# Unemployment Insurance Design: inducing moving and retraining*

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## Abstract

Evidence suggests that unemployed individuals sometimes can affect their job prospects by undertaking a costly action like deciding to move or retrain. Realistically, such an opportunity arises only for some individuals and the identity of those is unobservable. Unemployment insurance should then be designed to induce individuals to exploit existing opportunities to move or retrain without excessively diminishing the insurance value for the remaining unemployed. This problem has been neglected in previous literature on unemployment insurance design and we show that it may have important consequences. In particular, we derive closed-form solutions showing that when the moving/retraining incentive constraint binds, unemployment benefits should increase over the unemployment spell, having an initial period with low benefits and a substantial increase after this period has expired.

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# 1 Introduction

An important feature of the modern welfare state is the existence of an extensive unemployment insurance (UI) system. It is now well established that the design of the unemployment insurance affects the incidence of unemployment by distorting the incentives of unemployed to search for a job (see, e.g., Holmlund (1998) for a survey). This has motivated a growing literature on how the UI system should be designed to make an optimal trade-off between providing good insurance on the one hand, and not distorting the incentives too much, on the other. The seminal paper by Shavell and Weiss (1979) characterizes the optimal design of UI when search activity is unobservable. Since then, a line of papers that extend the analysis has appeared. For example, Hopenhayn and Nicolini (1997) allow a more general set of policies; specifically, they assume that in addition to UI-benefits, also taxes paid by employed can be made contingent on the employment history of the individual. An important assumption in Shavell and Weiss (1979) and Hopenhayn and Nicolini (1997), is that the insurer can fully control the individual's consumption – usually interpreted as the individual having no access to markets for saving and borrowing and no alternative sources of income. It has proven difficult to relax the assumption of no hidden savings, but recently important progress has been made in this respect (see Pavoni (2001), Arpad and Pavoni (2002) and Werning (2002)). Other important extensions of the analysis, for example allowing sequential search, endogenous wage formation, job-creation and production, has also been done in recent years (see, e.g., Shimer and Werning (2003) , Cahuc and Lehmann (2000), Abdulkadiroglu, Kuruscu and Sahin (2002), Fredriksson and Holmlund (2001) and Heer (2003)).

In this paper, we will maintain most of the standard assumptions in the literature but cast the focus on an important informational problem that has been largely neglected. Specifically, we will consider the case when some, but not all, unemployed can increase the probability of being hired by undertaking a costly investment, e.g., by retraining or moving to a location with better employment prospects. Under the realistic assumption that the insurer is unable to observe who has this option,

an incentive problem arises and a failure to take this into account may lead to sub-optimal UI-design. One conceivable way to mitigate the problem would be to offer subsidies to moving or retraining. However, as we will argue below, full cost-compensation is not feasible in the realistic case when the insurer can not fully distinguish voluntary and involuntary job-separations.

Although an empirical investigation is outside the scope of this paper, we argue that the consequences of not providing reasonable incentives for people to move or retrain may be of substantial quantitative importance. For instance, Bartel (1979) documents that the proportion of geographical mobility in the U.S. caused by the decision to change jobs is one-half of all migration decisions for young workers and one third of all migration decisions for workers above the age of 45. Furthermore, geographical mobility is substantially lower in continental Europe, and Hassler, Rodríguez Mora, Storesletten and Zilibotti (2004), hereafter called HRSZ, document in panel-data a negative correlation between geographical mobility and UI-generosity as well as between mobility and aggregate unemployment rates. Other empirical documentations of the link between unemployment and geographical mobility are DaVanzo (1978), Pissarides and Wadsworth (1989) and McCormick (1997).

In, HRSZ, a constant UI-benefit level is assumed and, of course, the higher this is, the weaker are the incentives to move. Following the tradition in the optimal UI-design literature, we will investigate if non-constant benefit rates can strengthen the incentives to move without reducing the insurance value of UI. Since we believe that also the standard moral hazard problem of providing incentives for a continuous job-search are important, we will include this in the analysis.

There is empirical evidence indicating that precautionary saving is used in order to self-insure against unemployment risk. Using PSID, Gruber (1997) finds that, in absence of UI, consumption falls by 22% when an individual become unemployed, showing that individuals are able to smooth consumption also when there is no UI. Similarly, Engen and Gruber (2001) show that UI crowds out financial savings, indicating that households use financial markets to self-insure against unemployment

risk.[1] The assumption that the insurer can perfectly control individual consumption is thus not entirely realistic. Building on the emerging tradition in the recent papers cited above, we will therefore allow the individual to make her own consumption decisions, allowing access to a market for saving and borrowing.

To facilitate understanding of the results, we will make assumptions that allow analytical characterizations and, specifically, graphical and closed form solutions for optimal benefits as well as for observables like the changes in individual consumptions levels associated with a change of job status. Our model also easily lends itself to allowing multiple incentive problems, e.g., adding a moral hazard problem in job-retention effort like in Wang and Williamson (1996).

Two important assumptions are key to analytical tractability; First, we assume constant absolute risk-aversion implying that search incentives are independent of asset holdings. Second; Individuals have access to a perfect market for borrowing and lending. As shown in Werning (2002), a constant benefit scheme is optimal under these assumptions when search efficiency is constant. This allows us to focus on simple benefit schemes with a limited number of benefit levels. Neither of the key assumptions is perfectly realistic, and the model does therefore not directly lend itself to quantitative policy recommendations. Our purpose is instead to illustrate a mechanism not previously explored in the literature, thereby providing guidance for future quantitative work.

The paper is structured in the following way. The model is presented in section 2, where in subsection 2.1 we derive the relevant value functions, in subsection 2.2 incentive compatibility constraints are derived. In section 3 and subsection 3.1 the main results are derived and discussed and section 4 concludes. Some proofs can be found in the text, others in the appendix and the remaining are available upon request from the authors.

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[1] Also if access to the formal capital market is limited, alternative means to smooth consumption may exist, see e.g., Cullen and Gruber (2000).

# 2 The model

Consider an economy in continuous time where individuals can be employed or unemployed. They have access to a market for safe saving and borrowing with an exogenous return $r$, equal to the subjective discount rate (possibly including a positive probability of dying). Unemployed individuals can affect their chances of finding a job. As noted in the introduction, we will focus on the case where some, but not necessarily all, individuals can make a costly investment increasing their chances of becoming employed. Allowing unobservable heterogeneity in this respect creates and informational problem similar to an adverse selection problem and makes full insurance infeasible.[2] In addition, we will allow a more standard moral hazard problem where search activity entails a flow cost.

