A Generalized Spatial Panel Data Model with Random Effects

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Abstract

This paper proposes a generalized specification for the panel data model with random effects and first-order spatially autocorrelated residuals that encompasses two previously suggested specifications. The first one is described in Anselin's (1988) book and the second by Kapoor, Kelejian, and Prucha (2004). Our encompassing specification allows us to test these models as restricted specifications. In particular, we derive three LM and LR tests that restrict our generalized model to obtain (i) the Anselin model, (ii) the Kapoor, Kelejian, and Prucha model, and (iii) the simple random effects model that ignores the spatial correlation in the residuals. For two of these three LM

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tests, we obtain closed form solutions. Our Monte Carlo results show that the suggested tests are powerful in testing for these restricted specifications even in small and medium sized samples.

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1 Introduction

The recent literature on spatial panels distinguishes between two different spatial autoregressive error processes. One specification assumes that spatial correlation occurs only in the remainder error term, whereas no spatial correlation takes place in the individual effects (see Anselin, 1988, Baltagi, Song, and Koh, 2003, and Anselin, Le Gallo, and Jayet, 2005; henceforth referred to as the Anselin model). Another specification assumes that the same spatial error process applies to both the individual and remainder error components (see Kapoor, Kelejian, and Prucha, 2004; henceforth referred to as the KKP model).

While the two data generating processes look similar, they imply different spatial spillover mechanisms. For example, consider the question of firm productivity using panel data. Besides the deterministic components, firms differ also with respect to their unobserved know-how or their managerial ability to organize production processes efficiently. At least over a short time period, this managerial ability may be time-invariant. Beyond that there are innovations that vary from period to period like random firm-specific technology shocks, capacity utilization shocks, etc. Under this scenario, it seems reasonable to assume that firm productivity may be spatially correlated due to spillovers. Such spillovers can occur, e.g., through information flows (transmission of process technologies) embodied in worker flows between firms at local labor markets or through input-output channels (technology requirements and interdependence of capacity utilization). Whereas the Anselin model assumes that spillovers are inherently time-varying, the KKP process assumes the spillovers to be time-invariant as well as time-variant. For example, firms located in the neighborhood of highly productive firms may get time-invariant permanent spillovers affecting their productivity in addition to the time-variant spillovers as in the Anselin model. While the Anselin model seems restrictive in that it does not allow permanent spillovers through the individual firm effects, the KKP approach is restrictive in the sense that it does not allow for a differential intensity of spillovers of the permanent and transitory shocks.

This paper introduces a generalized spatial panel model which encompasses these two models and allows for spatial correlation in the individual and remainder error components that may have different spatial autoregressive parameters. We derive the maximum likelihood estimator (MLE) for this more general spatial panel model when the individual effects are assumed to be random. This in turn allows us to test the restrictions on our generalized model to obtain (i) the Anselin model, (ii) the Kapoor, Kelejian, and Prucha model, and (iii) a simple random effects model that ignores the spatial correlation in the residuals. We derive the corresponding LM and LR tests for these three hypotheses and we compare their size and power performance using Monte Carlo experiments.

2 A Generalized Model

Econometric models for panel data with spatial error processes have been proposed by Anselin (1988), Baltagi, Song, and Koh (2003), Kapoor, Kelejian, and Prucha (2004) and Anselin, Le Gallo, and Jayet (2005), to mention a few. A generalized spatial panel model that encompasses these previous specifications as special cases is given by:

$$\begin{aligned}
\mathbf{y} &= \mathbf{X}\beta + \mathbf{u} \\
\mathbf{u} &= \mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2 \\
\mathbf{u}_1 &= \rho_1 \mathbf{W}_N \mathbf{u}_1 + \mu \\
\mathbf{u}_2 &= \rho_2 \mathbf{W} \mathbf{u}_2 + \nu,
\end{aligned} \tag{1}$$

where $\mathbf{y}$ is the dependent variable of dimension $(n \times 1)$ with $n = NT$ denoting the number of observations. N is the number of unique cross-sectional units, while T is the number of time periods. The $(n \times K)$ matrix $\mathbf{X}$ comprises the set of exogenous variables. β is the corresponding $(K \times 1)$ parameter vector. The error component structure is given by the second equation with $\mathbf{Z}_\mu = \iota_T \otimes \mathbf{I}_N$ denoting the selector matrix for the $(N \times 1)$ random vector of individual effects $\mathbf{u}_1$. Here ι_T is a vector of ones of dimension T and $\mathbf{I}_N$ is an identity matrix of dimension N. The vector of individual effects μ is assumed to be $i.i.d.(0, \sigma^2_\mu \mathbf{I}_N)$, while the $(n \times 1)$ vector of remainder disturbances ν is assumed to be $i.i.d.(0, \sigma^2_\nu \mathbf{I}_n)$. In addition, μ and ν are assumed to be independent of each other. Both $\mathbf{u}_1$ and $\mathbf{u}_2$ are spatially correlated with the same spatial weight matrix $\mathbf{W}_N$ for each time period, but with different spatial autocorrelation parameters $\rho_1$and ρ_2, respectively. Ordering the data first by time (with index $t = 1, ..., T$) and then by individual units (with index $i = 1, ..., N$), we get $\mathbf{W} = \mathbf{I}_T \otimes \mathbf{W}_N$, where the $(N \times N)$ spatial weight matrix $\mathbf{W}_N$ has zero diagonal elements and is row-normalized with its entries usually declining with distance. This in turn results in row and column sums of $\mathbf{W}_N$ that are uniformly bounded in absolute value.[1] We also assume that

[1]Row-normalization is sufficient but not necessary to achieve uniform boundedness of the row sums of $\mathbf{W}_N$. Kelejian and Prucha (2005) argue that one can alternatively

ρ_r is bounded in absolute value, i.e., $|\rho_r| < 1$ for $r = 1, 2$.

This model encompasses both the KKP model, which assumes that $\rho_1 = \rho_2$, and the Anselin model, which assumes that $\rho_1 = 0$. When $\rho_1 = \rho_2 = 0$, i.e., there is no spatial correlation, this model reduces to the familiar random effects (RE) panel data model; see Baltagi (2005).

Let $\mathbf{A} = (\mathbf{I}_N - \rho_1 \mathbf{W}_N)$ and $\mathbf{B} = (\mathbf{I}_N - \rho_2 \mathbf{W}_N)$, then

$$\mathbf{u}_1 = \mathbf{A}^{-1}\mu \sim (0, \sigma_\mu^2 (\mathbf{A}'\mathbf{A})^{-1}) \tag{2}$$

$$\mathbf{u}_2 = (\mathbf{I}_T \otimes \mathbf{B}^{-1})\nu \sim (0, \sigma_\nu^2 (\mathbf{I}_T \otimes (\mathbf{B}'\mathbf{B})^{-1})). \tag{3}$$

Observe that $tr(\mathbf{A}) = tr(\mathbf{B}) = N$, since $tr(\mathbf{W}_N) = 0$. Furthermore, $\mathbf{A}'\mathbf{A} = \mathbf{I}_N - \rho_1 \mathbf{W}'_N - \rho_1 \mathbf{W}_N + \rho_1^2 \mathbf{W}'_N \mathbf{W}_N$, with $tr(\mathbf{A}'\mathbf{A}) = N + \rho_1^2 tr(\mathbf{W}'_N \mathbf{W}_N)$ and $tr(\mathbf{B}'\mathbf{B}) = N + \rho_2^2 tr(\mathbf{W}'_N \mathbf{W}_N)$. The variance-covariance matrix of the spatial random effects panel data model is given by

$$\begin{aligned} \mathbf{\Omega}_u &= E(\mathbf{u}\mathbf{u}') = E[(\mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2)(\mathbf{Z}_\mu \mathbf{u}_1 + \mathbf{u}_2)'] \qquad (4) \\ &= \sigma_\mu^2 (\mathbf{J}_T \otimes (\mathbf{A}'\mathbf{A})^{-1}) + \sigma_\nu^2 (\mathbf{I}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}) \\ &= \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 (\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2 (\mathbf{B}'\mathbf{B})^{-1}] + \sigma_\nu^2 (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}) = \sigma_\nu^2 \mathbf{\Sigma}_u. \end{aligned}$$

