Education, employment and participation to the labour market in a matching model of unemployment

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Abstract The ageing of workers is a key factor to explain their different performance on the labour market. Employment rates by age generally exhibit a bell-shaped path, whereas unemployment rates by age are generally continuous decreasing. Over the past decades, unemployment rates have increased for all age categories, especially the youngest, whereas employment and participation rates have particularly decreased for two different age categories : the youngest and the oldest. We build a matching model of unemployment aimed at understanding these features.

Keywords Job Matching; Human capital; Retraining; Age discrimination

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1. Introduction

The ageing of workers is a key factor to explain their performance on the labour market. For instance, it is usually argued that unemployment is primarily harmful to the youth: as the following table shows, there is a large gap between the youth unemployment rate and the aggregate unemployment rate, which has considerably widened over the past decades.

Changes in unemployment, employment and activity rates by age in France (male pop.)

	1979				1998			
Age Category	15/24	25/54	55/64	Total	15/24	25/54	55/64	Total
Unemployment rate (%)	9.3	3.2	4.1	6.0	21.9	9.3	8.3	12.6
Employment rate (%)	47.6	93.3	67	79.2	24.2	85.8	37.9	66.8
Activity rate (%)	52.2	96.3	69.9	85.2	30.9	94.5	41.3	74.4

Source : OECD, (1995), (2001)

However, one should not conclude from this first evidence that this necessarily reflects a privileged situation for the eldest: a closer look at employment and activity rates by age reveals that these two variables are bell-shaped. Behind the low unemployment rate of the 55/64 years old hides the weakness of their employment and activity rates.

At the aggregate level, we also observe a collapse in both employment and participation to the labour market. This pattern seems to have more deeply affected two age categories: the youngest and the eldest[1].

The purpose of this paper is to account for the changes we have depicted so far. To this aim, we build a matching model of unemployment à la Pissarides (1990) where we have incorporated two main features: workers undertake endogeneous educational investment and firms discriminate against workers according to their age as hiring older workers is assumed to be more costly. In this non stationary environment, we show that a rise in job instability, captured here by an exogenous rise in the job destruction rate, may cause a simultaneously: (i) a rise in unemployment, (ii) a rise in the incentives to schooling,

[1] Givord (2001) provides straightforward evidence for France. See also OECD (1995, 2001) for additional evidence on some other European countries, such as Sweden, Spain, or Belgium.

(iii) a collapse in activity rates. Consistently with the evidence, we show that when labour market conditions worsen, it is essentially at the expense of two age categories: the youngest and the eldest.

The explanation we propose is the following: the downward sloping profile of the unemployment rate by age naturally stems from the existence of matching frictions: everybody starts his working life by being unemployed, and finding a job takes time, so that workers only gradually exit from unemployment. The bell-shaped profile of the employment rate by age suggests the existence of two different steps in workers' working life: during the first step workers leave the educational system and start to prospect on the labour market, where they progressively move into employment as matching takes time. The employment rate reaches then a stationary level, and would not decline without the occurence of a second step. However, workers do not stay infinitely on the labour market: above a certain age threshold, they start to retire from the labour force. Here, retirement from the labour force is due to workers' discouragement, who rationally drop the search above a certain age: as their recruitment would involve an outstanding amount of hiring/retraining costs compared to the expected profit it would generate, no firm is willing to hire them. Therefore, their skills are no more marketable and they retire from the labour force. The changes we have depicted previously are then accounted for by a permanent rise in job destruction: the fall in job tenures reduces the expected value of a filled job, which reduces the labour demand and raises the age threshold set by firms. Finding a job now takes longer, but in addition, a larger number of workers retire from the labour force: workers loose their jobs more frequently, and old workers exit earlier from the labour market as they are more heavily discriminated. In turn, there are higher incentives to schooling: the fall in employment opportunities reduces the opportunity cost of education, and one of the benefits of education is to increase one's employability.

Our model is based on three main assumptions: hiring/retraining a worker is costly - the older the worker, the higher the cost-, besides, jobs can be destroyed for exogenous reasons, and finally, education is a time-consuming activity. We now justify these modeling

choices in more details.

The introduction of hiring/retraining costs can be justified by two main empirical statements: first, the older the workers, the longer their unemployment spells (see e.g. Machin and manning, 1999, pp. 3093-3094, or also Chan and Stevens, 2001). Second, workers' activity rates fall sharply above a certain age. This suggests that retirement from the labour force can be partly due the collapse in employment perspectives above a certain age. This phenomenon, of course, can also be explained by some institutional features, such as the existence of a legal (exogenous) constraint on the retirement age. However, there is a trend towards earlier retirement, which cannot be easily accounted for by institutional constraints such as balanced budget constraints. Our view is that retirement from the labour force is endogeneous; it occurs as old workers would be unemployable otherwise. This comes close to Lazear's (1979, 1986) influential contribution on the analysis of retirement, or also to Sala-i-Martin's one (1996). In these papers, workers retire from the labour force as hiring them above a certain age threshold would become non profitable. In Lazear, this is a consequence of the existence of implicit contracts between firms and workers, while in Sala-i-Martin, it is assumed that productivity goes down above a certain age. Here, we assume hiring/retraining retraining an applicant is costly, and in addition the older he/she is, the larger the cost[2]. This directly implies that above a certain age threshold, matches become non profitable. Firms therefore refuse the applications of those above the age threshold. As employment perspectives for those 'old' unemployed are nil, they rationally choose to retire from the labour force.