Specifically, we assume that employed individuals loose their jobs at rate $q$. A share $p \in [0, 1]$ of those who loose their job can undertake a costly investment. We will interpret this as representing a cost of moving, denoted $m > 0$ (for example between geographical locations or between occupations that require some retraining). For simplicity, we assume that if the unemployed pays this cost ("moves"), she is immediately rehired. Unemployed who cannot, or decide not to move and who search for a job find one at rate $h$. Searching has a cost of $s \geq 0$ per unit of time. We may consider this cost as representing the opportunity cost of searching, arising from, for example, some alternative economic activity. Whether the agent actually searches or not and whether she has the opportunity to move are assumed to be her own private information. To make the problem interesting, we assume that it is optimal to induce individuals to search and move (if they have the opportunity). It is easy to show that under this assumption, agents who have the option to move should be induced to do so immediately. Therefore, in the optimal solution, no mass

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[2]There are few papers on UI deal with adverse selection. One recent paper is Hagedorn, Kaul and Memmel (2003), where individuals have different hiring rates are separated by being offered different menues of benefits.

of agents should be unemployed while having the opportunity to move.

An employed individual is said to be in state 1, receiving an exogenous gross wage $w$. An individual who looses her job and do not move enters into state 2 and is then called short-term unemployed, receiving benefits denoted $b_2$. To analyze the issue of whether unemployment benefits should be increasing or decreasing, we allow two benefit levels, $b_2$ and $b_3$, the latter being given to individuals in state 3, who are denoted long-term unemployed. The assumption of only two unemployment states is not important for the results.[3] To facilitate a simple presentation of the results, we assume that an individual in state 2 enters state 3 with a constant instantaneous probability $f$.[4] Since state 3 is an administrative state associated with long unemployment duration, we assume individuals who search to have the same hiring rates, $h$, in the two unemployment states.[5] Motivated by practical considerations, and in contrast to, e.g., Hopenhayn and Nicolini (1997), we assume that benefit levels can be given conditional only on current unemployment status (2 or 3), not not on employment history or asset holdings.

Individuals maximize their intertemporal utility, given by

$$E \int_0^\infty e^{-rt} U\left(c_t\right) dt,$$

where $c_t$ is consumption at time $t$ and $r$ the subjective discount rate. In order to facilitate analytical solutions when individuals have access to markets for saving and borrowing, we choose the CARA utility function

$$U\left(c_t\right) \equiv -e^{-\gamma c_t},$$

where $\gamma$ is the coefficient of absolute risk aversion.[6] All individuals are born (enter

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[3]Proof available upon request.

[4]This assumption implies that seach incentives remain constant as long as the individual remain in state 2. An alternative would be to use discrete time and assume that short-term UI benefits are paid for one period only as done by e.g., Cahuc and Lehmann (2000). Assuming that UI benefits change after some fixed period of time would make search incentives depend on the remaining time of current benefits and considerably complicate the analysis with little gain.

[5]This assumption could, however, easily be relaxed.

[6]Given this, assets will not affect individual decisions. For other utility functions, the decision

the labor market) as employed without assets and are identical at that point.

The purpose of this paper is to discuss how an unemployment insurance system should be constructed when there are incentive problems. To his end, we want to remove other motives for unemployment benefits than providing insurance. In particular, we are in this paper not interested in motives to use the UI system to create non-actuarial transfers between individuals with different characteristics.[7] Therefore, we assume that individuals face an actuarially fair insurance. This means that when an individual enters the labor force, the expected present discounted value of the benefits she will receive during her life-time exactly balances the expected present discounted value of her contributions. An alternative interpretation of actuarial fairness is that in a decentralized equilibrium, where individuals can sign binding insurance contracts with competitive insurance companies when entering their first job, actuarial fairness is identical to a break-even condition for the insurance companies, which would be satisfied under perfect competition.[8]

Without loss of generality, we let individuals pay lump-sum taxes, denoted $\tau$, implying that

$$\dot{A}_t = rA_t + \omega - c_t - \tau, \tag{1}$$

except at the points in time when the cost of moving is paid, and where $\omega \in \{w, b_1 - s, b_2 - s\}$, depending on the employment state. We define the average discounted probabilities (ADP's) of being in state 2 and 3, respectively, by

$$\Pi_2 \equiv r \int_0^\infty e^{-rt} \mu_{2,t} dt,$$

$$\Pi_3 \equiv r \int_0^\infty e^{-rt} \mu_{3,t} dt.$$

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to move and to search for a job would depend on the individual asset level. Then, asset dependent benefits would be required to satisfy the incentive constraints exactly. We believe that some asset dependence, like a means-tested UI-system might be reasonable, but we leave such schemes for future research.

[7]For positive implications, the redistributive elements of unemployment insurance are, however, central. See e.g., Wright (1986).

[8]Since we use the CARA specification, individual assets do not affect preference over insurance so older employed agents with non-zero asset holdings would not want to renegotiate their contract.

where $\mu_{2,t}$ and $\mu_{3,t}$ are the probabilities of being short term and long term unemployed at time $t$, respectively, conditional on being employed at time zero. Solving for the ADP's assuming that individuals who can move do so and that unemployed search for a job yields

$$\Pi_2 \equiv \tilde{q} \frac{h+r}{(r+h+\tilde{q})(r+h+f)},$$

$$\Pi_3 \equiv \Pi_2 \frac{f}{h+r},$$

$$(2)$$

where $\tilde{q} \equiv q(1-p)$ equals the rate of flow into unemployment.

The actuarial fairness requirement of the UI system can then be written

$$\tau = \Pi_2 b_2 + \Pi_3 b_3. \tag{3}$$

## 2.1 Value functions and consumption

It is well known that the value functions for the three states can be written as

$$V_j(A_t) = -\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma \sigma_j}, j \in \{1,2,3\}, \tag{4}$$

where $\sigma_j$ are state-dependent constants and where the state dependent consumption functions are

$$c_j(A_t) = r A_t + \sigma_j, \ j \in \{1,2,3\}. \tag{5}$$

Note from (5) that individuals optimally consume the permanent income from their asset holdings $(rA_t)$ plus a state dependent constant $\sigma_j$. As we see from (5), the differences in consumption between employed and unemployed individuals with identical assets holdings, i.e., $c_1(A_t) - c_2(A_t)$ and $c_1(A_t) - c_3(A_t)$, are independent of $A_t$ and given $\sigma_1 - \sigma_2$ and $\sigma_1 - \sigma_3$, respectively. These differences will be of key importance for our analysis and we therefore define

$$\sigma_1 - \sigma_2 \equiv \Delta_2 \text{ and } \sigma_1 - \sigma_3 \equiv \Delta_3. \tag{6}$$

Using these definition, it is straightforward to check that the Bellman equation

for the individuals who search and move is satisfied if the constants $\sigma_j$, satisfy

$$\sigma_1 = w - \tau - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma(\Delta_2)} - 1}{\gamma r},$$

(7)

$$\sigma_2 = b_2 - s - \tau + h\frac{1 - e^{-\gamma(\Delta_2)}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r},$$

$$\sigma_3 = b_3 - s - \tau + h\frac{1 - e^{-\gamma(\Delta_3)}}{\gamma r},$$

in which case individual intertemporal utility is maximized under (1) and a No-Ponzi condition.