This uses the fact that $E[\mathbf{u}_1 \mathbf{u}'_2] = \mathbf{0}$ since μ and ν are assumed to be independent. Note that $\mathbf{Z}_\mu \mathbf{Z}'_\mu = \mathbf{J}_T \otimes I_N$ where $\mathbf{J}_T$ is a matrix of ones of dimension T. Let $\mathbf{E}_T = \mathbf{I}_T - \overline{\mathbf{J}}_T$, where $\overline{\mathbf{J}}_T = \mathbf{J}_T / T$ is the averaging matrix, the last equality replaces $\mathbf{J}_T$ by $T\overline{\mathbf{J}}_T$ and $\mathbf{I}_T$ by $\mathbf{E}_T + \overline{\mathbf{J}}_T$; see Wansbeek and Kapteyn (1982). It is easy to show that the inverse of the $(n \times n)$ matrix $\mathbf{\Omega}_u$ can be obtained from the inverse of smaller dimension $(N \times N)$ matrices as follows:

$$\mathbf{\Omega}_u^{-1} = \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 (\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2 (\mathbf{B}'\mathbf{B})^{-1}]^{-1} + \frac{1}{\sigma_\nu^2} (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})) = \frac{1}{\sigma_\nu^2} \mathbf{\Sigma}_u^{-1},$$

normalize by the maximum row sum of the spatial weights matrix.

where

$$\mathbf{\Sigma}_u^{-1} = (\bar{\mathbf{J}}_T \otimes [T\frac{\sigma_\mu^2}{\sigma_\nu^2}(\mathbf{A}'\mathbf{A})^{-1} + (\mathbf{B}'\mathbf{B})^{-1}]^{-1}) + (\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})). \tag{6}$$

Also, $\det[\mathbf{\Omega}_u] = \det[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]\det[\sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{T-1}$. Under the assumption of normality of the disturbances the log likelihood function of the unrestricted model is given by

$$\begin{aligned} L_U(\beta, \sigma_\nu^2, \sigma_\mu^2, \rho_1, \rho_2) &= -\frac{NT}{2}\ln 2\pi - \frac{1}{2}\ln\det[T\sigma_\mu^2(\mathbf{A}'\mathbf{A})^{-1} + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}] \\ &\quad -\frac{T-1}{2}\ln\det(\sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-\mathbf{1}}) - \frac{1}{2}\mathbf{u}'\mathbf{\Omega}_u^{-1}\mathbf{u}, \end{aligned}$$

where $\mathbf{u} = \mathbf{y} - \mathbf{X}\beta$. For the special case, when $\rho_1 = 0$, this implies that $\mathbf{A} = \mathbf{I}_N$ and the restricted log likelihood function reduces to the one considered by Anselin (1988, p.154).

The hypotheses under consideration in this paper are the following:

(1) $H_0^A : \rho_1 = \rho_2 = 0$, and the alternative H_1^A is that at least one component is not zero. The restricted model is the standard random effects (RE) panel data model with no spatial correlation; see Baltagi (2005).

(2) H_0^B: $\rho_1 = 0$, and the alternative is H_1^B: $\rho_1 \neq 0$. The restricted model is the Anselin (1988) spatial panel model with random effects.

(3) $H_0^C : \rho_1 = \rho_2 = \rho$ and the alternative is H_1^C: $\rho_1 \neq \rho_2$. The restricted model is the KKP spatial panel model with random effects.

In the next subsections, we derive the corresponding LM tests for these hypotheses and we compare their performance with the corresponding LR tests using Monte Carlo experiments.[2]

[2]LM tests for spatial models are surveyed in Anselin (1988, 1999) and Anselin and Bera (1998), to mention a few. For a joint test of the absence of spatial correlation and random effects in a panel data model, see Baltagi, Song, and Koh (2003).

2.1 LM and LR Tests for $H_0^A : \rho_1 = \rho_2 = 0$

The joint LM test statistic for the null hypothesis of no spatial correlation, $H_0^A : \rho_1 = \rho_2 = 0$, is derived in Appendix 1 and is given by

$$LM_A = \frac{(T-1)\widetilde{\sigma}_1^4 + \widetilde{\sigma}_\nu^4}{2b_A T^2 (T-1)\widetilde{\sigma}_1^8} G_A^2 - \frac{\widetilde{\sigma}_\nu^2}{b_A T(T-1)\widetilde{\sigma}_1^4} G_A M_A + \frac{1}{2b_A(T-1)} M_A^2, \quad (7)$$

where $\widetilde{\sigma}_1^2 = T\widetilde{\sigma}_\mu^2 + \widetilde{\sigma}_\nu^2$, $b_A = tr[(\mathbf{W}_N' + \mathbf{W}_N)^2]$, $G_A = \widetilde{\mathbf{u}}'\{\mathbf{J}_T \otimes (\mathbf{W}_N' + \mathbf{W}_N)\}\widetilde{\mathbf{u}}$, and $M_A = \widetilde{\mathbf{u}}'\{(\frac{\widetilde{\sigma}_\nu^2}{\widetilde{\sigma}_1^4}\bar{\mathbf{J}}_T + \frac{1}{\widetilde{\sigma}_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}_N' + \mathbf{W}_N)\}\widetilde{\mathbf{u}}$. In this case, $\widetilde{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widetilde{\beta}$ denotes the vector of restricted ML residuals. Under H_0^A, the restricted model is the simple random effects (RE) panel data model with no spatial autocorrelation. In fact, $\widetilde{\sigma}_\nu^2 = \frac{\widetilde{\mathbf{u}}'\{(\mathbf{E}_T \otimes \mathbf{I}_N)\}\widetilde{\mathbf{u}}}{N(T-1)}$ and $\widetilde{\sigma}_1^2 = \frac{\widetilde{\mathbf{u}}'\{(\bar{\mathbf{J}}_T \otimes \mathbf{I}_N)\}\widetilde{\mathbf{u}}}{N}$. The ML-estimates under H_0^A are labelled by a tilde. Under H_0^A, the LM_A statistic is expected to be asymptotically distributed as χ_2^2. The large sample distribution of the LM test statistics derived in this paper are not formally established. However, they are likely to hold under a set of low level assumptions developed in Kelejian and Prucha (2001) for the Moran I statistic and in Kapoor, Kelejian, and Prucha (2004) for the spatial random effects model.

One can also derive the corresponding LR test for H_0^A: $\rho_1 = \rho_2 = 0$ as

$$LR_A = 2(L_U - L_R^A),$$

using the restricted log likelihood

$$L_R^A = -\frac{NT}{2}\ln 2\pi\widetilde{\sigma}_\nu^2 - \frac{N}{2}\ln\frac{\widetilde{\sigma}_1^2}{\widetilde{\sigma}_\nu^2} - \frac{1}{2}\widetilde{\mathbf{u}}'\widetilde{\mathbf{\Omega}}_u^{-1}\widetilde{\mathbf{u}}.$$

This test statistic is likewise expected to be asymptotically distributed as χ_2^2.

2.2 LM and LR Tests for H_0^B: $\rho_1 = 0$

Under H_0^B: $\rho_1 = 0$, the restricted model is the spatial panel data model with random effects described in Anselin (1988). The corresponding LM test for H_0^B is a conditional test for zero spatial correlation in the individual effects, allowing for the possibility of spatial correlation in the remainder error term, i.e., $\rho_2 \neq 0$. Appendix 2 gives the formal derivation of this LM statistic. In fact, under H_0^B, the information matrix is block diagonal with the lower block being independent of β. Let $\mathbf{d}_\theta$ be the (4×1) score vector referring to the parameter vector $\theta = (\sigma_\mu^2, \sigma_\nu^2, \rho_1, \rho_2)$ and denote the 4×4 lower block of the information matrix by $\mathbf{J}_\theta$. The ML-estimates under H_0^B are labelled by a hat. The LM test for H_0^B makes use of the estimated score $\widehat{\mathbf{d}}_\theta = [0, 0, \widehat{d}_{\rho_1}, 0]'$ with

$$\widehat{d}_{\rho_1} = \left.\frac{\partial L}{\partial \rho_1}\right|_{H_0^B} = -\frac{1}{2} T \widehat{\sigma}_\mu^2 tr[\mathbf{C}_1 \mathbf{C}_2] + \frac{1}{2} \widehat{\sigma}_\mu^2 \widehat{\mathbf{u}}' \{\mathbf{J}_T \otimes \mathbf{C}_1 \mathbf{C}_2 \mathbf{C}_1\} \widehat{\mathbf{u}},$$

where $\mathbf{C}_1 = [T\widehat{\sigma}_\mu^2 \mathbf{I}_N + \widehat{\sigma}_\nu^2 (\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}]^{-1}$ and $\mathbf{C}_2 = (\mathbf{W}_N' + \mathbf{W}_N)$. An estimate of the lower (4×4) block of the information matrix $\widehat{\mathbf{J}}_\theta$ under H_0^B is given by

