-1}}+\frac{W^B\tau^{\sigma-1}}{N_B+N_A\tau^{\sigma-1}}\right),\\
z^B &= \frac{\mu}{q}\left(\frac{W^B}{N_B+N_A\tau^{\sigma-1}}+\frac{W^A\tau^{\sigma-1}}{N_A+N_B\tau^{\sigma-1}}\right),\\
W^A &= \left(\bar{L}-\frac{1}{2}fN\right)+\frac{w\beta+q-2\beta}{2}N_A z^A+\frac{q-\beta w}{2}N_B z^B,\\
W^B &= \left(\bar{L}-\frac{1}{2}fN\right)+\frac{w\beta+q-2\beta}{2}N_B z^B+\frac{q-\beta w}{2}N_A z^A.
\end{aligned}
$$

This yields a unique solution with

$$
\begin{aligned}
z^A &= \frac{\mu\left(2\bar{L}-fN\right)}{2\Phi}\left(q\left(N_B\left(1+\tau^{2(\sigma-1)}\right)+2N_A\tau^{\sigma-1}\right)-\mu N_B\beta\left(1-\tau^{2(\sigma-1)}\right)(w-1)\right) \quad (23)\\
z^B &= \frac{\mu\left(2\bar{L}-fN\right)}{2\Phi}\left(q\left(N_A\left(1+\tau^{2(\sigma-1)}\right)+2N_B\tau^{\sigma-1}\right)-\mu N_A\beta\left(1-\tau^{2(\sigma-1)}\right)(w-1)\right) \quad (24)\\
W^A &= \frac{1}{2\Phi}q\left(2\bar{L}-fN\right)\left(N_A+N_B\tau^{\sigma-1}\right)\left(N_B\left(q-\mu\beta(w-1)\right)+N_A\tau^{\sigma-1}\left(q+\mu\beta(w-1)\right)\right),\\
W^B &= \frac{1}{2\Phi}q\left(2\bar{L}-fN\right)\left(N_B+N_A\tau^{\sigma-1}\right)\left(N_A\left(q-\mu\beta(w-1)\right)+N_B\tau^{\sigma-1}\left(q+\mu\beta(w-1)\right)\right),
\end{aligned}
$$

where

$$
\Phi=\left(q\left(1-\mu\right)+\mu\beta\right)\left(N_AN_B\left(q\left(1+\tau^{2\sigma-2}\right)-\mu\beta\left(1-\tau^{2\sigma-2}\right)(w-1)\right)+q\tau^{\sigma-1}\left(N_A^2+N_B^2\right)\right) \quad (25)
$$

Note that $\Phi > 0$ when q maximizes profits and w maximizes the Nash product.

We can now compute

$$
\begin{aligned}
\Delta\pi &= \pi^A-\pi^B=(q-w\beta)\left(z^A-z^B\right)\\
&= \left(N_A-N_B\right)\left(\tau^{\sigma-1}-\frac{q-\mu w\beta+\mu\beta}{q+\mu w\beta-\mu\beta}\right)\Psi\\
&= \left(N_A-N_B\right)\left(\tau^{\sigma-1}-\frac{\sigma^2-\sigma\left(1-\phi+\mu\phi\right)+\mu\phi}{\sigma^2-\sigma\left(1-\phi-\mu\phi\right)-\mu\phi}\right)\Psi
\end{aligned}
$$

where

$$
\Psi=\frac{1}{2\Phi}\mu\left(2L-fN\right)\left(1-\tau^{\sigma-1}\right)\left(q+\mu\beta\left(w-1\right)\right)>0.
$$

In the symmetric equilibrium, $N_A = N/2 = N_B$. Using (23) and (25) one finds

$$z^* = z_A^* = z_B^* = \frac{\mu (\sigma - 1)^2 \left(2\bar{L} - fN\right)}{\beta N \left(\sigma^2 - \sigma (1 + \mu) + \sigma\phi (1 - \mu) + \mu\right)}.$$

In the equilibrium with agglomeration, $N_A = N$ whereas $N_B = 0$. Using (23) and (25) one also finds

$$z_A^* = \frac{\mu (\sigma - 1)^2 \left(2\bar{L} - fN\right)}{\beta N \left(\sigma^2 - \sigma (1 + \mu) + \sigma\phi (1 - \mu) + \mu\right)}.$$