Our objective is to maximize welfare of an individual entering the labor market with no assets, $V_1(0)$, subject to incentive constraints and actuarial fairness. From (4), we note that i) this is equivalent to maximizing $\sigma_1$, subject to the constraints, and ii) the solution will maximize welfare of all employed, regardless of their asset holdings and previous employment history.

Our procedure will be done in two steps. First, we will maximize $\sigma_1$ over the consumption differences subject to incentive constraints (to be defined shortly) and actuarial fairness, thus finding the optimal $\Delta_2$ and $\Delta_3$. Second, we characterize the unique combination of benefits $b_2$ and $b_3$ that implements the optimal allocation. To do this and to express actuarial fairness (3) in terms of $\Delta_2$ and $\Delta_3$, we subtract the second and third line of (7), respectively, from the first, yielding

$$\Delta_2 = w - (b_2 - s) - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma\Delta_2} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r},$$

(8)

$$\Delta_3 = w - (b_3 - s) - q\frac{pe^{\gamma rm} + (1-p)\, e^{\gamma\Delta_2} - 1}{\gamma r} - h\frac{1 - e^{-\gamma\Delta_3}}{\gamma r}.$$

Notice that (8) establishes a one-to-one relationship between $\{\Delta_2, \Delta_3\}$ and $\{b_2, b_3\}$. If we subtract the two equations in (8) it is easy to see that $(\Delta_2 - \Delta_3)$ is a monotonously increasing function of $b_3 - b_2$ that crosses the origin. Furthermore, whenever $\Delta_2$ is larger than $\Delta_3$, benefits are necessarily larger for long run than for short run unemployed.

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## 2.2 Incentive constraints

### 2.2.1 Incentives to move

Now, consider a person who has lost her job and has the ability to move. She should be induced to do so voluntarily. If her assets at separation were $A_t$, her value immediately after moving is

$$V_1 \left( A_t - m \right) = -\frac{1}{r} e^{-\gamma r (A_t - m)} e^{-\gamma \sigma_1}.$$

We compare this to the value of a one-period deviation, i.e., the value if she does not move during this unemployment spell, given by

$$V_2 \left( A_t \right) = -\frac{1}{r} e^{-\gamma r A_t} e^{-\gamma \sigma_2}.$$

To induce moving we need $V_1 \left( A_t - m \right) \geq V_2 \left( A_t \right)$. It follows immediately that this can be written

$$\Delta_2 \geq rm. \tag{9}$$

We label (9) the *ICM-condition.* Note that ICM-condition is independent of assets, implying that it can never be individually rational to wait in the short-term unemployment state and move later, while still in state 2.[9]

Note that the ICM is independent of $\Delta_3$. This does not mean that the incentives to move are independent of long-run benefits. On the contrary, as seen in (8), $\Delta_2$ depends on $\Delta_3$, which, in turn, depends on $b_3$. However, an important advantage of focusing on the incentives $\Delta_2$ and $\Delta_3$, is that incentive constraints in state $j$ can be expressed in terms of $\Delta_j$ only. As we will see, this orthogonality will hold also for the remaining incentive constraints, discussed in the next subsection, and will make the analysis simple.

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[9]Of course, when $b_2 > b_3$, it *could* be individually rational not to move in state 2 but move when state 3 is entered.

### 2.2.2 Incentives to search

Let us now consider the incentives for searching during unemployment. A long-term unemployed who does not search will remain unemployed for ever, consuming her permanent income, given by $b_3 - \tau + rA_t$. This yields an intertemporal utility of $-\frac{1}{r}e^{-\gamma rA_t}e^{-\gamma(b_3-\tau)}$. The long-term unemployed will search if this is less than her intertemporal utility when searching, i.e., if $V_3(A_t) \geq -\frac{1}{r}e^{-\gamma rA_t}e^{-\gamma(b_3-\tau)}$. This, again, is independent of assets, and can be written as

$$\sigma_3 \geq b_3 - \tau. \tag{10}$$

Using this in the individual dynamic budget constraint yields,

$$
\begin{aligned}
\dot{A}_t &= rA_t + b_3 - \tau - s - \sigma_3 - rA_t \\
&\leq rA_t + b_3 - \tau - s - (b_3 - \tau) - rA_t = -s
\end{aligned}
$$

As we see, (10) requires that consumption for a long term unemployed ($c_3(A_t) = \sigma_3 + rA_t$) must be larger than or equal to her income net of taxes ($b_3 - \tau + rA_t$). This means that incentives have to be at least large enough to make the individual willing to borrow to finance her search cost. This, in turn, means that consumption necessarily falls as long as the individual remains long-term unemployed. Using (7), (10) can be written as

$$\Delta_3 \geq -\frac{\ln\left(1 - \frac{\gamma rs}{h}\right)}{\gamma} \equiv \hat{\Delta}\left(\frac{rs}{h}; \gamma\right). \tag{11}$$

which we label the *IC3-condition*. As we see, the increase in consumption a long-term unemployed achieves by finding a job needs to be larger than $\hat{\Delta}\left(\frac{rs}{h}; \gamma\right)$ where $\hat{\Delta}$ is a strictly increasing function of $rs/h$ and, in addition, only dependent on $\gamma$. Since the gain from searching comes in the future, it is intuitive that the potential reward, $\Delta_3$, has to be larger as discounting increases. It is also intuitive that increased search cost and reduced search effectiveness requires a larger reward for individuals to want to search. The sign of the derivative, $\partial\hat{\Delta}\left(\frac{rs}{h}; \gamma\right)/\partial\gamma$ is on the other hand non-monotonic, being positive for low values of $\gamma$ and becoming negative as $\gamma$ approach $h/rs$.

Note that while the search incentive in general depends on the extent to which the value function increases when employment is gained, in this case, the incentive constraint can be written as only depending on the extent to which *consumption* increases at re-employment. This is due to the stationary risk-environment we impose in combination with the CARA utility.

Now, we turn to search incentives for short-run unemployed. For the short term unemployed, we compute the value associated with a one-period deviation, i.e., no search in the current employment state, conditional on searching in future states. In the appendix, we show that this value is $-\frac{e^{-\gamma r A_t} e^{-\gamma c_{2,n}}}{r}$ where $\sigma_{2,n}$ satisfies

$$\sigma_{2,n} = b_2 - \tau + \frac{f \left(1 - e^{-\gamma(\sigma_3 - \sigma_{2,n})}\right)}{\gamma r}.$$

The IC2 constraint is $\sigma_2 \geq \sigma_{2,n}$, which can be written as

$$\Delta_2 \geq \hat{\Delta} \left(\frac{rs}{h}; \gamma\right), \tag{12}$$

which we label the *IC2-condition*.