$$\begin{bmatrix} \frac{1}{2} tr\mathbf{C}_3^2 + \frac{N(T-1)}{2\widehat{\sigma}_\nu^4} & \frac{T}{2} tr\,[\mathbf{C}_3\mathbf{C}_1] & \frac{T\widehat{\sigma}_\mu^2}{2} tr[\mathbf{C}_3\mathbf{C}_1\mathbf{C}_2] & \frac{\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_3\mathbf{C}_1\mathbf{C}_4] + \frac{(T-1)}{2\widehat{\sigma}_\nu^2} tr[\mathbf{C}_5] \\ \frac{T}{2} tr\,[\mathbf{C}_3\mathbf{C}_1] & \frac{T^2}{2} tr\,[\mathbf{C}_1^2] & \frac{T^2\widehat{\sigma}_\mu^2}{2} tr[\mathbf{C}_1^2\mathbf{C}_2] & \frac{T\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_1^2\mathbf{C}_4] \\ \frac{T\widehat{\sigma}_\mu^2}{2} tr[\mathbf{C}_3\mathbf{C}_1\mathbf{C}_2] & \frac{T^2\widehat{\sigma}_\mu^2}{2} tr[\mathbf{C}_1^2\mathbf{C}_2] & \frac{T^2\widehat{\sigma}_\mu^4}{2} tr[\mathbf{C}_1\mathbf{C}_2]^2 & \frac{T\widehat{\sigma}_\mu^2\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_1\mathbf{C}_2\mathbf{C}_1\mathbf{C}_4] \\ \frac{\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_3\mathbf{C}_1\mathbf{C}_4] + \frac{(T-1)}{2\widehat{\sigma}_\nu^2} tr[\mathbf{C}_5] & \frac{T\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_1^2\mathbf{C}_4] & \frac{T\widehat{\sigma}_\mu^2\widehat{\sigma}_\nu^2}{2} tr[\mathbf{C}_1\mathbf{C}_2\mathbf{C}_1\mathbf{C}_4] & \frac{\widehat{\sigma}_\nu^4}{2} tr[\mathbf{C}_1\mathbf{C}_4]^2 + \frac{(T-1)}{2} tr[\mathbf{C}_5^2] \end{bmatrix},$$

where $\widehat{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widehat{\beta}$, $\mathbf{C}_3 = [T\widehat{\sigma}_\mu^2(\widehat{\mathbf{B}}'\widehat{\mathbf{B}}) + \widehat{\sigma}_\nu^2 \mathbf{I}_N]^{-1}$, $\mathbf{C}_4 = (\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}(\mathbf{W}_N'\widehat{\mathbf{B}} + \widehat{\mathbf{B}}'\mathbf{W}_N)(\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}$, $\mathbf{C}_5 = (\mathbf{W}_N'\widehat{\mathbf{B}} + \widehat{\mathbf{B}}'\mathbf{W}_N)(\widehat{\mathbf{B}}'\widehat{\mathbf{B}})^{-1}$. The LM test for H_0^B has no simple closed form representation and is calculated as

$$LM_B = \widehat{\mathbf{d}}_\theta' \widehat{\mathbf{J}}_\theta^{-1} \widehat{\mathbf{d}}_\theta = \widehat{d}_{\rho_1}^2 \widehat{\mathbf{J}}_{33}^{-1}, \tag{8}$$

where $\widehat{\mathbf{J}}_{33}^{-1}$ is the $(3,3)$ element of the inverse of the information matrix $\widehat{\mathbf{J}}_{\theta}^{-1}$ evaluated at H_0^B. This test statistic is expected to be asymptotically distributed as χ_1^2. The corresponding LR test is based upon the restricted log likelihood

$$\begin{aligned} L_R^B = &-\frac{NT}{2}\ln 2\pi\widehat{\sigma}_\nu^2 - \frac{1}{2}\ln\det[\mathbf{C}_1] \\ &+\frac{T-1}{2}\ln\det(\widehat{\mathbf{B}}'\widehat{\mathbf{B}}) - \frac{1}{2}\widehat{\mathbf{u}}'\widehat{\mathbf{\Omega}}_u^{-1}\widehat{\mathbf{u}}. \end{aligned} \tag{9}$$

This restricted log-likelihood is the same as that given by Anselin (1988, p. 154).

2.3 LM and LR Tests for $H_0^C : \rho_1 = \rho_2 = \rho$

Under $H_0^C : \rho_1 = \rho_2 = \rho$, the true model is that suggested by KKP. In this case, $\mathbf{B} = \mathbf{A}$ and the parameter estimates under H_0^C are labelled by a bar. The score and the information matrix needed for this test are derived in Appendix 3. The joint LM test statistic for H_0^C is given by

$$LM_C = \frac{(T-1)\overline{\sigma}_1^4 + \overline{\sigma}_\nu^4}{2b_C T^2(T-1)\overline{\sigma}_1^8} G_C^2 - \frac{\overline{\sigma}_\nu^2}{b_C T(T-1)\overline{\sigma}_1^4} G_C M_C + \frac{1}{2b_C(T-1)} M_C^2 \tag{10}$$

where $G_C = -T\overline{\sigma}_1^2 tr(\mathbf{D}) + \overline{\mathbf{u}}'\{\mathbf{J}_T \otimes \mathbf{F}\}\overline{\mathbf{u}}$, $M_C = -\left[\frac{\overline{\sigma}_\nu^2}{\overline{\sigma}_1^2} + (T-1)\right] tr(\mathbf{D}) + \overline{\mathbf{u}}'\{(\frac{\overline{\sigma}_\nu^2}{\overline{\sigma}_1^4}\overline{\mathbf{J}}_T + \frac{1}{\overline{\sigma}_\nu^2}\mathbf{E}_T) \otimes \mathbf{F}\}\overline{\mathbf{u}}$, $\mathbf{D} = (\mathbf{W}_N'\overline{\mathbf{A}} + \overline{\mathbf{A}}'\mathbf{W}_N)(\overline{\mathbf{A}}'\overline{\mathbf{A}})^{-1}$ and $\mathbf{F} = \mathbf{W}_N'\overline{\mathbf{A}} + \overline{\mathbf{A}}'\mathbf{W}_N$. Also, $b_C = tr(\mathbf{D}^2) - (tr(\mathbf{D}))^2/N$, $\overline{\sigma}_1^2 = \frac{\overline{\mathbf{u}}'\{\overline{\mathbf{J}}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})\}\overline{\mathbf{u}}}{N}$ and $\overline{\sigma}_\nu^2 = \frac{\overline{\mathbf{u}}'\{\mathbf{E}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})\}\overline{\mathbf{u}}}{N(T-1)}$. Under H_0^C, the LM_C statistic is asymptotically distributed as χ_1^2.

The LR test is based on the following restricted likelihood:

$$L_R^C = -\frac{NT}{2}\ln 2\pi\overline{\sigma}_\nu^2 - \frac{N}{2}\ln(\frac{\overline{\sigma}_1^2}{\overline{\sigma}_\nu^2}) + \frac{T}{2}\ln\det(\overline{\mathbf{B}}'\overline{\mathbf{B}}) - \frac{1}{2}\overline{\mathbf{u}}'\overline{\mathbf{\Omega}}_u^{-1}\overline{\mathbf{u}}.$$

Kapoor, Kelejian, and Prucha (2004) consider a Generalized Moments method of estimation, rather than MLE, for their spatial random effects panel data model. Nevertheless, L_R^C is the restricted likelihood for the KKP model.

3 Monte Carlo Results

In the Monte Carlo analysis, we use a simple panel data model that includes one explanatory variable and a constant ($K = 2$)

$$y_{it} = \beta_0 + \beta_1 x_{it} + u_{it}, \quad i = 1, ..., N \text{ and } t = 1, ..., T,$$

where $\beta_0 = 5$ and $\beta_1 = 0.5$. x_{it} is generated by $x_{it} = \zeta_i + z_{it}$, where $\zeta_i \sim i.i.d.$ $U[-7.5, 7.5]$ and $z_{it} \sim i.i.d.$ $U[-5, 5]$. The individual specific effects are drawn from a normal distribution so that $\mu_i \sim i.i.d.$ $N(0, 20\theta)$, while for the remainder error we assume $\nu_{it} \sim i.i.d.$ $N(0, 20(1-\theta))$ with $0 < \theta < 1$. This implies $\sigma_\mu^2 + \sigma_\nu^2 = 20$ and $\theta = \frac{\sigma_\mu^2}{\sigma_\mu^2 + \sigma_\nu^2}$ is the proportion of the total variance due to the heterogeneity of the individual specific effects. We generate the spatial weighting matrix by allocating observations randomly on a grid of $2N$ squares. Consequently, as the number of observations N increases, the number of squares in the grid grows larger, too. The probability that an observation is located on a particular coordinate is equal for all coordinates on the grid. The spatial weighting scheme is based on the Queens design and the corresponding spatial weighting matrix is normalized so that its rows sum to one. ρ_1 and ρ_2 vary over the set $\{-0.8, -0.5, -0.2, 0, 0.2, 0.5, 0.8\}$. We also vary the cross-sectional and time dimensions such that $N = 50, 100$ and $T = 3, 5, 10$. Lastly, θ, the proportion of the variance due to the random individual effects, takes the values $0.25, 0.50$, and 0.75. In total, this amounts to 882 experiments. For each experiment, we calculate the three LM and LR tests as derived above using 2000 replications.[3]