Our second assumption is that jobs can terminate for some (exogeneous) reasons. Though it introduces a bit of analytical complexity, there are two main reason to con-

[2] There are many reasons why hiring/retraining an old worker is more costly: the older the worker, the lower her ability to learn, but also the older the workers, the less able she is to implement the existing technology. In addition Hutchens (1986), and Scott, Berger and Garen (1995): show on US data that the probability of being employed is significantly reduced for old workers in companies providing for instance health insurance schemes to their employees. They also show that the more costly these schemes, the lower the probability of employing old workers.

sider this variable: first, it gives a 'touch of realism' to the demographics of the model. Second, it is quite convenient for the need of our comparative statics exercise. Even in environments characterized by massive unemployment such as European labour markets, the average duration of an unemployment spell is generally below a year. This implies that most of the individuals entering the labour market will presumably end up with a job at least after a few years. Therefore, in order to get some broadly realistic values for the unemployment rates by age, we need to introduce job destruction. Job destruction also implies that those who become unemployable are not only job seekers who have failed to find a job during their lifetime[3], but also old, dismissed workers. What's more, a rise in job destruction can simultaneously account for all the changes we have depicted so far. Evidence from the occurence of such shocks come from time series on unemployment and long-term unemployment rates. Considering time series for France (see e.g. Machin and Manning, 1999, pp3103-3107), it seems that two such shocks have probably occured during the 70's: one between 1973-74 and another one between 1979-80. During these periods, unemployment has gone up while long-term unemployment (i.e. the share of unemployment whose unemployment spell is shorter than 6 months) has gone down. In the longer run, however, both unemployment and long-term unemployment have increased. This clearly seems to indicate a rise in job destruction.

Finally, a fall in job stability may also well raise the incentives to schooling. Education is beneficial to workers in at least two ways: it increases workers' earnings as usual in the human capital theory (Becker, 1964), but it also improves workers' employability: the more educated the workers, the more profitable they are to the firms. For this reason, higher educated the workers can be profitably retrained for longer and tend to retire later from the labour force[4]. This seems to be empirically consistent with the observation that

[3] See for instance Coles and Masters (2000), Decreuse and Granier (2000) or also Laing, Palivos and Wang (2001).

[4] Alternatively, one may have assumed that higher educated workers can be more easily trained, as in Thurow (1975). This introduces slight differences in the economic meaning, but does not change our main results.

activity rates increase in workers' education attainment.

Activity rates and education attainment : France 1999

Education attainment	Lower secondary	Secondary	Higher education
Activity rates	66.6 %	82.8 %	87.2 %

Source: OECD (2001), p.239

Besides education takes time. In this way, the marginal schooling cost depends on labour market variables such as tightness and job destruction rate, since workers forego earnings during the schooling period. In this case, a rise in unemployment due to a fall in tightness and/or a rise in job destruction reduces the shadow cost of education. Simultaneously, a rise in unemployment may also increase the return to education: when workers loose their job more frequently, or when finding a job takes longer, there are higher incentives to spend time in the educational system to improve one's employability. For this reason, a rise in job destruction may account for the rise in unemployment and in education attainment, and a fall in participation to the labour market. This latter fall is particularly important for two age categories: the eldest workers who are more strictly discriminated against by firms, but also the youngest who have higher incentives to increase their schooling duration.

This paper relates workers' incentives to schooling to their employability. In this perspective, the closest contribution to ours are Decreuse and Granier (2000), and Laing, Palivos and Wang (2001) who consider the nexus between schooling and the labour market in a vintage human capital framework with matching frictions. There are two main differences with these previous studies : we allow for job turnover, and we consider human capital investment as a duration spent in the educational system.

The outline of this paper is as follows: section 2 highlights the demographics of the model, given the education attainment, the duration of employability and the hazard rates into and out of unemployment. Section 3 proposes a theoretical framework to endogenize these variables. Section 4 describes firms' hiring behaviour and workers' education attainment while in section 5, we show that a structural shock on the job destruction rate can account for a rise in unemployment and in education attainment, a fall in the

participation to the labour market, this latter being more pronounced for the eldest and the youngest. Section 6 concludes.

2. Demographics

The demographic structure of the model hinges on a perpetual youth assumption as in Blanchard (1985): workers face a constant, age-independent risk of dying δ. At any time, $\delta > 0$ agents are born, and directly enter the educational system, so that the population is constant and normalized to unity. For the sake of simplicity, let us start by assuming that the schooling duration T is fixed and common to the whole population. Let us further assume that there exist a threshold age A above which workers become unemployable and therefore retire from the labour force. These two variables will be endogenized in the next section. Let us also denote by q and μ respectively the hazard rates into and out of unemployment.

2.1. Flows and stocks

Across the working age population (i.e. workers aged $a < A$), there are S students, U unemployed and L employed individuals. Let us denote by $U(a)$ and $L(a)$ the respective mass numbers of unemployed and employed workers with age less than a. Workers discourage and drop the search once they have reached the threshold age A. However, some of them remain employed above A, and will retire from the labour force in case of job loss. Therefore, there are two different possible states for workers with age a above the threshold A : there are I inactive workers, namely retirees, and L_I employed workers.