As noted in the previous subsection, the incentive constraints for the two state, IC2 and IC3, are orthogonal, only depending on the relevant incentive ($\Delta_2$ or $\Delta_3$) and exogenous variables. To repeat, this does, of course, not mean that only $b_2$ ($b_3$) matters for search incentives of the short-term (long-term) unemployed. On the contrary, both $b_2$ and $b_3$ affect consumption in all states, as seen in (7). However, individual optimization and access to markets for saving and borrowing imply the value function to be a monotonous transformation of consumption. Thus, the wedge between consumption in the current state and during employment is a sufficient statistic to determine if search incentives are sufficiently strong.

Furthermore, note that the RHS of IC2 and IC3 are identical. In other words, given that the hiring probability and search costs are the same for short-term unemployed and long-term unemployed, individuals in these states need the same reward in terms of consumption increases after a successful job-search to be willing to search. Allowing different search costs and/or hiring probabilities, would simply

change the argument of the $\hat{\Delta}(.)$ function, while maintaining orthogonality between the two constraints.

The optimal insurance contract should then be chosen to maximize $\sigma_1 = w - \tau - q\frac{pe^{\gamma rm}+(1-p)e^{\gamma \Delta_2}-1}{\gamma r}$, over $\Delta_2$ and $\Delta_3$, subject to the incentive constraints (9), (11) and (12), and the actuarial fairness constraint (defined by (3) and (8)).

# 3   Characterization of the preferred UI-scheme

Since the focus of this paper is the incentive problems associated with moving, we start the analysis with the assumption that search costs are zero, while moving costs are strictly positive. Specifically, we first assume that only the ICM condition binds, i.e., that $\Delta_2 = rm > 0$. In addition, we require $\Delta_3 \geq \hat{\Delta}(0;\gamma) = 0$. If this constraint were violated, long-term unemployed would strictly prefer to remain unemployed.

To provide an understanding of our analytical results below, we start by deriving a graphical representation of the problem. By substituting for $\tau$ in the objective function, solving (8) for benefits and substituting into (3), and dividing by $\Pi_2$ the problem can be written

$$\max_{\Delta_2,\Delta_3} \left\{ K + \Delta_2 + \frac{f}{h+r}\Delta_3 - \frac{1}{\gamma r}\left((r+h+f)e^{\gamma\Delta_2} + he^{-\gamma\Delta_2} + fe^{\gamma(\Delta_3-\Delta_2)} + \frac{fh}{h+r}e^{-\gamma\Delta_3}\right)\right\}$$

(13)

$$s.t.\ \Delta_2 \geq rm, \Delta_3 \geq 0.$$

where $K$ is a constant.[10]

For $\Delta_3, \Delta_2 \geq 0$ and $\{\Delta_3, \Delta_2\} \neq \{0,0\}$, the indifference curve for this problem

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[10]The constant is given by

$$\begin{aligned} K &\equiv w\frac{(1-\Pi_2-\Pi_3)}{\Pi_2} - \frac{(\Pi_2+\Pi_3)}{\Pi_2}s \\ &\quad - \frac{1-\Pi_2-\Pi_3}{\Pi_2}\frac{q}{r}\frac{pe^{\gamma rm}-1}{\gamma} + \frac{1}{\gamma r}\left(h+f\left(1+\frac{h}{h+r}\right)\right) \end{aligned}$$

has a slope given by

$$\frac{d\Delta_2}{d\Delta_3}|_{\sigma_1 \text{ constant}} = -\frac{\frac{f}{h+r} - \frac{1}{r}\left(fe^{\gamma(\Delta_3-\Delta_2)} - \frac{fh}{h+r}e^{-\gamma\Delta_3}\right)}{1 - \frac{1}{r}\left((r+h+f)e^{\gamma\Delta_2} - he^{-\gamma\Delta_2} - fe^{\gamma(\Delta_3-\Delta_2)}\right)} \quad (14)$$

In figure 1, we make a graphical representation of the problem. The bliss point is at full insurance, when $\{\Delta_3, \Delta_2\} = \{0,0\}$. The indifference curves have elliptical shapes around this point, of which we are only interested in the segment in the positive quadrant since incentive compatibility certainly requires $\Delta_3, \Delta_2 \geq 0$. Specifically, the slope of an indifference curve is; i) negative at $\Delta_3 = \Delta_2$, and ii) positive at $\Delta_3 = 0$ and $\Delta_2 > 0$. The ICM constraint is satisfied for all values of $\Delta_2$ above the ICM-curve. Since the ICM-curve is horizontal, the first fact implies that benefits at the tangency must satisfy $\Delta_3 < \Delta_2 \Leftrightarrow b_3 > b_2$. The second fact implies that at the tangency, $\Delta_3 > 0$, implying a strictly positive search incentive also for the long-term unemployed.

To conclude, the tangent to the ICM constraint ($\Delta_2 \geq rm$) must be at a point where $\Delta_2 > \Delta_3 > 0$, implying $b_2 - s < b_3 - s < w$. To understand these results, note that when $\Delta_3 = 0$ while $\Delta_2 = rm$, long-term unemployed are as well of as the employed (given assets) and their expected marginal utility is relatively low. A reallocation from long-term to short-term benefits therefore increases the value of the insurance so the tax-cost of providing a given insurance value can be reduced. Thus, indifference curves have positive slopes at $\Delta_3 = 0, \Delta_2 > 0$. On the other hand, when $\Delta_2 = \Delta_3$ (i.e., when $b_2 = b_3$) the opposite happens. The expected marginal utility of a long-run unemployed is larger than for a short term unemployed, as assets are depleted during the unemployment spell (see Hassler and Rodríguez Mora (1999) for more on this). A reallocation from long-term to short-term benefits therefore increases the overall value of the insurance.

The economic reason for our results can now be phrased in the following way; To separate individuals who have the option to move from those who have not, a positive $\Delta_2$ is required. However, this does not call for an inefficient structure of the benefit schedule. Specifically, starting from a flat benefit schedule, the welfare
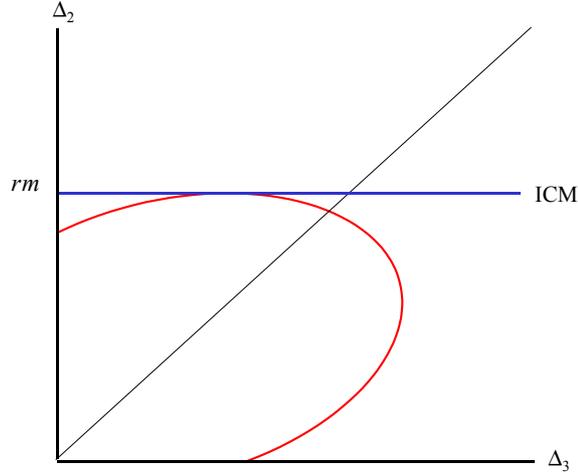
Figure 1: Indifference curve (constant $\sigma_1$) and Incentive Constraint for Moving

in all states can be increased, while maintaining the necessary wedge $\Delta_2 = rm$, by increasing benefits for long-term unemployed and reducing benefits for short-term unemployed. The reason for this is that expected marginal utility is higher for individuals who have been unemployed for a long time. The optimum is, however, reached before benefits to long-term unemployed are high enough to make the latter indifferent between having a job and staying unemployed.