===== Tables 1-3 =====

Table 1 reports the frequency of rejections for $N = 50$, $T = 5$, and $\theta = 0.5$ in 2000 replications. This means that $\sigma_\mu^2 = \sigma_\nu^2 = 10$. The size of each test is denoted in bold figures and is not statistically different from the 5% nominal size. The only exception where the LM test might be undersized is for the KKP model, for high absolute values of ρ_1 and ρ_2, both equal to 0.8. The size adjusted power of the LR and LM tests is reasonably high for all three hypotheses considered. The performance of the LM test is almost the same as that of the LR test, except for a few cases. For $H_0^A : \rho_1 = \rho_2 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 61.4% as compared to 64.6% for LR. However, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70% as compared to 66.4% for LR. Similarly, for H_0^B: $\rho_1 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70.2% as compared to 72.9% for LR. However, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 76.7% as compared to 74.6% for LR. For $H_0^C : \rho_1 = \rho_2 = \rho$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 66.1% as compared to 68.5% for LR.

[3] In a few cases, we got negative LR test statistics due to numerical imprecision. These cases occur mainly with the Anselin model at $\rho_1 = 0$. However, this happened in less than 0.5 percent of the Monte Carlo experiments. We drop the corresponding experiments in the subsequent calculations of the size and power of the tests.

However, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test is 70.6% as compared to 65% for LR.

Tables 2 and 3 repeat the same experiments but now for $\theta = 0.25$ and 0.75, respectively. These tables show that as we increase θ, we increase the power of these tests. In fact, the power of all three tests is higher, the higher the variance of the individual specific effect as a proportion of the total variance. For example, for $H_0^A : \rho_1 = \rho_2 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 61.4% for $\theta = 0.5$ (in Table 1) to 68% for $\theta = 0.75$ (in Table 3), while the size adjusted power of the LR test increases from 64.6% to 74.8%. Similarly, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70% for $\theta = 0.5$ to 78.4% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 66.4% to 77.4%. For H_0^B: $\rho_1 = 0$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70.2% for $\theta = 0.5$ to 81% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 72.9% to 83.4%. However, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 76.7% for $\theta = 0.5$ to 86.6% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 74.6% to 84.9% for LR. For $H_0^C : \rho_1 = \rho_2 = \rho$, when $\rho_1 = -0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 66.1% for $\theta = 0.5$ to 73% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 68.5% to 74.8%. However, when $\rho_1 = 0.5$ and $\rho_2 = 0$, the size adjusted power of the LM test increases from 70.6% for $\theta = 0.5$ to 80.4% for $\theta = 0.75$, while the size adjusted power of the LR test increases from 65% to 77.3%.

Things also improve if the number of observations increases. The increase in power is larger when we double N from 50 to 100 as compared to doubling T from 5 to 10.[4] We conclude that the three LM and LR tests perform reasonably well in testing the restrictions underlying the simple random effects model without spatial correlation, the Anselin model and the KKP model in small and medium sized samples.

Figures 1-4 plot the size adjusted power for the various hypotheses considered. In Figure 1, the pure random effects model is true, whereas in Figure 2, the Anselin model is true. In Figures 3 and 4, the KKP-type model is true with different values for the common ρ.

===== Figures 1-2 =====

Let us start with a comparison of the panels given in Figure 1, which assumes that the random effects model is true ($\rho_1 = \rho_2 = 0$). On the left hand side, we plot the size adjusted power of the LM test for deviations of ρ_1 from 0, maintaining that $\rho_2 = 0$. On the right hand side it is the other way around. Observe that the power of the LM test is higher for deviations of ρ_2 from 0 as compared to deviations of ρ_1 from 0. Keep in mind that the estimates of ρ_2 are based on NT observations, while those of ρ_1 rely on only N observations. The top two panels show that the power increases for deviations in ρ_1 as θ increases. However, for deviations in ρ_2, the power of the test is insensitive to θ. The two panels at the center of Figure 1 illustrate

[4] We do not include the corresponding Tables $4-9$, for ($N = 50, T = 10$) and ($N = 100$, $T = 5$), for $\theta = 0.25, 0.50$, and 0.75, in order to save space. However, these Tables are available upon request from the authors. Below, we summarize the corresponding information by means of size adjusted power plots.

that both the size and the power of the LM test improve as the sample size increases, especially as N becomes larger. A comparison of the two panels at the center with those at the bottom of Figure 1 provides information on the interaction of sample size (N, T) and the relative importance of (θ). It is obvious that for deviations of ρ_1 from 0 (on the left), the size and power improve with N, especially as θ increases.

Figure 2 assumes that the Anselin-type process of the error term is the true model ($\rho_1 = 0$). One important difference when compared to Figure 1 is that ρ_2 is now a nuisance parameter. The qualitative effects of an increase in N, T, and θ are similar to those in Figure 1 on the left hand side. The right hand side panels of Figure 2 show that the size adjusted power of the LM test is lower if ρ_2 is high (0.5 compared to 0), especially for low θ (0.25 compared to 0.75).

===== Figures 3-4 =====

Figures 3 and 4 assume that the KKP model is the true one. Note that an assessment of the performance of the LM test is different here, since the KKP model assumes that $\rho_1 = \rho_2$. The null hypothesis in Figure 3 is $\rho_1 = \rho_2 = 0.2$ and the one in Figure 4 is $\rho_1 = \rho_2 = 0.5$. The major difference between the two figures is that assuming a null that is different from $\rho_1 = \rho_2 = 0$ shifts the size adjusted power function and renders it skewed to the right. Otherwise, the conclusions regarding the impact of θ, N, and T are qualitatively similar to those of the random effects model. A major difference from the random effects model is that for the KKP model the power is lower in the ρ_2 direction, especially for small θ.

4 Conclusions

The recent literature on first-order spatially autocorrelated residuals (SAR(1)) with panel data distinguishes between two data generating processes of the error term. One process described in Anselin (1988) and Anselin, Le Gallo and Jayet (2005) assumes that only the remainder error component is spatially correlated. In an alternative process put forward by Kapoor, Kelejian, and Prucha (2004) both the individual and remainder components of the disturbances are characterized by the same spatial autocorrelation pattern. This paper formulates a SAR(1) process of the residuals with panel data that encompasses these two processes. In particular, this paper derives three LM tests based upon the more general model, testing its restricted counterparts: the Anselin model, the Kapoor, Kelejian, and Prucha model, and the random effects model without spatial correlation. For the latter two tests, closed-form expressions for the LM statistics can be obtained.

Our Monte Carlo study assesses the small sample performance of the derived tests. We find that the tests are properly sized and powerful even in relatively small samples. The LM tests are easy to calculate and their power is reasonably high for all three tests considered. The power of these LM tests matches that of the corresponding LR tests except in a few cases. In general, the power of the tests increases with the relative importance of the individual effects' variance as a proportion of the total variance, as well as with increasing N and T. Hence, these LM and LR tests are recommended for the applied researcher to test the restrictions imposed by the RE model with no spatial correlation, the Anselin model, and the Kapoor, Kelejian, and Prucha model.

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Appendix 1

This Appendix derives the joint LM test for the null hypothesis of no spatial correlation in model (1). This is given by $H_0^A : \rho_1 = \rho_2 = 0$. Denote the vector of parameters of interest by $\theta' = (\sigma_\nu^2, \sigma_\mu^2, \rho_1, \rho_2)$. We can focus on the part of the information matrix corresponding to θ. Note that the part of the information matrix corresponding to β can be ignored in computing this LM statistic since the information matrix is block diagonal between θ and β, and the first derivative with respect to β evaluated at the restricted MLE is zero. The LM statistic is given by:

$$LM_A = \widetilde{\mathbf{D}}_\theta' \widetilde{\mathbf{J}}_\theta^{-1} \widetilde{\mathbf{D}}_\theta \tag{11}$$

where $\widetilde{\mathbf{D}}_\theta = (\partial L/\partial\theta)(\widetilde{\theta})$ is a 4×1 vector of partial derivatives of the likelihood function with respect to the elements of θ, evaluated at the restricted MLE $\widetilde{\theta}$. $\widetilde{\mathbf{J}}_\theta = E[-\partial^2 L/\partial\theta\partial\theta'](\widetilde{\theta})$ is the part of the information matrix corresponding to θ, evaluated at the restricted MLE $\widetilde{\theta}$.