[Figure 1: demographic structure of the model]

We now determine these various populations : our perpetual youth assumption implies that the size of a cohort aged z is worth $\delta e^{-\delta z}$. The number of individuals involved in the education system S is equal to the sum of the various cohorts aged between 0 and T.

Thus:

$$S = \int_0^T \delta e^{-\delta z} dz = 1 - e^{-\delta T} \tag{2.1}$$

Besides, the flow equilibrium of the model implies:

$$\delta e^{-\delta T} + qL(a) = (\delta + \mu) U(a) + dU(a)/da \tag{2.2}$$

$$\mu U(a) = (\delta + q) L(a) + dL(a)/da \tag{2.3}$$

The inflow into the population of unemployed aged less than a corresponds to the mass $\delta e^{-\delta T}$ of individuals who enter the labour market immediately after having completed their schooling period, plus an additional mass of $qL(a)$ previously employed who have separated from their employer. The outflow is composed of $(\delta + \mu) U(a)$ unemployed who were either successful in their search or deceased, plus $dU(a)/da$ unemployed who now become older than a. The inflow into the category of employed workers aged less than a is composed of $\mu U(a)$ successful job-seekers, while the outflow is made of $(\delta + q) L(a)$ previously employed workers who either separate from their employer or deceased, plus $dL(a)/da$ employees who are now older than a. In the appendix, we also show that the masses of unemployed $U(a)$ and employed $L(a)$ can be written:

$$U(a) = e^{-\delta T} \left[\frac{\delta + q}{\mu + q + \delta} - \frac{q e^{-\delta(a-T)}}{\mu + q} - \frac{\delta \mu e^{-(\delta+\mu+q)(a-T)}}{(\mu + q)(\mu + q + \delta)} \right] \tag{2.4}$$

$$L(a) = e^{-\delta T} \left[\frac{\mu}{\mu + q + \delta} - \frac{\mu e^{-\delta(a-T)}}{\mu + q} + \frac{\delta \mu e^{-(\delta+\mu+q)(a-T)}}{(\mu + q)(\mu + q + \delta)} \right] \tag{2.5}$$

Differentiating these expressions with respect to age, we get the following masses of unemployed $dU(a)/da$ and employed $dL(a)/da$ exactly aged a:

$$dU(a)/da = e^{-\delta T} \frac{\delta}{\mu + q} \left[q e^{-\delta(a-T)} + \mu e^{-(\delta+\mu+q)(a-T)} \right] \tag{2.6}$$

$$dL(a)/da = e^{-\delta T} \frac{\delta \mu}{\mu + q} \left[e^{-\delta(a-T)} - e^{-(\delta+\mu+q)(a-T)} \right] \tag{2.7}$$

The expressions of the mass L_I of employed "old" workers and of the mass of retirees I are shown in the appendix. We now concentrate on the unemployment, employment and activity rates by age.

2.2. Unemployment, employment and participation rates by age

Let us denote respectively by $u(a)$, $l(a)$ and $p(a)$ the unemployment, employment and activity rates by age in the population.

For any $a \in [T, A]$, we define the unemployment rate by age $u(a)$ as the mass of unemployed aged a, $dU(a)/da$, to the size of a cohort of the same age $\delta e^{-\delta a}$. Thus $u(a) = \frac{dU(a)/da}{\delta e^{-\delta a}}$. Owing to equation (2.4), we get:

$$u(a) = \begin{cases} \frac{q+\mu e^{-(\mu+q)(a-T)}}{q+\mu} \text{ for } T \leq a \leq A \\ 0 \text{ otherwise} \end{cases} \tag{2.8}$$

Similarly, the employment rate by age is worth $l(a) = \frac{dL(a)/da}{\delta e^{-\delta a}}$ for any $a \in [T, A]$. Above A, there is also a mass of workers who are still employed, though they will retire in case of separation. Since workers drop the search above A, those who are employed above A were necessarily employed at age A. These workers gradually exit from employment as they separate from their employer at rate q or die at rate δ, i.e. the probability that an individual employed at age A is still employed at age $a \geq A$ is equal to $e^{-(\delta+q)(a-A)}$. Therefore, the employment rate above A is worth:

$$l(a) = \underbrace{\delta e^{-\delta A} l(A)}_{\text{mass of employed individuals at age } A} \times \underbrace{e^{-(\delta+q)(a-A)}}_{\text{probability of being employed at } a \geq A} / \underbrace{\delta e^{-\delta a}}_{\text{size of a cohort aged } a} \quad \text{if } a > A \tag{2.9}$$

Taking account of (2.5), this yields:

$$l(a) = \begin{cases} \frac{\mu\left(1-e^{-(\mu+q)(a-T)}\right)}{q+\mu} \text{ for } T \leq a \leq A \\ l(A)\, e^{-q(a-A)} \text{ for } a > A \\ 0 \text{ otherwise} \end{cases} \tag{2.10}$$

Unemployment and employment rates as a function of workers' age are displayed in the following panel of figures :

[figure 2a et 2b: unemployment and employment rates by age]