Now, let us derive closed-form solutions to our problem. Using the binding ICM condition $\Delta_2 = rm$ to substitute for $\Delta_2$, the objective function, $\sigma_1$, can be rewritten as $w - \tau - q\frac{e^{\gamma rm}-1}{\gamma r}$ where everything except $\tau$ is exogenous. In other words, the problem is to minimize taxes over $\Delta_3$, respecting actuarial fairness and that benefits must be consistent with the chosen $\Delta_3$ and $\Delta_2 = rm$. After removing constants from the objective function, the problem can then be written

$$\max_{\Delta_3 \in R^+} \left\{ \Pi_3 \left( \Delta_3 - h\frac{e^{-\gamma\Delta_3}}{\gamma r} \right) - \Pi_2 f\frac{e^{\gamma(\Delta_3-rm)}}{\gamma r} \right\}. \tag{15}$$

These terms have straightforward interpretations; the first term is due to the benefit of reducing the tax-cost of long-term benefits. This term is increasing in $\Delta_3$ since higher $\Delta_3$ is achieved by lower benefits for long-term unemployed, which

reduces taxes in proportion to the ADP of long term unemployment $\Pi_3$. Note that this tax reduction comes from two sources; there is a direct effect that is proportional to $\Delta_3$ but there is also a indirect effect, captured by the second term inside the parenthesis. Long-term unemployed find jobs at a positive rate $h$. The prospect of finding a job keeps up consumption so that it falls less than proportionally to the reduction in benefits. Conversely, given an increase in $\Delta_3$, benefits can be reduced more than proportionally.

The second term in (15) is due to the benefit of reducing tax cost of short-term benefits. It is decreasing in $\Delta_3$ since less consumption for long-term unemployed has a negative impact on consumption also of the short-term unemployed, proportional to $f$. As $\Delta_3$ increases, benefits to the short-term unemployed must therefore increase to keep $\Delta_2 = rm$. This has a tax-cost proportional to the ADP of short-run unemployment $\Pi_2$.

The second derivative of (15) with respect to $\Delta_3$ is strictly negative, the first derivative is strictly positive when $\Delta_3 = 0$, and strictly negative for $\Delta_3 = rm$. Thus, the unique solution to the problem is obtained by the solution to the first-order condition, given by

$$\Delta_3^* = -\frac{\ln\left(\sqrt{\left(\frac{r}{2h}\right)^2 + e^{-\gamma rm}\left(\frac{h+r}{h}\right)} - \frac{r}{2h}\right)}{\gamma} > 0,$$

which implies (from 8), that

$$b_3^* - b_2^* = rm - \Delta_3^* + \left(f + he^{-\gamma\Delta_3}\right)\frac{1 - e^{-\gamma\left(rm - \Delta_3^*\right)}}{\gamma r} > 0, \qquad (16)$$

where stars denote optimal values.

Notice also that since the solutions for $\Delta_3$ and $b_3$ are independent of $f$, we see that $b_2$ falls monotonically in $f$. That is, as the duration of the short-term unemployment spell falls, the difference $b_3 - b_2$ should increase.[11]

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[11]It can be shown that the derivative of the objective function with respect to $f$ is always positive. Low values of $f$ is an inefficient way of inducing separation between those who can move and those who cannot, as agents expect to spend a longer stochastic time suffering the low short-

As is clear from the analysis above, a reduction in $m$ reduces $\Delta_2$ and allows more generous unemployment insurance. Such a reduction could be achieved by subsidies to moving or retraining. However, full compensation is unlikely to be optimal in reality. Suppose, realistically, that individuals with a job sometimes experience a preference or productivity shock, making another job or a job in another location more attractive than the current. Suppose also, that these shocks are not large enough to induce voluntary separation and moving if the individual has to pay the moving cost herself. Clearly, such moves are then not socially optimal.

The insurer would now like to fully subsidize the moving cost of individuals who involuntarily are separated from their job, but not subsidize it for individuals who voluntary separate in order to claim the subsidy. This, however, is infeasible if the insurer cannot distinguish voluntary and involuntary separations. Therefore, we argue that although partial subsidies may be feasible and, in fact, optimal, full subsidization is not. More specifically, it seems clear that subsidies should be as large as possible, without inducing inefficient voluntary separation. We could thus interpret $m$ as the cost of moving or retraining net of the optimal subsidy. Although this point could be formalized by introducing preference shocks along the lines just described, we abstain in order not to complicate the analysis unnecessarily.

## 3.1 Search costs

We can now easily analyze the conditions such that IC2 and IC3 are satisfied despite positive search costs. Graphically, the constraints are simply horizontal and vertical lines and all values of $\Delta_2(\Delta_3)$ above (to the right of) these lines imply that the respective constraints are satisfied. If search costs are sufficiently small, specifically,

run benefits. Without showing this formally, we conjecture that if lump-sum benefits were allowed, the best policy would be to punish unemployment by a lump-sum unemployment tax when an individual becomes unemployed. In reality, however, it may be politically diffcult or even infeasible to implement a lump-sum punishment on those who loose their jobs. Similarly, a lower bound on $b_2$ might be imposed for political reasons, in which case this would pin down $f$ from (16).

if

$$\Delta_3^* \geq \hat{\Delta}\left(\frac{rs}{h}; \gamma\right) \tag{17}$$

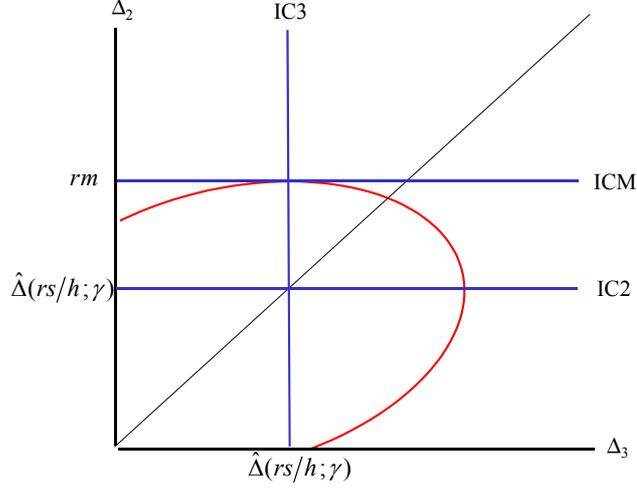none of the search constraints bind, as shown in figure 2.



Figure 2: Low search costs.