Under $H_0^A : \rho_1 = \rho_2 = 0$ and $\mathbf{B} = \mathbf{A} = \mathbf{I}_N$.

$$\mathbf{\Omega}_u = \sigma_1^2(\overline{\mathbf{J}}_T \otimes \mathbf{I}_N) + \sigma_\nu^2(\mathbf{E}_T \otimes \mathbf{I}_N)$$

$$\mathbf{\Omega}_u^{-1} = \frac{1}{\sigma_1^2}(\overline{\mathbf{J}}_T \otimes \mathbf{I}_N) + \frac{1}{\sigma_\nu^2}(\mathbf{E}_T \otimes \mathbf{I}_N).$$

Hartley and Rao (1971) and Hemmerle and Hartley (1973) give a general useful formula that helps in obtaining the score:

$$\frac{\partial L}{\partial\theta_r} = -\frac{1}{2}tr[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\theta_r}] + \frac{1}{2}\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\theta_r}\mathbf{\Omega}_u^{-1}\right]\mathbf{u}$$

for $r = 1, ..., 4$.

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2} &= \frac{1}{\sigma_1^2}(\overline{\mathbf{J}}_T\otimes\mathbf{I}_N)+\frac{1}{\sigma_\nu^2}(\mathbf{E}_T\otimes\mathbf{I}_N)\\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{(\frac{1}{\sigma_1^4}\overline{\mathbf{J}}_T+\frac{1}{\sigma_\nu^4}\mathbf{E}_T)\otimes\mathbf{I}_N\}\mathbf{u}\\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2}\right) &= \frac{N}{\sigma_1^2}+\frac{N(T-1)}{\sigma_\nu^2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2} &= \frac{1}{\sigma_1^2}(\mathbf{J}_T\otimes\mathbf{I}_N)\\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{\frac{1}{\sigma_1^4}(\mathbf{J}_T\otimes\mathbf{I}_N)\}\mathbf{u}\\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2}\right) &= \frac{NT}{\sigma_1^2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_1} &= \frac{\sigma_\mu^2}{\sigma_1^2}(\mathbf{J}_T\otimes(\mathbf{W}_N'+\mathbf{W}_N))\\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_1}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \frac{\sigma_\mu^2}{\sigma_1^4}\mathbf{u}'(\mathbf{J}_T\otimes(\mathbf{W}_N'+\mathbf{W}_N))\mathbf{u}\\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_1}\right) &= \frac{T\sigma_\mu^2}{\sigma_1^2}\ tr(\mathbf{W}_N'+\mathbf{W}_N)=0
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_2} &= (\frac{\sigma_\nu^2}{\sigma_1^2}\overline{\mathbf{J}}_T+\mathbf{E}_T)\otimes(\mathbf{W}_N'+\mathbf{W}_N)\\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{(\frac{\sigma_\nu^2}{\sigma_1^4}\overline{\mathbf{J}}_T+\frac{1}{\sigma_\nu^2}\mathbf{E}_T)\otimes(\mathbf{W}_N'+\mathbf{W}_N)\}\mathbf{u}\\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\rho_2}\right) &= \left[\frac{\sigma_\nu^2}{\sigma_1^2}+(T-1)\right]\ tr(\mathbf{W}_N'+\mathbf{W}_N)=0.
\end{aligned}$$

Therefore, the score under H_0^A is given by

$$\begin{aligned}
\left.\frac{\partial L}{\partial\sigma_\nu^2}\right|_{H_0^A} &= -\frac{N}{2\sigma_1^2}-\frac{N(T-1)}{2\sigma_\nu^2}\\
&\quad+\frac{1}{2}\mathbf{u}'\{(\frac{1}{\sigma_1^4}\overline{\mathbf{J}}_T+\frac{1}{\sigma_\nu^4}\mathbf{E}_T)\otimes\mathbf{I}_N\}\mathbf{u}
\end{aligned}$$

$$\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{H_0^A} = -\frac{NT}{2\sigma_1^2} + \frac{1}{2}\mathbf{u}'\{\frac{1}{\sigma_1^4}(\mathbf{J}_T \otimes \mathbf{I}_N)\}\mathbf{u}$$

$$\left.\frac{\partial L}{\partial \rho_1}\right|_{H_0^A} = \frac{\sigma_\mu^2}{2\sigma_1^4}\mathbf{u}'(\mathbf{J}_T \otimes (\mathbf{W}_N' + \mathbf{W}_N))\mathbf{u}$$

$$\left.\frac{\partial L}{\partial \rho_2}\right|_{H_0^A} = \frac{1}{2}\mathbf{u}'\{(\frac{\sigma_\nu^2}{\sigma_1^4}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}_N' + \mathbf{W}_N)\}\mathbf{u}.$$

Using the following matrix differentiation formula given in Harville (1977), the elements of the information matrix can be obtained as:

$$J_{rs} = E\left[-\frac{\partial^2 L}{\partial \theta_r \theta_s}\right] = \frac{1}{2}tr\left[\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_r}\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \theta_s}\right] \qquad r, s = 1, ..., 4$$

$$J_{11} = \frac{N}{2\sigma_1^4} + \frac{N(T-1)}{2\sigma_\nu^4}$$

$$J_{12} = \frac{NT}{2\sigma_1^4}$$

$$J_{13} = \frac{T\sigma_\mu^2}{2\sigma_1^4}tr(\mathbf{W}_N' + \mathbf{W}_N) = 0$$

$$J_{14} = \left(\frac{\sigma_\nu^2}{2\sigma_1^4} + \frac{(T-1)}{2\sigma_\nu^2}\right)tr(\mathbf{W}_N' + \mathbf{W}_N) = 0$$

$$J_{22} = \frac{NT^2}{2\sigma_1^4}$$

$$J_{23} = \frac{T^2\sigma_\mu^2}{2\sigma_1^4}tr(\mathbf{W}_N' + \mathbf{W}_N) = 0$$

$$J_{24} = \frac{T\sigma_\nu^2}{2\sigma_1^4}tr(\mathbf{W}_N' + \mathbf{W}_N) = 0$$

$$J_{33} = \frac{T^2\sigma_\mu^4}{2\sigma_1^4}tr(\mathbf{W}_N' + \mathbf{W}_N)^2$$

$$J_{34} = \frac{T\sigma_\mu^2\sigma_\nu^2}{2\sigma_1^4}tr(\mathbf{W}_N' + \mathbf{W}_N)^2$$

$$J_{44} = \left(\frac{\sigma_\nu^4}{2\sigma_1^4} + \frac{(T-1)}{2}\right)tr(\mathbf{W}_N' + \mathbf{W}_N)^2.$$

Note that the restricted MLE of β under H_0^A is the MLE of a standard random effects (RE) error component model with no spatial correlation $\widetilde{\beta}$ and $\widetilde{\mathbf{u}} = \mathbf{y} - \mathbf{X}\widetilde{\beta}$. From the second score equation one gets

$$\widetilde{\sigma}_1^2 = \frac{\widetilde{\mathbf{u}}'\{(\bar{\mathbf{J}}_T \otimes \mathbf{I}_N)\}\widetilde{\mathbf{u}}}{N}$$

and substituting this into the first score equation, one gets

$$\widetilde{\sigma}_\nu^2 = \frac{\widetilde{\mathbf{u}}'\{(\mathbf{E}_T \otimes \mathbf{I}_N)\}\widetilde{\mathbf{u}}}{N(T-1)}.$$