Finding a job takes time because of the existence of matching frictions. This is reflected by the shape of the unemployed rate, which is a continuously decreasing function of age. The unemployment rate by age is worth 1 for workers who have just entered the labour market at age T, and it is strictly decreasing until $\frac{q+\mu e^{-(q+\mu)A}}{q+\mu}$ as job seekers are matched to employers. Conversely, the employment rate is continuously increasing until A. Above A, it is then decreasing as workers die or separate from their employers. Consistently with the empirical evidence, the employment rate by age exhibits a bell-shaped profile,.with a strong dissymmetry to the left. The bell-shaped profile crucially stems from the existence of a threshold age above which workers drop the search. The dissymmetric profile of this curve is due to the fact that workers tend to exit relatively fast (at rate $\delta + q$) from unemployment above A, whereas the flow into employment is exhausted above this age.

Finally, the participation rate $p(a)$ is worth $p(a) = u(a) + l(a)$, which writes down:

$$p(a) = \begin{cases} 1 \text{ for } T \leq a < A \\ l(a) \text{ for } a \geq A \\ 0 \text{ otherwise} \end{cases} \tag{2.11}$$

[figure 2c: participation rates by age]

The profile of the participation rate is less smooth than the empirically observed one. This is mainly due to the fact that heterogeneous workers undertake heterogeneous educational investment in reality, and get heterogeneous durations of activity from their investment. The introduction of ex ante heterogeneity would imply a smoother participation rate. However, the participation rate we obtain is broadly consistent with the empirical observation, in the sense that it is characterized by a succession of three different steps: in the first one, individuals are involved in the education system. Having completed their schooling period, they then enter the labour market and remain in activity, at least until A. In a third step, they progressively retire from the labour force.

Up to now, we have considered that the duration of schooling as well as the activity threshold were given. We now build a model aimed at endogenizing these variables.

3. Economic Modelling

The most natural framework to perform our analysis is that of a matching model a la Pissarides (1990), where we have incorporated two main features: hiring/retraining costs increasing with workers' age and endogeneous (time consuming) schooling choices.

The economy is composed of an endogeneously sized continuum of small firms producing a single good. Each firm is endowed with a unique job slot which can be either vacant or filled. Preferences are linear and ρ stands for the individual rate of time preference as well as the interest rate of the economy. Individuals are involved successively in two different theatres of activity: the education sector, then the labour market

3.1. The Education sector

Immediately after birth, agents enter the education system. We assume schooling is a time consuming activity. We therefore assume the existence of a relationship between the individual schooling duration T and productivity y. The properties of the technology y are standard, in line with the human capital theory (Becker, 1964, Mincer, 1974, Psacharopoulos, 1994). Formally, the function $y(T) : [0, \infty) \rightarrow [0, \infty)$ is strictly increasing, strictly concave, and twice continuously differentiable, satisfying the Inada conditions and the boundary properties $y(0) = 0$ and $y(\infty) = \infty$.

3.2. The matching market

Once they have completed their schooling, agents enter the labour market and start searching for a job. Employers endowed with vacancies and unemployed searching for a job are brought together in pair via an imperfect matching process. This process is captured by the customary matching function, which relates the total number of contacts M to the number of protagonists actively searching on each side of the market. If U stands for the total number of workers searching for a job and V for the number of vacancies, the number of contacts M is worth $M(U, V)$. The matching function M is strictly increasing, concave and twice continuously differentiable in each of its arguments, linearly homogenous. Linear

homogeneity allows us to express the hiring probability of a worker μ as well as the rate at which vacancies become filled η as functions of the labour market tightness $\theta \equiv V/U$. Thus the hiring probability of a job-seeker is $\mu = M/U = M(1,\theta)$, whereas the rate at which vacancies become filled is $\eta = M/V = M(1,\theta)/\theta$. The matching probability μ increases with tightness θ, while η is a decreasing function of θ.

3.3. Hiring costs

Hiring and retraining a worker is costly. We assume this cost is increasing with the elapsed duration on the labour market (experience) $\alpha = a - T$[5]. This leads us to neglect a potential effect of the duration of schooling, which may well reduce the individual activity horizon on the labour market[6]. We assume the function $H(\alpha)$ to be strictly increasing, continuous and satisfying $H(0) = 0$ and $H(\infty) = \infty$. This latter condition implies that some of the workers will never be hired, i.e. retraining the unemployed above a certain duration threshold $\Delta = A - T$ will not be profitable.

3.4. Firms and workers

Let $r \equiv \rho + \delta$, and q denote respectively the effective discount rate and the job destruction rate of this economy. Let us denote by μ the unemployment hazard rate. Let V^v and $V^e(T)$ be the respective value of holding a vacancy and a job slot filled with a worker whose education attainment is T. Let us denote $W^u(\alpha, T)$ and $W^e(\alpha, T)$ the expected utilities from being respectively searching for a job and employed for an individual whose experience on the labour market is α with education attainment T. These values satisfy respectively the two arbitrage equations:

$$(\rho + \eta) V^v = -\gamma + \eta E_{\alpha,T} \left[V^e(T) - H(\alpha) \left| V^e(T) - H(\alpha) \geq V^v \right. \right] \tag{3.1}$$

$$\rho V^e(T) = (1 - \beta) y(T) + (\delta + q)(V^v - V^e(T)) \tag{3.2}$$

[5] The careful reader may also notice that all expressions provided in the previous section depend on $a - T$, so that this change of variable is innocuous for the demographics.