Increasing search costs shift out IC2 and IC3 and eventually, (17) is no longer satisfied at the point where the ICM constraint is tangent to the indifference curve. This situation is depicted in figure 3. Here, the point where the ICM is tangent to the dotted indifference curve satisfies the IC2 constraint, but not the IC3 constraint. Thus, $\Delta_3$ must be increased but since the IC3 and the ICM constraint are orthogonal, $\Delta_2$ need not be changed. The optimal point is where the ICM and the IC3 constraint cross and, clearly, $\Delta_3$ remains smaller than $\Delta_2$ implying $b_2 < b_3$. Specifically, $\Delta_2$ should be set equal to $rm$ and equal $\Delta_3$ to $\hat{\Delta}\left(\frac{rs}{h}; \gamma\right)$. This means that individuals will be indifferent in the choice of moving and that long-term unemployed are indifferent to searching, while the short-term unemployed strictly prefer to search.

A further increase in search costs will eventually call for a situation like in graph 4. Here both search constraints bind, while the moving constraint is slack. Benefits
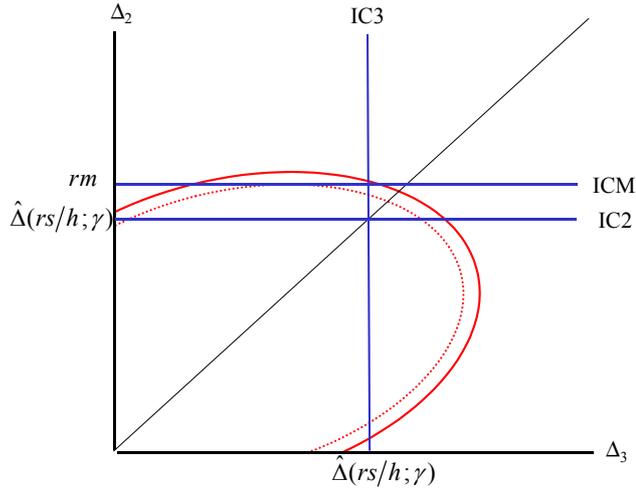
18

Figure 3: Moderate search costs.

are constant over time since $\Delta_2 = \Delta_2 = \hat{\Delta}\left(\frac{rs}{h}; \gamma\right).$[12]

# 4    Conclusion

In this paper, we have argued that there are reasons to believe that an important moral hazard problem associated with unemployment insurance has been neglected in the previous literature. This problem stems from the fact that unemployed individuals sometimes have the option to make an up-front investment that could increase their chances of finding a job. Examples of such investments are retraining and moving to another location. Since it is reasonable to assume that it is difficult or impossible to observe who has these options, the UI system should give incentives for people to take advantage of any reasonable option to increase their labor market prospect. By deriving analytical closed-form solutions for the optimal two-tier system, we have shown that such incentives can be provided without reducing the value of the unemployment insurance excessively. This requires an initial

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[12]This is special case of the result in Werning (2002) who shows that constant benefits are optimal under CARA utility in a general class of UI-schemes.
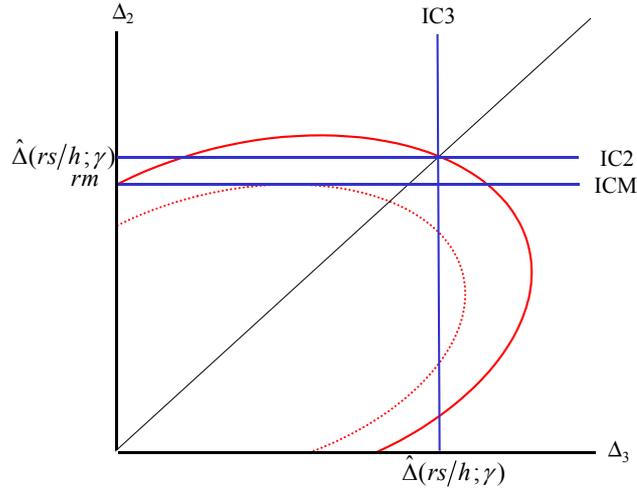
19

Figure 4: High search costs.

period of relatively low benefits. The intuition here is straightforward, by setting initial benefits at a low level, individuals with good opportunities to get new jobs are induced to exploit these. On the other hand, individuals with worse opportunities value insurance against long-term unemployment more than insurance against short-term unemployment. The value of the UI system can therefore be maintained by providing generous benefits after the initial period.

We have assumed that individuals can self-insure via unobservable savings, i.e., that individual consumption is unobservable or uncontractable. If the insurer has control over the consumption of the individual, it is well known that there would be a tendency to provide a downward sloping path of consumption (and benefits, if the individual has no other income)to provide good search incentives. Nevertheless, the point of this paper, that a period of low initial UI benefits is an efficient way to separate individuals who can move from those who cannot would still be true. Which of the two effects dominates would depend on how important the two different incentive constraints are. In a working paper version of this paper we provide a model in which both effects cancel, so that constant benefits are optimal.

We also assume constant absolute risk-aversion in this paper. This representa-

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tion of individual preferences is not necessarily the most realistic. Let us therefore speculate on the consequences of allowing constant relative risk-aversion. In such a case, the analysis is greatly complicated by the fact that, in general, search incentives would depend on asset holdings. Therefore, incentive compatibility would not in general be consistent with a finite number of benefits that are independent of individual asset holdings. However, the intuition for the results in this paper appear not to be related to such effects. In our model the preference for increasing benefits arises from the need to separate between the two types of workers and the fact that individual assets are depleted during unemployment, (which is true for general specifications of utility, in particular for CRRA, as shown in e.g., Hassler and Rodríguez Mora (1999)). Both mechanisms are likely to be present also under more general preference specifications. However, since search incentives in general depend on asset holdings and the duration of unemployment is likely to be correlated with the individual's asset holdings, unobservability of the latter may have consequences for optimal benefit time profiles. For example, if the search incentives are reinforced as wealth decumulates and individuals with long unemployment spells are likely to have less wealth, this might call for increasing benefits. The analysis of optimal UI design with hidden savings when individual behavior depends on asset holdings is likely to demand numerical models. We leave this for future research.

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# 5 Appendix

## 5.1 The IC2 condition

The IC2 constraint is given by

$$\sigma_2 - \sigma_{2,n} \geq 0.$$

Furthermore,

$$
\begin{aligned}
\sigma_2 - \sigma_{2,n} &= \left( -s + \frac{h\left(1 - e^{-\gamma\Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - e^{-\gamma(\sigma_3 - \sigma_{2,n})}\right)}{\gamma r} \right) \quad (18) \\
&= \left( -s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r}e^{\gamma(\Delta_3 - \Delta_2)}\left(1 - e^{-\gamma(\sigma_2 - \sigma_{2,n})}\right) \right) \\
&\equiv R\left(\sigma_2 - \sigma_{2,n}\right)
\end{aligned}
$$