The score with respect to each element of θ evaluated at the restricted MLE $\widetilde{\theta}$ is given by

$$\widetilde{\mathbf{d}}_\theta = \begin{bmatrix} 0 \\ 0 \\ \frac{\widetilde{\sigma}_\mu^2}{2\widetilde{\sigma}_1^4}\widetilde{\mathbf{u}}'\{\mathbf{J}_T \otimes (\mathbf{W}_N' + \mathbf{W}_N)\}\widetilde{\mathbf{u}} \\ \frac{1}{2}\widetilde{\mathbf{u}}'\{(\frac{\widetilde{\sigma}_\nu^2}{\widetilde{\sigma}_1^4}\bar{\mathbf{J}}_T + \frac{1}{\widetilde{\sigma}_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}_N' + \mathbf{W}_N)\}\widetilde{\mathbf{u}} \end{bmatrix}$$

and the information matrix evaluated at the restricted MLE $\widetilde{\theta}$ is given by

$$\widetilde{\mathbf{J}}_\theta = \begin{bmatrix} \frac{N}{2\widetilde{\sigma}_1^4} + \frac{N(T-1)}{2\widetilde{\sigma}_\nu^4} & \frac{NT}{2\widetilde{\sigma}_1^4} & 0 & 0 \\ \frac{NT}{2\widetilde{\sigma}_1^4} & \frac{NT^2}{2\widetilde{\sigma}_1^4} & 0 & 0 \\ 0 & 0 & \frac{T^2\widetilde{\sigma}_\mu^4}{2\widetilde{\sigma}_1^4}b_A & \frac{T\widetilde{\sigma}_\mu^2\widetilde{\sigma}_\nu^2}{2\widetilde{\sigma}_1^4}b_A \\ 0 & 0 & \frac{T\widetilde{\sigma}_\mu^2\widetilde{\sigma}_\nu^2}{2\widetilde{\sigma}_1^4}b_A & \left(\frac{\widetilde{\sigma}_\nu^4}{2\widetilde{\sigma}_1^4} + \frac{(T-1)}{2}\right)b_A \end{bmatrix}$$

where $b_A = tr((\mathbf{W}_N' + \mathbf{W}_N)^2)$. The determinant of the submatrix $\widetilde{\mathbf{J}}_{\rho_1,\rho_2}$ is given by

$$\left|\widetilde{\mathbf{J}}_{\rho_1,\rho_2}\right| = \left(\frac{b_A}{2}\right)^2 \frac{T^2(T-1)\widetilde{\sigma}_\mu^4}{\widetilde{\sigma}_1^4}$$

and the inverse of $\widetilde{\mathbf{J}}_{\rho_1,\rho_2}$ is given by

$$\widetilde{\mathbf{J}}_{\rho_1,\rho_2}^{-1} = \frac{2}{b_A}\frac{1}{T^2(T-1)\widetilde{\sigma}_\mu^4}\begin{bmatrix} (T-1)\widetilde{\sigma}_1^4 + \widetilde{\sigma}_\nu^4 & -T\widetilde{\sigma}_\mu^2\widetilde{\sigma}_\nu^2 \\ -T\widetilde{\sigma}_\mu^2\widetilde{\sigma}_\nu^2 & T^2\widetilde{\sigma}_\mu^4 \end{bmatrix}.$$

Define

$$G_A = \widetilde{\mathbf{u}}'\{\mathbf{J}_T \otimes (\mathbf{W}'_N + \mathbf{W}_N)\}\widetilde{\mathbf{u}}$$
$$M_A = \widetilde{\mathbf{u}}'\{(\frac{\widetilde{\sigma}_\nu^2}{\widetilde{\sigma}_1^4}\overline{\mathbf{J}}_T + \frac{1}{\widetilde{\sigma}_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}'_N + \mathbf{W}_N)\}\widetilde{\mathbf{u}}.$$

Then, the resulting LM statistic for H_0^A is given by

$$LM_A = \frac{(T-1)\widetilde{\sigma}_1^4 + \widetilde{\sigma}_\nu^4}{2b_A T^2 (T-1)\widetilde{\sigma}_1^8} G_A^2 - \frac{\widetilde{\sigma}_\nu^2}{b_A T(T-1)\widetilde{\sigma}_1^4} G_A M_A + \frac{1}{2b_A(T-1)} M_A^2.$$

Under H_0^A, this LM_A statistic is asymptotically distributed as χ_2^2.

Appendix 2

This Appendix derives the LM test for the null hypothesis that the spatial panel correlation follows the specification described in Anselin(1988). This is given by $H_0^B : \rho_1 = 0$.

Under $H_0^B : \rho_1 = 0; \mathbf{A} = \mathbf{I}_N$ and $\mathbf{\Omega}_u = \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}] + \sigma_\nu^2[\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})^{-1}]$, $\mathbf{\Omega}_u^{-1} = \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1} + \frac{1}{\sigma_\nu^2}(\mathbf{E}_T \otimes \mathbf{B}'\mathbf{B})$

$$\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\nu^2} = \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N]^{-1} + \frac{1}{\sigma_\nu^2}(\mathbf{E}_T \otimes \mathbf{I}_N)$$
$$\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\nu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} = \mathbf{u}'\{\overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N]^{-1}[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}$$
$$+\frac{1}{\sigma_\nu^4}[\mathbf{E}_T \otimes \mathbf{B}'\mathbf{B}]\}\mathbf{u}$$
$$tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\nu^2}\right) = tr[T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N]^{-1} + \frac{N(T-1)}{\sigma_\nu^2}$$

$$\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\mu^2} = \mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}$$
$$\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\mu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} = \mathbf{u}'\{\mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-2}\}\mathbf{u}$$
$$tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial \mathbf{\Omega}_u}{\partial \sigma_\mu^2}\right) = T\ tr[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}$$

$$
\begin{aligned}
\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1} &= \sigma_\mu^2 \mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{W}_N' + \mathbf{W}_N) \\
\mathbf{u}'\left[\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1}\boldsymbol{\Omega}_u^{-1}\right]\mathbf{u} &= \sigma_\mu^2 \mathbf{u}'\{\mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{W}_N' + \mathbf{W}_N) \\
&\quad [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}\}\mathbf{u} \\
tr\left(\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1}\right) &= T\sigma_\mu^2\ tr\{[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{W}_N' + \mathbf{W}_N)\}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2} &= \sigma_\nu^2 \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1} \\
&\quad +(\mathbf{E}_T \otimes (\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}) \\
\mathbf{u}'\left[\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2}\boldsymbol{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{\sigma_\nu^2 \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1} \\
&\quad [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1} + \frac{1}{\sigma_\nu^2}[\mathbf{E}_T \otimes (\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)]\}\mathbf{u} \\
tr\left(\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2}\right) &= \sigma_\nu^2\ tr\{[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}\} \\
&\quad +(T-1)tr\{(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}\}.
\end{aligned}
$$

Therefore, the score under H_0^B, is given by

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \sigma_\nu^2}\right|_{H_0^B} &= -\frac{1}{2}tr[T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N]^{-1} - \frac{N(T-1)}{2\sigma_\nu^2} \\
&\quad +\frac{1}{2}\mathbf{u}'\{\overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N]^{-1}[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1} \\
&\quad +\frac{1}{\sigma_\nu^4}[\mathbf{E}_T \otimes (\mathbf{B}'\mathbf{B})]\}\mathbf{u}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{H_0^B} &= -\frac{1}{2}T\ tr[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1} \\
&\quad +\frac{1}{2}\mathbf{u}'\{\mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-2}\}\mathbf{u}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \rho_1}\right|_{H_0^B} &= -\frac{1}{2}T\sigma_\mu^2\ tr\{[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{W}_N' + \mathbf{W}_N)\} \\
&\quad +\frac{1}{2}\sigma_\mu^2 \mathbf{u}'\{\mathbf{J}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{W}_N' + \mathbf{W}_N)[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}\}\mathbf{u}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \rho_2}\right|_{H_0^B} = & -\frac{1}{2}\sigma_\nu^2\ tr\{[T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}\} \\
& -\frac{(T-1)}{2} tr\{(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}\} \\
& +\frac{1}{2}\mathbf{u}'\{\sigma_\nu^2 \overline{\mathbf{J}}_T \otimes [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1} \\
& [T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1}]^{-1} + \frac{1}{\sigma_\nu^2}[\mathbf{E}_T \otimes (\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)]\}\mathbf{u}
\end{aligned}
$$

and the elements of the information matrix are given by

$$
J_{11} = \frac{1}{2} tr\left[(T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N)^{-1}\right]^2 + \frac{N(T-1)}{2\sigma_\nu^4}
$$

$$
J_{12} = \frac{T}{2} tr\left[(T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N)^{-1}(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}\right]
$$

$$
J_{13} = \frac{T\sigma_\mu^2}{2} tr[(T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N)^{-1}(T\sigma_\mu^2 + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(\mathbf{W}_N' + \mathbf{W}_N)]
$$

$$
\begin{aligned}
J_{14} = & \frac{\sigma_\nu^2}{2} tr[(T\sigma_\mu^2(\mathbf{B}'\mathbf{B}) + \sigma_\nu^2 \mathbf{I}_N)^{-1}(T\sigma_\mu^2 + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1} \\
& (\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}] + \frac{(T-1)}{2\sigma_\nu^2} tr[(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}]
\end{aligned}
$$

$$
J_{22} = \frac{T^2}{2} tr\left[(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}\right]^2
$$

$$
\begin{aligned}
J_{23} = & \frac{T^2\sigma_\mu^2}{2} tr[(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1} \\
& (\mathbf{W}_N' + \mathbf{W}_N)]
\end{aligned}
$$

$$
\begin{aligned}
J_{24} = & \frac{T\sigma_\nu^2}{2} tr[(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1} \\
& (\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}]
\end{aligned}
$$

$$
J_{33} = \frac{T^2\sigma_\mu^4}{2} tr[(T\sigma_\mu^2 \mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(\mathbf{W}_N' + \mathbf{W}_N)]^2
$$

$$J_{34} = \frac{T\sigma_\mu^2\sigma_\nu^2}{2} tr[(T\sigma_\mu^2\mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(\mathbf{W}_N' + \mathbf{W}_N)$$
$$(T\sigma_\mu^2\mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}]$$

$$J_{44} = \frac{\sigma_\nu^4}{2} tr[(T\sigma_\mu^2\mathbf{I}_N + \sigma_\nu^2(\mathbf{B}'\mathbf{B})^{-1})^{-1}(\mathbf{B}'\mathbf{B})^{-1}(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}]^2$$
$$+\frac{(T-1)}{2} tr[(\mathbf{W}_N'\mathbf{B} + \mathbf{B}'\mathbf{W}_N)(\mathbf{B}'\mathbf{B})^{-1}]^2.$$