Derivative of $\widehat{\tau}$ with respect to σ. We here compute the derivative of $\widehat{\tau}$ with respect to σ. We have

$$\widehat{\tau} = \left(\frac{\sigma^2 - \sigma (1 - \phi + \mu\phi) + \mu\phi}{\sigma^2 - \sigma (1 - \phi - \mu\phi) - \mu\phi}\right)^{\frac{1}{\sigma-1}} \equiv \left(\frac{N}{D}\right)^{\frac{1}{\sigma-1}} \equiv x^{\frac{1}{\sigma-1}}$$

with $x \leq 1$. Let us analyze the derivative of $\log \widehat{\tau}$ with respect to σ.

$$\frac{d \log \widehat{\tau}}{d\sigma} = \frac{1}{(\sigma - 1)^2} \left[- \log x + \frac{x_\sigma}{x} (\sigma - 1)\right].$$

By properties of logs we have $-\log(x) \geq 1 - x$ for $x \leq 1$. Hence, a sufficient condition to get a positive derivative of $\widehat{\tau}$ with respect to σ is

$$1 - x + \frac{x_\sigma}{x} (\sigma - 1) > 0.$$

We can compute

$$\frac{x_\sigma}{x} = \frac{2\mu\phi}{DN} \left[(\sigma - 1)^2 - \phi\right].$$

Hence, the sufficient condition writes

$$\begin{aligned} 1 - \frac{N}{D} + \frac{2\mu\phi}{DN} (\sigma - 1) \left[(\sigma - 1)^2 - \phi\right] &> 0, \\ \text{or, } N (D - N) + 2\mu\phi (\sigma - 1) \left[(\sigma - 1)^2 - \phi\right] &> 0. \end{aligned}$$

Using the definitions of N and D we have $D - N = 2\mu\phi (\sigma - 1)$. The sufficient condition becomes

$$2\mu\phi (\sigma - 1) \left[N + (\sigma - 1)^2 - \phi\right] > 0.$$

One can check that $N > \phi$ which proves the result.

Proof that $2\overline{L} > fN$. The fixed costs in the manufacturing firms require fN agricultural goods and the number of farmers should be such that $2\overline{L} > fN$. Using (17), one can check that, in equilibrium, $2\overline{L} - fN^*$ takes the sign of $(\sigma - 1)(\sigma - \mu) + \phi\sigma(1 - \mu)$ which is positive.

Proof of Proposition 7. We need to show (a) that $V_a > V_s$ if and only if $\tau > \bar{\tau}$, where $\bar{\tau}$ is unique, and (b) that $\bar{\tau} < \hat{\tau}$.

(a) For the first item we prove that $V_a > V_s$ if $\mu > \sigma - 1$ and, if $\mu < \sigma - 1$ and $\tau > \bar{\tau}$, where $\bar{\tau}$ is unique.

It is straightforward to check that $(V_a - V_s)$ takes the same sign as

$$2Y\left\{1 - \left(\frac{1+\tau^{\sigma-1}}{2}\right)^{\frac{\mu}{\sigma-1}}\right\} - (1 - \tau^{\mu}).$$

The derivative of this expression with respect to τ is equal to

$$\mu\tau^{\mu-1}\left\{1 - Y\left(\frac{1+\tau^{\sigma-1}}{2\tau^{\sigma-1}}\right)^{\frac{\mu}{\sigma-1}-1}\right\}.$$

The term $\tau^{\mu-1}$ is decreasing in τ. Moreover, the term $\left(1 + \tau^{\sigma-1}\right)/2\tau^{\sigma-1}$ always takes values above 1 and is decreasing in τ. On the one hand, since $Y > 1$, the square bracket term is always negative when $\mu/(\sigma - 1) > 1$. The difference between V_a and V_s decreases when $\mu > \sigma - 1$. On the other hand, the curly bracket term, and thus the derivative of $V_a - V_s$, are decreasing functions of τ when $\mu/(\sigma - 1) < 1$. The difference $V_a - V_s$ is a concave function of τ when $\mu < \sigma - 1$. We know that it has zero value with negative derivative at $\tau = 1$. It can therefore have a second zero (with positive derivative) at $\tau \equiv \overline{\tau} < 1$. For any $\tau > \overline{\tau}$, $V_a - V_s > 0$.

To sum up, if $\mu > \sigma - 1$, $V_a > V_s$ $\forall\tau \in [0, 1]$. Otherwise, there exists a unique $\overline{\tau}$ such that $V_a > V_s \Leftrightarrow \tau > \overline{\tau}$.

(b) $\bar{\tau} < \hat{\tau}$.

We first prove that at $\tau = \hat{\tau}$, $(V_a - V_s) \geq 0$. Once this result established, we use the definition of $\bar{\tau}$, which states that $(V_a - V_s)$ is positive for all $\tau \in [\bar{\tau}, 1]$. Since $(V_a - V_s) \geq 0$ at $\tau = \hat{\tau}$, it follows that $\bar{\tau} < \hat{\tau}$.