Clearly, $R$ is a monotonously decreasing function that has an horizontal asymptote at $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) - \frac{f}{\gamma r}e^{\gamma(\Delta_3 - \Delta_2)}$ (achieved as $\sigma_2 - \sigma_{2,n}$ approaches infinity), approaches infinity as $\sigma_2 - \sigma_{2,n}$ approaches minus infinity and $R(0) = -s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right)$. The solution to (18) is the unique fixed-point of $R$. This value is non-negative if and only if $-s + \frac{h}{\gamma r}\left(1 - e^{-\gamma\Delta_2}\right) \geq 0$. So

$$\sigma_2 \geq \sigma_{2,n} \Leftrightarrow \Delta_2 \geq -\frac{\ln\left(1 - \frac{\gamma r s}{h}\right)}{\gamma} = \hat{\Delta}(h)$$

is true

QED

# 6 Proofs not intended for publication

## 6.1 Proof that results extend to n unemployment states

Suppose we have $n$, states, then the consumption constants are

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$$\sigma_1 = w - \tau - q\frac{pe^{\gamma rm} + (1-p)\,e^{\gamma(\Delta_2)} - 1}{\gamma r}, \tag{19}$$

$$\sigma_2 = b_2 - s - \tau + h\frac{1 - e^{-\gamma(\Delta_2)}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r},$$

$$\sigma_3 = b_3 - s - \tau + h\frac{1 - e^{-\gamma(\Delta_3)}}{\gamma r} - f_3\frac{e^{\gamma(\Delta_4 - \Delta_3)} - 1}{\gamma r} \tag{20}$$

$$\ldots \tag{21}$$

$$\sigma_{n-1} = b_{n-1} - s - \tau + h\frac{1 - e^{-\gamma(\Delta_{n-1})}}{\gamma r} - f_{n-1}\frac{e^{\gamma(\Delta_n - \Delta_{n-1})} - 1}{\gamma r} \tag{22}$$

$$\sigma_n = b_n - s - \tau + h\frac{1 - e^{-\gamma(\Delta_n)}}{\gamma r},$$

Now, $\tau = \sum_{s=2}^{n} b_s \Pi_s$, and the ICM constraint is binding, so $\Delta_2 = rm$, implying that we should minimize taxes. Using the above, and $\Delta_2 = rm$ we have

$$\Delta_2 = w - b_2 + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma rm}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r}$$

$$\Delta_3 = w - b_3 + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma \Delta_3}}{\gamma r} + f_3\frac{e^{\gamma(\Delta_4 - \Delta_3)} - 1}{\gamma r}$$

$$\ldots$$

$$\Delta_{n-1} = w - b_{n-1} + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma \Delta_{n-1}}}{\gamma r} + f_{n-1}\frac{e^{\gamma(\Delta_n - \Delta_{n-1})} - 1}{\gamma r}$$

$$\Delta_n = w - b_n + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma(\Delta_n)}}{\gamma r}$$

or

$$b_2 = w - \Delta_2 + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma rm}}{\gamma r} + f_2\frac{e^{\gamma(\Delta_3 - \Delta_2)} - 1}{\gamma r}$$

$$b_3 = w - \Delta_3 + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma \Delta_3}}{\gamma r} + f_3\frac{e^{\gamma(\Delta_4 - \Delta_3)} - 1}{\gamma r}$$

$$\ldots$$

$$b_{n-1} = w - \Delta_{n-1} + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma \Delta_{n-1}}}{\gamma r} + f_{n-1}\frac{e^{\gamma(\Delta_n - \Delta_{n-1})} - 1}{\gamma r}$$

$$b_n = w - \Delta_n + s - q\frac{e^{\gamma rm} - 1}{\gamma r} - h\frac{1 - e^{-\gamma(\Delta_n)}}{\gamma r}$$

$$\tau \;=\; \Pi_2\left(w - rm + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma rm}}{\gamma r} + f\frac{e^{\gamma(\Delta_3-rm)}-1}{\gamma r}\right)$$

$$+\sum_{i=3}^{n-1}\Pi_3\left(w - \Delta_i + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma\Delta_i}}{\gamma r} + f_s\frac{e^{\gamma(\Delta_{i+1}-\Delta_i)}-1}{\gamma r}\right)$$

$$+\Pi_n\left(w - \Delta_n + s - q\frac{e^{\gamma rm}-1}{\gamma r} - h\frac{1-e^{-\gamma\Delta_n}}{\gamma r}\right)$$

Removing constants,

$$\tau \;=\; \text{constant} + \Pi_2\left(f\frac{e^{\gamma(\Delta_3-rm)}}{\gamma r}\right)$$

$$+\sum_{i=3}^{n-1}\Pi_i\left(-\Delta_i + h\frac{e^{-\gamma\Delta_i}}{\gamma r} + f_s\frac{e^{\gamma(\Delta_{i+1}-\Delta_i)}}{\gamma r}\right)$$

$$+\Pi_n\left(-\Delta_n + h\frac{e^{-\gamma\Delta_n}}{\gamma r}\right)$$

First order conditions are

$$\Delta_{i\in\{3,n-1\}};\Pi_{i-1}\frac{f_{i-1}}{r}e^{\gamma(\Delta_i-\Delta_{i-1})} - \Pi_i\left(1 + \frac{h}{r}e^{-\gamma\Delta_{i-1}} + \frac{f_i}{r}e^{\gamma(\Delta_i-\Delta_{i-1})}\right) \;=\; 0$$

$$\Delta_n;\Pi_{n-1}\frac{f_{n-1}}{r}e^{\gamma(\Delta_n-\Delta_{n-1})} - \Pi_n\left(1 + \frac{h}{r}e^{-\gamma\Delta_n}\right) \;=\; 0,$$

where $\Delta_2 = rm$.

Suppose this is satisfied for $\Delta_3 = \Delta_4 = ...\Delta_n = \Delta$.Then,

$$e^{\gamma(\Delta-rm)} \;=\; \frac{r\Pi_3}{f_2\Pi_2}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{f_3}{r}\right)$$

$$\frac{f_{i-1}}{r} \;=\; \frac{\Pi_i}{\Pi_{i-1}}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{f_i}{r}\right)$$

$$\frac{f_{n-1}}{r} \;=\; \frac{\Pi_n}{\Pi_{n-1}}\left(1 + \frac{h}{r}e^{-\gamma\Delta}\right)$$

or

$$e^{\gamma(\Delta-rm)} \;=\; \frac{r\Pi_3}{f_2\Pi_2}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{\Pi_4}{\Pi_3}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{f_4}{r}\right)\right)$$

$$=\; \frac{r\Pi_3}{f_2\Pi_2}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{\Pi_4}{\Pi_3}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{\Pi_5}{\Pi_4}\left(1 + \frac{h}{r}e^{-\gamma\Delta} + \frac{f_5}{r}\right)\right)\right)$$

$$=\; \frac{r}{f_2}\left(1 + \frac{h}{r}e^{-\gamma\Delta}\right)\left(\left(\frac{\Pi_3}{\Pi_2} + \frac{\Pi_4}{\Pi_3} + ... + \frac{\Pi_{n-1}}{\Pi_{n-2}}\right) + \frac{\Pi_n}{\Pi_2}\right)$$

$$=\; \frac{r}{f_2}\left(1 + \frac{h}{r}e^{-\gamma\Delta}\right)\left(\sum_{i=3}^{n-1}\frac{\Pi_i}{\Pi_{i-1}} + \frac{\Pi_n}{\Pi_2}\right)$$

Clearly, there exists a $\Delta^*$ such that this is satisfied, consequently $\Delta_i = \Delta^* \forall i \in \{3, 4, ..., n\}$ satisfies all the first-order conditions. This allocation is implemented by a $\tilde{b}_2^*$ and a constant benefit sequence $\tilde{b}_3^* = \tilde{b}_4^* = ... \tilde{b}_n^*$ thereafter. Finally, we note that since individuals face identical conditions in states $3, ...n$, the allocation would not change if the number of states is reduced as long as $n > 3$. Thus, the optimal value of $b_2$ is independent of $n$ if $n > 3$. Consequently, the optimal benefit schedule is to have $b_2 = b_2^*$ and a constant benefit level $b_3 = b_3^*$ thereafter.