Appendix 3

This Appendix derives the LM test for the null hypothesis $H_0^C : \rho_1 = \rho_2 = \rho$, i.e., that the spatial panel correlation follows the specification proposed by KKP.

Under $H_0^C : \rho_1 = \rho_2 = \rho; \mathbf{B} = \mathbf{A}$; $\mathbf{\Omega}_u = (\sigma_1^2\overline{\mathbf{J}}_T + \sigma_\nu^2\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})^{-1}$, $\mathbf{\Omega}_u^{-1} = (\frac{1}{\sigma_1^2}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})$.

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2} &= \frac{1}{\sigma_1^2}(\overline{\mathbf{J}}_T \otimes \mathbf{I}_N) + \frac{1}{\sigma_\nu^2}(\mathbf{E}_T \otimes \mathbf{I}_N) \\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{\frac{1}{\sigma_1^4}(\overline{\mathbf{J}}_T \otimes \mathbf{I}_N)(\mathbf{A}'\mathbf{A}) + \frac{1}{\sigma_\nu^4}\mathbf{E}_T \otimes (\mathbf{A}'\mathbf{A})\}\mathbf{u} \\
&= \mathbf{u}'\{(\frac{1}{\sigma_1^4}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^4}\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})\}\mathbf{u} \\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\nu^2}\right) &= \frac{N}{\sigma_1^2} + \frac{N(T-1)}{\sigma_\nu^2}
\end{aligned}$$

$$\begin{aligned}
\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2} &= \frac{1}{\sigma_1^2}(\mathbf{J}_T \otimes \mathbf{I}_N) \\
\mathbf{u}'\left[\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2}\mathbf{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{\frac{1}{\sigma_1^2}(\mathbf{J}_T \otimes \mathbf{I}_N)(\sigma_1^2(\mathbf{A}'\mathbf{A})^{-1})^{-1}\}\mathbf{u} \\
&= \mathbf{u}'\{\frac{1}{\sigma_1^4}[\mathbf{J}_T \otimes (\mathbf{A}'\mathbf{A})]\}\mathbf{u} \\
tr\left(\mathbf{\Omega}_u^{-1}\frac{\partial\mathbf{\Omega}_u}{\partial\sigma_\mu^2}\right) &= \frac{NT}{\sigma_1^2}
\end{aligned}$$

$$
\begin{aligned}
\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1} &= \frac{\sigma_\mu^2}{\sigma_1^2}\mathbf{J}_T \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1} \\
\mathbf{u}'\left[\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1}\boldsymbol{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{\frac{\sigma_\mu^2}{\sigma_1^4}\mathbf{J}_T \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)\}\mathbf{u} \\
tr\left(\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_1}\right) &= \frac{T\sigma_\mu^2}{\sigma_1^2}\ tr\{(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}\}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2} &= \sigma_\nu^2\overline{\mathbf{J}}_T \otimes [\frac{1}{\sigma_1^2}(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}] + \\
&\quad \mathbf{E}_T \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1} \\
&= (\frac{\sigma_\nu^2}{\sigma_1^2}\overline{\mathbf{J}}_T + \mathbf{E}_T) \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1} \\
\mathbf{u}'\left[\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2}\boldsymbol{\Omega}_u^{-1}\right]\mathbf{u} &= \mathbf{u}'\{(\frac{\sigma_\nu^2}{\sigma_1^4}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)\}\mathbf{u} \\
tr\left(\boldsymbol{\Omega}_u^{-1}\frac{\partial \boldsymbol{\Omega}_u}{\partial \rho_2}\right) &= \left[\frac{\sigma_\nu^2}{\sigma_1^2} + (T-1)\right]\ tr\{(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}\}.
\end{aligned}
$$

Therefore, the score under H_0^C is given by

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \sigma_\nu^2}\right|_{H_0^C} &= -\frac{N}{2\sigma_1^2} - \frac{N(T-1)}{2\sigma_\nu^2} \\
&\quad + \frac{1}{2}\mathbf{u}'\{(\frac{1}{\sigma_1^4}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^4}\mathbf{E}_T) \otimes (\mathbf{A}'\mathbf{A})\}\mathbf{u} \\
\left.\frac{\partial L}{\partial \sigma_\mu^2}\right|_{H_0^B} &= -\frac{NT}{2\sigma_1^2} + \frac{1}{2}\mathbf{u}'\{\frac{1}{\sigma_1^4}[\mathbf{J}_T \otimes (\mathbf{A}'\mathbf{A})]\}\mathbf{u}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \rho_1}\right|_{H_0^C} &= -\frac{T\sigma_\mu^2}{2\sigma_1^2}\ tr\{(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}\} \\
&\quad + \frac{1}{2}\mathbf{u}'\{\frac{\sigma_\mu^2}{\sigma_1^4}\mathbf{J}_T \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)\}\mathbf{u}
\end{aligned}
$$

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \rho_2}\right|_{H_0^C} &= -\frac{1}{2}\left[\frac{\sigma_\nu^2}{\sigma_1^2} + (T-1)\right]\ tr\{(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}\} \\
&\quad + \frac{1}{2}\mathbf{u}'\{(\frac{\sigma_\nu^2}{\sigma_1^4}\overline{\mathbf{J}}_T + \frac{1}{\sigma_\nu^2}\mathbf{E}_T) \otimes (\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)\}\mathbf{u}
\end{aligned}
$$

and the elements of the information matrix

$$J_{11} = \frac{N}{2\sigma_1^4} + \frac{N(T-1)}{2\sigma_\nu^4}$$

$$J_{12} = \frac{NT}{2\sigma_1^4}$$

$$J_{13} = \frac{T\sigma_\mu^2}{2\sigma_1^4} tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]$$

$$J_{14} = \left(\frac{\sigma_\nu^2}{2\sigma_1^4} + \frac{(T-1)}{2\sigma_\nu^2}\right) tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]$$

$$J_{22} = \frac{NT^2}{2\sigma_1^4}$$

$$J_{23} = \frac{T^2\sigma_\mu^2}{2\sigma_1^4} tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]$$

$$J_{24} = \frac{T\sigma_\nu^2}{2\sigma_1^4} tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]$$

$$J_{33} = \frac{T^2\sigma_\mu^4}{2\sigma_1^4} tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]^2$$

$$J_{34} = \frac{T\sigma_\mu^2\sigma_\nu^2}{2\sigma_1^4} tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]^2$$

$$J_{44} = \left(\frac{\sigma_\nu^4}{2\sigma_1^4} + \frac{(T-1)}{2}\right) tr[(\mathbf{W}_N'\mathbf{A} + \mathbf{A}'\mathbf{W}_N)(\mathbf{A}'\mathbf{A})^{-1}]^2.$$

The restricted MLE estimates under H_0^C are labelled by a bar. In fact, this gives the MLE of the KKP model and $\overline{\mathbf{u}} = \mathbf{y} - \mathbf{X}\overline{\beta}$. From the second score equation, we have

$$\overline{\sigma}_1^2 = \frac{\overline{\mathbf{u}}'\{\overline{\mathbf{J}}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})\}\overline{\mathbf{u}}}{N}$$

and substituting this into the first score equation, one obtains

$$\overline{\sigma}_\nu^2 = \frac{\overline{\mathbf{u}}'\{\mathbf{E}_T \otimes (\overline{\mathbf{A}}'\overline{\mathbf{A}})\}\overline{\mathbf{u}}}{N(T-1)}.$$