[6] See for instance Boucekine, de la Croix and Licandro (2001) for a different analysis.

Holding a vacancy is costly and involves a flow advertisement cost $\gamma > 0$ paid continuously. A vacant job may become filled with probability η. Hiring a worker implies a retraining/recruitment cost $H(\alpha)$ so that the worker can become fully productive. Since both the education attainment and the experience of the incoming worker is a priori unknown to the potential employer, the value of a filled job has to be taken with an expectation operator in (3.1). Only matches yielding a net profit higher than the option value of searching for another worker V^v are acceptable to the firm, so that the expectation is conditional on this event. We assume that wages are the outcome of a simple sharing game between firms and workers, splitting the output flow $y(T)$ in two shares β to the worker and $1-\beta$ to the employer. Therefore a filled job brings an instantaneous revenue $(1-\beta)\,y(T)$, the output flow net of the wage accruing to the worker. With probability $\delta + q$, jobs are destroyed , either by the worker's death or by exogenous separation.

$$rW^u(\alpha, T) = \mu\left[\overline{W}^e(\alpha, T) - W^u(\alpha, T)\right] + \partial W^u/\partial\alpha \tag{3.3}$$

$$rW^e(\alpha, T) = \beta y(T) - q\left[W^e(\alpha, T) - W^u(\alpha, T)\right] + \partial W^e/\partial\alpha \tag{3.4}$$

where $\overline{W}^e(\alpha, T)$ stands for the felicity level expected on average by an individual whose experience on the labour market is α and whose education attainment is T. With 'probability' μ, a job seeker is matched to a vacancy, in which case he gets a corresponding rise in utility $\overline{W}^e(\alpha, T) - W^u(\alpha, T)$. Job seekers face a non stationary environment: their ability to be trained diminishes as they get older, and therefore their employment perspectives fall. This is captured by the term $dW^u/d\alpha$ in (3.3). An employed worker with education T is paid a wage $\beta y(T)$. With probability q, jobs are destroyed and the worker steps into unemployment, where he gets the utility level associated with job search $W^u(\alpha, T)$. The existence of the job destruction rate also implies that employed workers face a non stationary environment as they take into account that their employment perpsectives decay, which is captured by $dW^e/d\alpha$.

The next section describes the properties of the model in partial equilibrium, i.e. firms' hiring behaviour and workers' schooling choices for a given level of the labour demand.

4. Partial equilibrium of the labour market

We start by describing firms' hiring decisions and the corresponding activity threshold for a given education attainment in the workforce T. We then characterize optimal educational choices for a given level of the labour market tightness.

4.1. Education and activity

The existence of recruitment costs increasing with age/experience imply that firm set non trivial hiring standards with respect to the age/experience of an applicant. Firms proceed to the recruitment of job seekers on condition that hiring them is profitable, i.e. provided the expected profit flow generated by a recruitment is higher than the cost of hiring them. Formally, this defines an activity threshold Δ such that $V^e(T) = H(\Delta)$. Above this threshold, the application of a job seeker will be systematically rejected by all firms, so that the unemployed who have spent more than Δ on the labour market discourage: they finally drop the search and retire from the labour force. Taking account of (3.2) this yields the following expression for Δ:

$$\frac{(1-\beta)\, y(T)}{r+q} = H(\Delta) \tag{4.1}$$

The activity threshold decreases in the share of the output flow accruing to a worker β, as well as with the job destruction rate q. Since the expected profit stream of a firm decreases with these parameters, employers become more selective and set stricter hiring standard as these variables increase. For opposite reasons, the activity threshold Δ increases in workers' education attainment: higher educated workers are more productive, which makes employers more willing to retrain them. Therefore the effect of schooling on workers' activity rates is twofold: it tends to increase their participation to the labour market when old but reduces it when young since investing in human capital takes time.

4.2. Optimal schooling duration

To determine the optimal schooling duration, we preliminary need to establish the expression of the value of job search to an unemployed. Solving the system of differential equations (3.3) and (3.4), imposing the two conditions $W^u(\Delta, T) = 0$ and $W^e(\Delta, T) = \beta y(T) / (r+q)$, yields the following expression to the value of searching for a job $W^u(\alpha, T)$ to a worker with labour market experience α and education attainment T:

$$W^u(\alpha, T) = \frac{\beta\mu y(T)}{r(r+\mu+q)}\left(1 - \frac{r\mu e^{(r+\mu+q)(\alpha-\Delta(T))}}{(\mu+q)(r+q)} - \frac{q(r+\mu+q)e^{r(\alpha-\Delta(T))}}{(\mu+q)(r+q)}\right) \tag{4.2}$$

When the activity threshold Δ tends to infinity, workers' environment becomes stationary. We thus get the following usual expression:

$$W^u(\alpha, T) = \frac{\beta\mu y(T)}{r(r+\mu+q)} \text{ for any } \alpha \geq 0 \tag{4.3}$$