## 6.2 Finding value functions

Guessing that the value function is $-e^{-\gamma(rA_t+\sigma_j)}$ for $j \in \{1, 2, 3\}$, the Bellman equation for the employed is,

$$-\frac{1}{r}e^{-\gamma(rA_t+\sigma_1)} = \max_{\sigma} -e^{-\gamma(rA_t+\sigma)}dt$$
$$- (1 - rdt)\left[(1 - qdt)\frac{1}{r}e^{-\gamma(rA_{t+dt}+\sigma_1)} + qdt\frac{1}{r}e^{-\gamma(rA_{t+dt}+\sigma_2)}\right].$$

Using first-order linear approximations and dividing by $e^{-\gamma rA_t}$, this becomes

$$-\frac{1}{r}e^{-\gamma\sigma_1} = \max_{\sigma} -e^{-\gamma\sigma}dt$$
$$- (1 - rdt)\left[(1 - qdt)\frac{1}{r}e^{-\gamma\sigma_1}(1 - \gamma r(w - \tau - \sigma)dt) + qdt\frac{1}{r}e^{-\gamma\sigma_2}(1 - \gamma r(w - \tau - \sigma)dt)\right]$$

Adding $\frac{1}{r}e^{-\gamma\sigma_1}$ to both sides, dividing by $dt$ and letting $dt$ approach zero, yields

$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_1)} + r + \gamma r(w - \tau - \sigma) + q\left(1 - e^{-\gamma(\sigma_2-\sigma_1)}\right)\right\}. \qquad (23)$$

Similarly, for the short-term and long-run unemployed, we obtain

$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_2)} + r + \gamma r(b_2 - s - \tau - \sigma) + h + f - he^{-\gamma(\sigma_1-\sigma_2)} - fe^{-\gamma(\sigma_3-\sigma_2)}\right\},$$
$$(24)$$

$$0 = \max_{\sigma}\left\{-re^{-\gamma(\sigma-\sigma_3)} + r + \gamma r(b_3 - s - \tau - \sigma) + h\left(1 - e^{-\gamma(\sigma_1-\sigma_3)}\right)\right\}.$$

Equations (23) and (24) are maximized at $\sigma = \sigma_j$, implying that for the Bellman

equation to be satisfied, the constants $\sigma_j$, must satisfy

$$\sigma_1 = w - \tau - \frac{q\left(e^{\gamma \Delta_2} - 1\right)}{\gamma r}$$

$$\sigma_2 = b_2 - s - \tau + \frac{h\left(1 - e^{-\gamma \Delta_2}\right)}{\gamma r} - \frac{f\left(e^{\gamma(\Delta_3 - \Delta_2)} - 1\right)}{\gamma r}$$

$$\sigma_3 = b_3 - s - \tau + \frac{h\left(1 - e^{-\gamma \Delta_3}\right)}{\gamma r}.$$

## 6.3 Derivation of 13

Doing the substitution in the text and collecting endogenous terms, we have

$$
\begin{aligned}
\sigma_1 &= w - \Pi_2\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - (h + f)\frac{1}{\gamma r}\right) - \Pi_3\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r}\right) - q\frac{pe^{\gamma rm} - 1}{\gamma r} \\
&\quad - \Pi_2\left(-\Delta_2 - q\frac{(1 - p)e^{\gamma \Delta_2}}{\gamma r} + h\frac{e^{-\gamma \Delta_2}}{\gamma r} + f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r}\right) \\
&\quad - \Pi_3\left(-\Delta_3 - q\frac{(1 - p)e^{\gamma \Delta_2}}{\gamma r} + h\frac{e^{-\gamma \Delta_3}}{\gamma r}\right) - q\frac{(1 - p)e^{\gamma \Delta_2}}{\gamma r}
\end{aligned}
$$

Dividing by $\Pi_2$, and defining

$$
\begin{aligned}
K &\equiv \frac{w - \Pi_2\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - (h + f)\frac{1}{\gamma r}\right)}{\Pi_2} \\
&\quad - \frac{\Pi_3\left(w + s - q\frac{pe^{\gamma rm} - 1}{\gamma r} - \frac{h}{\gamma r}\right) + q\frac{pe^{\gamma rm} - 1}{\gamma r}}{\Pi_2} \\
&= w\frac{(1 - \Pi_2 - \Pi_3)}{\Pi_2} - \frac{(\Pi_2 + \Pi_3)}{\Pi_2}s \\
&\quad - \frac{1 - \Pi_2 - \Pi_3}{\Pi_2}\frac{q}{r}\frac{pe^{\gamma rm} - 1}{\gamma} + \frac{1}{\gamma r}\left(h + f\left(1 + \frac{h}{h + r}\right)\right)
\end{aligned}
$$

we get

$$
\begin{aligned}
\frac{\sigma_1}{\Pi_2} &= K + \Delta_2 + \frac{f}{h + r}\Delta_3 + q\frac{(1 - p)e^{\gamma \Delta_2}}{\gamma r}\left(1 - \frac{1}{\Pi_2} + \frac{f}{h + r}\right) \\
&\quad - h\frac{e^{-\gamma \Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r} - \frac{f}{h + r}h\frac{e^{-\gamma \Delta_3}}{\gamma r} \\
&= K + \Delta_2 + \frac{f}{h + r}\Delta_3 - q\frac{(1 - p)e^{\gamma \Delta_2}}{\gamma r}\frac{r + h + f}{(1 - p)q} \\
&\quad - h\frac{e^{-\gamma \Delta_2}}{\gamma r} - f\frac{e^{\gamma(\Delta_3 - \Delta_2)}}{\gamma r} - \frac{f}{h + r}h\frac{e^{-\gamma \Delta_3}}{\gamma r}
\end{aligned}
$$

In the following figure, we plot this, for $r = 0.05, f = 1, \gamma = 1, h = 1$ against $\Delta_2, \Delta_3$ viewing it from above an cutting all values above -100.2. As we see, the isoquant has an elliptical form.