Let us now prove that $(V_a - V_s) \geq 0$ at $\tau = \hat{\tau}$. We have

$$(V_a - V_s) = \frac{\mu_0 \bar{L}\left(N^*\right)^{\frac{\mu}{\sigma-1}}}{q^{\mu}}\left\{2Y\left[1 - \left(\frac{1+\tau^{\sigma-1}}{2}\right)^{\frac{\mu}{\sigma-1}}\right] - (1 - \tau^{\mu})\right\}.$$

One can also check that $2Y = \left(1+\widehat{\tau}^{\sigma-1}\right)/\widehat{\tau}^{\sigma-1}$. Hence, at $\tau = \hat{\tau}$, we can write, after some algebraic manipulations,

$$(V_a - V_s) = \frac{\mu_0 \bar{L}\,(N^*)^{\frac{\mu}{\sigma-1}}}{q^{\mu}\widehat{\tau}^{\sigma-1}} \left\{ \widehat{\tau}^{\mu+\sigma-1} + 1 - \left(\frac{1}{2}\right)^{\frac{\mu}{\sigma-1}} \left(1+\widehat{\tau}^{\sigma-1}\right)^{\frac{\mu}{\sigma-1}+1} \right\}.$$

The sign of $(V_a - V_s)$ is given by the sign of the expression in curly brackets. This expression decreases in $\hat{\tau}$ if and only if

$$(\mu+\sigma-1)\,\hat{\tau}^{\sigma-2}\left(\hat{\tau}^{\mu} - \left(\frac{1+\hat{\tau}^{\sigma-1}}{2}\right)^{\frac{\mu}{\sigma-1}}\right) \leq 0,$$

that is, if and only if

$$\left(\frac{1+\hat{\tau}^{\sigma-1}}{2\widehat{\tau}^{\sigma-1}}\right)^{\frac{1}{\sigma-1}} \geq 1,$$

which holds for all $\hat{\tau} \leq 1$. Therefore, at $\tau = \hat{\tau}$, $(V_a - V_s)$ takes its lowest value if $\hat{\tau} = 1$. At $\hat{\tau} = 1$, it is straightforward to check that $(V_a - V_s) = 0$. Hence, $(V_a - V_s) \geq 0$ at $\tau = \hat{\tau}$.

7 References

S. Bentolila (1997), Sticky Labor in Spanish Regions, *European Economic Review*, **41**, 591-598.

O. Blanchard and S. Fischer (1989), "Lectures on Macroeconomics", MIT Press, Cambridge, MA.

J. Decressin and A. Fatás (1995), Regional Labor Market Dynamics in Europe, *European Economic Review*, **39**, 1627-1655.

A.K. Dixit and J. Stiglitz (1977), Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, **67**, 297-308.

R. Faini, G. Galli, P. Gennari and F. Rossi (1997), An Empirical Puzzle: Falling Migration and Growing Unemployment Differentials among Italian Regions, *European Economic Review*, **41**, 571-579.

M. Fujita, P. Krugman and A. J. Venables (1999), "The Spatial Economy: Cities, Regions and International Trade", MIT Press, Cambridge, MA.

M. Fujita and J.F. Thisse (forthcoming), "Economics of Agglomeration", Cambridge University Press, Cambridge.

P.R. Krugman (1991a), Increasing Returns and Economic Geography, *Journal of Political Economy*, **99**, 483-499.

P.R. Krugman (1991b), "Geography and Trade", MIT Press, Cambridge, MA.

P.R. Krugman and A. J. Venables (1995), Globalization and The Inequality of Nations, *Quarterly Journal of Economics*, **110**, 857-880.

I.M. McDonald and R. Solow (1981), Wage Bargaining and Employment, *American Economic Review,* **71**, 896-908.

G.I.P. Ottaviano (2001), Monopolistic Competition, Trade, and Endogenous Spatial Fluctuations, *Regional Science and Urban Economics*, **31**, 51-77.

G.I.P. Ottaviano and D. Puga (1998), Agglomeration in the Global Economy: A Survey of the 'New Economic Geography', *World Economy*, **21**, 707-731.

A.J. Oswald (1985), The Economic Theory of Trade Unions: An Introductory Survey, *Scandinavian Journal of Economics*, **87**, 160-193.

D. Puga (1999), The Rise and Fall of Regional Inequalities, *European Economic Review*, **43**, 303-334.

A.J. Venables (1996), "Equilibrium Locations of Vertically Linked Industries", *International Economic Review*, **37**, 341-359.