The optimal duration of schooling $\widehat{T}$ then solves:

$$\widehat{T} = \arg\max_{T\geq 0} e^{-rT} W^u(0, T) \tag{4.4}$$

The first order condition is $rW^u\left(0, \widehat{T}\right) = dW^u\left(0, \widehat{T}\right)/dT$. The left-hand side stands for the marginal cost of an additional year of schooling, which corresponds to the value of foregone earnings during the period. The right-hand side represents the marginal benefit of schooling. Optimal schooling is then defined by:

$$\underbrace{d\Delta\left(\widehat{T}\right)/dT \frac{\mu e^{-(r+\mu+q)\Delta\left(\widehat{T}\right)} + qe^{-r\Delta\left(\widehat{T}\right)}}{\frac{\mu}{r+q+\mu}\left(1 - e^{-(r+\mu+q)\Delta\left(\widehat{T}\right)}\right) + \frac{q}{r}\left(1 - e^{-r\Delta\left(\widehat{T}\right)}\right)}} = r$$

Equation (4.2) states the equality between the return to schooling and the pure rate of time preference r[7]. This equation highlights two types of returns to schooling: wage and activity returns. The existence of a second type of return clearly implies that workers

[7] This is thus a generalisation of human capital earnings function as those surveyed in Willis (1986).

facing a non stationary environment undertake higher human capital investment than in a stationary world. In addition, this equation suggests that the usual estimations of the return to schooling may be downward biased as they generally concentrate on wage return to schooling and do not consider the existence of additional benefits of education.

The decision to invest in human capital or stay at school for one more period depends on two main (endogeneous) variables of the model: the exit rate from unemployment $\mu(\theta)$ and the age threshold Δ. It also crucially depends on one (exogenous) variable of the economy: the job destruction rate q. Let us also note that if education had no effect on workers' duration of activity, then the schooling duration would be independent of these variables. The reason is that when education is time consuming and does not feature any other benefit than raising one's earnings, the effects of these variables on marginal cost and benefit from education exactly offset each other. This is no more the case when we introduce activity returns to schooling.

How is then the return to schooling affected by changes in these various variables? At first sight, a rise in labour market tightness θ can cause a fall in the return to education. The interpretation here is straightforward: a tighter labour market guarantees a high exit rate from unemployment, which reduces the probability of being unemployed in Δ. This reduces the incentive to spend time in the schooling system to increase one's duration of activity. For similar but opposite reasons, a fall in the average job tenure (i.e. a rise in the job destruction rate) can cause an increase in the return to schooling. Namely, lower job tenures make employers less willing to hire old workers and therefore reduces the age threshold. It is then worth spending some time in the schooling system to improve one's employability. For this reason, the duration of schooling $\widehat{T}$ may be increasing in the job destruction rate q. Since it may also be decreasing in tightness θ, it may be the case that the schooling duration and unemployment together increase.

5. Effects of structural shocks

5.1. Closing the model

This section describes the effects of a structural shock, namely a permanent rise in the job destruction rate q. In the previous section, we have considered the labour demand as given. We now endogenize this latter by assuming firms are free to advertise vacancies on the matching market. Free entry drives the value of vacancies to zero, which writes down:

$$V^e(T) = \gamma/\eta(\theta) + \overline{H}(\Delta, \theta) \tag{5.1}$$

Free entry implies the exhaustion of all rents. Firms advertise new vacancies until the expected value of a match $V^e(T) = \frac{(1-\beta)y(T)}{r+q}$ is equal to the average cost to recruit a worker, i.e. the average search cost $\gamma/\eta(\theta)$ plus the average retraining/recruitment cost $\overline{H}(\Delta, \theta) = \int_0^\Delta H(\alpha)\phi(\alpha, \Delta, \theta)\, d\alpha$.

Solving the model then reduces to the determination of the activity threshold Δ^*, the equilibrium labour market tightness θ^*, the schooling duration T^*, and the distribution of experience across the unemployed $\widetilde{\phi}(\alpha)$.

An equilibrium is then defined by

(i) Duration of Activity

$$\frac{(1-\beta)\, y(T^*)}{r+q} = H(\Delta^*) \tag{DA}$$

(ii) Otimal Schooling Duration

$$\frac{y'(T^*)}{y(T^*)} + \frac{(1-\beta)\, y'(T^*)}{(r+q)\, H'(\Delta^*)} \frac{\mu(\theta^*)\, e^{-(r+\mu(\theta^*)+q)\Delta^*} + q e^{-r\Delta^*}}{\frac{\mu(\theta^*)}{r+q+\mu(\theta^*)}\left(1 - e^{-(r+\mu(\theta^*)+q)\Delta^*}\right) + \frac{q}{r}\left(1 - e^{-r\Delta^*}\right)} = r \tag{OS}$$

(iii) Free Entry

$$\frac{\gamma}{\eta(\theta^*)} + \int_0^{\Delta^*} H(\alpha)\, \phi(\alpha, \theta^*, \Delta^*)\, d\alpha = \frac{(1-\beta)\, y(T^*)}{r+q} \tag{FE}$$

(iv) Distribution of experience across the unemployed

$$\widetilde{\phi}(\alpha) = \phi(\alpha, \theta^*, \Delta^*) = \frac{\delta(\delta + \mu(\theta^*) + q)\left[q e^{-\delta\alpha} + \mu(\theta^*)\, e^{-(\mu(\theta^*)+q+\delta)\alpha}\right]}{(\delta+q)(\mu(\theta^*)+q) - \delta\mu(\theta^*)\, e^{-(\mu(\theta^*)\mu+q+\delta)\Delta^*} - q(\mu(\theta^*)+q+\delta)\, e^{-\delta\Delta^*}} \tag{Alpha}$$

Equation (DA) defines the activity threshold prevailing in the economy as a function of the education level of the workforce. With knowledge of the distribution of experience across the unemployed $\widetilde{\phi}(\alpha) = \frac{u(\alpha)}{U(\Delta^*)}$, equations (OS) and (FE) can be interpreted as the reaction functions of a game between employers who determine the labour demand according to the education attainment of the population, and workers whose schooling duration depends on the intensity of the labour demand.

When the equilibrium is unique, the Optimal Schooling locus (OS) depicts a decreasing relationship between schooling duration T^* and labour market tightness θ^*, while the Free Entry locus (FE) is downward sloping[8].

[figure 3a : unicity et multiplicity of equilibria]

In the remainder, we choose to focus on situations where the equilibrium is unique. This situation is depicted by the following figure.

[figure 3: education and labour market tightness]

5.2. Numerical Simulations

We now illustrate the effects of a structural shock, such as a permanent rise in the job destruction rate of the economy. We perform this analysis numerically. The following table displays our baseline parameters:

Baseline parameters

δ	r	ρ	y_0	A	ν	h	κ	B	β	γ
0.02	0.05	0.03	0.35	0.1	0.5	0.1	0.4	0.5	0.5	1

[8] However, the equilibrium is not necessarily unique. Two externalities are at play: as in Pissarides (1992), or Coles and Masters (2000), there may be increasing returns to the entry of firms, since advertising one more vacancy may reduce the amount of retraining costs expected by all firms. In addition, schooling choices generate externalities between firms and workers.

The inflow rate into the population δ is 2%, an average value between both birth and death rates of most OECD countries. The effective discount rate $r = \rho + \delta$ is worth 5%,so that the interest rate ρ is set to 3%. The technology in use in the education sector is such that $y(T) = y_0 (A * T + T^{\nu})$, while we assume the hiring/retraining technology is $H(\alpha) = \exp h\alpha$. With the chosen values of ν and h, Hiring an applicant at age 40 with some higher education, say 15 years of schooling, involves spending about 3.5 years of output in retraining/hiring costs, while hiring an applicant of the same age with some basic education, say 10 years of schooling, involves spending about 7 years of output in retraining. The matching function is Cobb Douglas, namely $m(U, V) = BV^{\kappa}U^{1-\kappa}$. We set the value of the parameter κ to 0.4, an admissible value with regard to most of the usual estimates, which range from 0.4 up to 0.6 (see Pissarides and Petrongolo, 2001 for a survey). Other parameters are set so as to broadly reproduce the situation of a 'typical' European country such as France, i.e. empirically plausible values of the activity threshold Δ, of the schooling duration T, of the average duration of unemployment $\frac{1}{\mu}$ as well as of the aggregate unemployment rate $u = \frac{\delta+q}{\delta+q+\mu}$.

5.3. Results

In order to highlight the impact of a rise in job destruction, we consider three possible values for the job destruction rate, ranging from 1 to 10%. The first value is purely hypothetic, but is is meant to represent a very steady environment, such as that prevailing before the first oil shock. The second one is slightly more realistic, and could correspond to that prevailing in the mid or late 70's, while the third one is broadly equal to the average value of the job destruction rate nowadays in France. Our results are now depicted by the following table:

Results

q	$\mu(\theta^*)$	Δ^*	T^*	u
0.01	1.7	26.5	12.9	1.7
0.05	1.13	22	14	5.8
0.1	0.84	18.5	15.3	12.4

The interpretation is as follows: the rise in job destruction shortens job tenures, which reduces the expected value of a filled job. This is detrimental to the labour demand, and makes employers less willing to hire old workers. Consequently, both tightness and age threshold go down. As job tenure reduces, there are now higher incentives to schooling: as the age threshold goes down, it is worth spending time in the schooling system to improve one's employability. Besides, the opportunity cost of one further year of schooling is reduced as unemployment hasards collapse (it now takes twice as long as previously to find a job). The schooling duration therefore goes up, rising from approximately 13 years in the first case, up to 15 years in the third one. At the aggregate level, a reduction in job tenures translates into higher unemployment (from 1.7 to 12.4% in our example), but also into lower participation to the labour market: as the expected value of a filled job falls, firms are less incited to advertise vacancies and it is less profitable to retrain an old unemployed. The effects of a rise in q on the unemployment, employment and participation rates by age are then depicted by the following panel of figures:

[figure 4: effects of structurals shocks on employment, unemployment and participation rates by age]

The continuous grey curves correspond to a relatively stable environment where the average job destruction rate is 1%, while the dashed grey lines corresponds to lower job tenures ($q = 5\%$). The third case, depicted in dark continuous lines represents the case where $q = 10\%$. In the first case, the schooling duration is relatively low compared to the other cases, since the employability constraint is not really binding (approximately 27 years in our example) and finding a job does not take so long (approximately 6 months in

our example). As job tenures decline, the environment becomes relatively more unstable: unemployment rates by age increase for all age considered, and activity rates fall particularly sharply for two types of workers: the youngest, who devote more time to schooling, and the eldest who retire earlier since they are more heavily discriminated, and they also loose their jobs more frequently. These changes are then reflected by the employment rate by age, where the sharpest fall is also the concern of the youngest, who move at a lower pace from school to work, as well as of the eldest, who move faster from employment to inactivity as job destruction increases, but also earlier as firms are less willing to hire them.

6. Conclusion

In this paper, we have considered the limks between education, unemployment and participation to the labour market in a matching model with ex post heterogeneous workers. The stylized facts we aimed to explain are the following: the unemployment rate is continuously decreasing in workers' age, whereas the employment rate exhibits a bell-shaped path. While the former can easily be accounted for by the existence of matching frictions, one needs to incorporate additional features into the standard matching model a la Pissarides (1990) to account for the latter. This can be done for instance by introducing hiring/retraining costs increasing with the job seekers' age, which directly implies the existence of an age or activity threshold above which workers retire from the labour force. Over the past decades, the fall in employment has been particularly pronounced for two age categories: the youngest and the eldest. These changes are also reflected by falling participation rates to the labour market. Our explanation is thus the following: for the youngest, a worsening in labour market conditions have raised the incentives to schooling, whereas for the eldest, falling participation reflects a lower employability.

DEMOGRAPHICS

The flow equilibrium equations write down:

$$\delta e^{-\delta T} + qL(a) = (\delta+\mu)U(a) + dU(a)/da \tag{0.1}$$

$$\mu U(a) = (\delta+q)L(a) + dL(a)/da \tag{0.2}$$

Since $u(a) + l(a) = \delta e^{-\delta a}$, one gets after some simple algebra

$$dU(a)/da = \left[\delta + q\left(1-e^{-\delta a}\right)\right]e^{-\delta T} - (\delta+\mu+q)U(a) \tag{0.3}$$

$$dL(a)/da = \mu\left(1-e^{-\delta a}\right)e^{-\delta T} - (\delta+\mu+q)L(a) \tag{0.4}$$

Integrating these expressions, we get:

$$\begin{aligned} U(a) &= e^{-\delta T}\int_T^a \left[\delta + q\left(1-e^{-\delta(z-T)}\right)\right]e^{-(\mu+q+\delta)(a-z)}dz \\ &= e^{-\delta T}\left[\frac{\delta+q}{\mu+q+\delta} - \frac{qe^{-\delta(a-T)}}{\mu+q} - \frac{\delta\mu e^{-(\delta+\mu+q)(a-T)}}{(\mu+q)(\mu+q+\delta)}\right] \end{aligned} \tag{0.5}$$

$$\begin{aligned} L(a) &= e^{-\delta T}\int_T^a \left[\mu\left(1-e^{-\delta(z-T)}\right)\right]e^{-(\mu+q+\delta)(a-z)}dz \\ &= e^{-\delta T}\left[\frac{\mu}{\mu+q+\delta} - \frac{\mu e^{-\delta(a-T)}}{\mu+q} + \frac{\delta\mu e^{-(\delta+\mu+q)(a-T)}}{(\mu+q)(\mu+q+\delta)}\right] \end{aligned} \tag{0.6}$$

Then, the mass numbers of unemployed and employed workers with age a $dU(a)/da$ $dL(a)/da$ can then be obtained by differentiating the expressions of (0.5) and (0.6). We get :

$$dU(a)/da = e^{-\delta T}\frac{\delta}{\mu+q}\left[qe^{-\delta(a-T)} + \mu e^{-(\delta+\mu+q)(a-T)}\right] \tag{0.7}$$

$$dL(a)/da = e^{-\delta T}\frac{\delta\mu}{\mu+q}\left[e^{-\delta(a-T)} - e^{-(\delta+\mu+q)(a-T)}\right] \tag{0.8}$$

It is also possible to check that these expressions are also consistent with those involved by the flow equilibrium equations, i.e. $dU(a)/da + dL(a)/da = \delta e^{-\delta a}$ and $U(a) + L(a) = e^{-\delta T}\left(1-e^{-\delta(a-T)}\right)$. The expressions of the unemployment, employment, and activity rates therefore stem from these equations.

Besides, there is also a mass number L_I of individuals with age above A who are still employed. Since workers retire from the labour market after A, workers

belonging to L_I where necessarily employed at age A. They then progressively exit from this population at rate $\delta + q$, as they die or loose their jobs. We have $L_I = \delta e^{-\delta A} l(A) \int_A^{+\infty} e^{-(q+\delta)(z-A)} dz$, thus :

$$L_I = \frac{\delta \mu e^{-\delta T}}{(\mu + q)(q + \delta)} \left[e^{-\delta(A-T)} - e^{-(\mu+q+\delta)(A-T)} \right] \tag{0.9}$$

Finally, the mass number of "old" retirees I is then determined residually by $I = e^{-\delta T} - U(A) - L(A) - L_I$. We have:

$$I = e^{-\delta T} \frac{q(\mu + q + \delta) e^{-\delta(A-T)} + \delta \mu e^{-(\mu+q+\delta)(A-T)}}{(\delta + q)(\mu + q)} \tag{0.10}$$

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