The score with respect to each element of θ evaluated at the restricted MLE $\overline{\theta}$ is given by

$$\overline{\mathbf{d}}_{\theta}=\begin{bmatrix} 0 \\ 0 \\ \frac{\overline{\sigma}_{\mu}^{2}}{2\overline{\sigma}_{1}^{4}}[-T\overline{\sigma}_{1}^{2}\ tr(\mathbf{D})+\overline{\mathbf{u}}'\{\mathbf{J}_{T}\otimes\mathbf{F}\}\overline{\mathbf{u}}] \\ -\frac{1}{2}\left[\frac{\overline{\sigma}_{\nu}^{2}}{\overline{\sigma}_{1}^{2}}+(T-1)\right]\ tr(\mathbf{D})+\frac{1}{2}\overline{\mathbf{u}}'\{(\frac{\overline{\sigma}_{\nu}^{2}}{\overline{\sigma}_{1}^{4}}\overline{\mathbf{J}}_{T}+\frac{1}{\overline{\sigma}_{\nu}^{2}}\mathbf{E}_{T})\otimes\mathbf{F}\}\overline{\mathbf{u}} \end{bmatrix}$$

where $\mathbf{D}=(\mathbf{W}_N'\overline{\mathbf{A}}+\overline{\mathbf{A}}'\mathbf{W}_N)(\overline{\mathbf{A}}'\overline{\mathbf{A}})^{-1}$ and $\mathbf{F}=\mathbf{W}_N'\overline{\mathbf{A}}+\overline{\mathbf{A}}'\mathbf{W}_N$. The lower (4×4) block of the estimated information matrix evaluated at the restricted MLE $\overline{\theta}$ is given by

$$\begin{aligned}\overline{\mathbf{J}}_{\theta} &= \frac{1}{2\overline{\sigma}_{1}^{4}}\begin{bmatrix} NT\begin{bmatrix}\frac{(T-1)\overline{\sigma}_{1}^{4}+\overline{\sigma}_{\nu}^{4}}{T\overline{\sigma}_{\nu}^{4}} & 1 \\ 1 & T\end{bmatrix} & Ttr(\mathbf{D})\begin{bmatrix}\overline{\sigma}_{\mu}^{2} & \frac{(T-1)\overline{\sigma}_{1}^{4}+\overline{\sigma}_{\nu}^{4}}{T\overline{\sigma}_{\nu}^{2}} \\ T\overline{\sigma}_{\mu}^{2} & \overline{\sigma}_{\nu}^{2}\end{bmatrix} \\ Ttr(\mathbf{D})\begin{bmatrix}\overline{\sigma}_{\mu}^{2} & T\overline{\sigma}_{\mu}^{2} \\ \frac{(T-1)\overline{\sigma}_{1}^{4}+\overline{\sigma}_{\nu}^{4}}{T\overline{\sigma}_{\nu}^{2}} & \overline{\sigma}_{\nu}^{2}\end{bmatrix} & Ttr(\mathbf{D}^2)\begin{bmatrix}T\overline{\sigma}_{\mu}^{4} & \overline{\sigma}_{\mu}^{2}\overline{\sigma}_{\nu}^{2} \\ \overline{\sigma}_{\mu}^{2}\overline{\sigma}_{\nu}^{2} & \frac{(T-1)\overline{\sigma}_{1}^{4}+\overline{\sigma}_{\nu}^{4}}{T}\end{bmatrix}\end{bmatrix} \\ &= \frac{1}{2\overline{\sigma}_{1}^{4}}\begin{bmatrix}\mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22}\end{bmatrix}.\end{aligned}$$

To derive the lower right block of the inverse $\overline{\mathbf{J}}_{\theta}^{-1}$, we use the formulae for the partitioned inverse $\overline{\mathbf{J}}_{\rho_1,\rho_2}^{-1}=2\overline{\sigma}_{1}^{4}(\mathbf{M}_{22}-\mathbf{M}_{21}\mathbf{M}_{11}^{-1}\mathbf{M}_{12})^{-1}$. The determinant of $\mathbf{M}_{11}$ is given by

$$|\mathbf{M}_{11}|=NT\frac{(T-1)\overline{\sigma}_{1}^{4}}{\overline{\sigma}_{\nu}^{4}}$$

and the inverse of $\mathbf{M}_{11}$ is

$$\mathbf{M}_{11}^{-1}=\frac{1}{NT}\frac{\overline{\sigma}_{\nu}^{4}}{(T-1)\overline{\sigma}_{1}^{4}}\begin{bmatrix}T & -1 \\ -1 & \frac{(T-1)\overline{\sigma}_{1}^{4}+\overline{\sigma}_{\nu}^{4}}{T\overline{\sigma}_{\nu}^{4}}\end{bmatrix}.$$

Next we calculate

$$\mathbf{M}_{21}\mathbf{M}_{11}^{-1}\mathbf{M}_{12} = \frac{T(tr(\mathbf{D}))^2}{N}\begin{bmatrix} T\overline{\sigma}_\mu^4 & \overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 \\ \overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 & \frac{(T-1)\overline{\sigma}_1^4+\overline{\sigma}_\nu^4}{T} \end{bmatrix}$$

so that

$$\begin{aligned} \overline{\mathbf{J}}_{\rho_1,\rho_2}^{-1} &= \frac{2\overline{\sigma}_1^4}{T\left[tr(\mathbf{D}^2) - \frac{(tr(\mathbf{D}))^2}{N}\right]}\begin{bmatrix} T\overline{\sigma}_\mu^4 & \overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 \\ \overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 & \frac{(T-1)\overline{\sigma}_1^4+\overline{\sigma}_\nu^4}{T} \end{bmatrix}^{-1} \\ &= \frac{2}{T\left[tr(\mathbf{D}^2) - \frac{(tr(\mathbf{D}))^2}{N}\right](T-1)\,\overline{\sigma}_\mu^4}\begin{bmatrix} \frac{(T-1)\overline{\sigma}_1^4+\overline{\sigma}_\nu^4}{T} & -\overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 \\ -\overline{\sigma}_\mu^2\overline{\sigma}_\nu^2 & T\overline{\sigma}_\mu^4 \end{bmatrix}. \end{aligned}$$

Defining $b_C = tr(\mathbf{D}^2) - (tr(\mathbf{D}))^2/N$ and

$$\begin{aligned} G_C &= -T\overline{\sigma}_1^2\, tr(\mathbf{D}) + \overline{\mathbf{u}}'\{\mathbf{J}_T \otimes \mathbf{F}\}\overline{\mathbf{u}} \\ M_C &= -\left[\frac{\overline{\sigma}_\nu^2}{\overline{\sigma}_1^2} + (T-1)\right]\, tr(\mathbf{D}) + \overline{\mathbf{u}}'\{(\frac{\overline{\sigma}_\nu^2}{\overline{\sigma}_1^4}\overline{\mathbf{J}}_T + \frac{1}{\overline{\sigma}_\nu^2}\mathbf{E}_T) \otimes \mathbf{F}\}\overline{\mathbf{u}} \end{aligned}$$

the resulting LM statistic for H_0^C is given by

$$LM_C = \frac{(T-1)\overline{\sigma}_1^4 + \overline{\sigma}_\nu^4}{2b_C T^2(T-1)\overline{\sigma}_1^8}G_C^2 - \frac{\overline{\sigma}_\nu^2}{b_C T(T-1)\overline{\sigma}_1^4}G_C M_C + \frac{1}{2b_C(T-1)}M_C^2.$$

Under H_0^C the LM_C statistic is asymptotically distributed as χ_1^2.

Appendix 4

We use the constrained quasi-Newton method involving the constraints $\sigma_\mu^2 > 0$, $\sigma_\nu^2 > 0$, $-1 < \rho_1 < 1$ and $-1 < \rho_2 < 1$ to estimate the parameters of the four models (the unrestricted model and the three restricted ones: random effects, Anselin, and KKP). The quasi-Newton method calculates the gradient of the likelihood numerically. We use the optimization routine *fmincon* available from Matlab which uses the sequential quadratic programming method. This method guarantees superlinear convergence by accumulating second order information regarding the Kuhn-Tucker equations using a quasi-Newton updating procedure. An estimate of the Hessian of the Lagrangian is updated at each iteration using the BFGS formula. However, all tests are based on the analytically derived formulas for both the gradient and the information matrix, using the estimated parameters.

Table 1: Monte carlo simulations for size and power of LM and LR tests of the Anselin, the random effects and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=10, σ^2_v=10)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.938	0.964	**0.039**	**0.041**
-0.80	-0.50	1.000	1.000	0.985	0.992	0.590	0.565
-0.80	-0.20	0.997	0.998	0.989	0.991	0.919	0.922
-0.80	0.00	0.979	0.982	0.989	0.991	0.982	0.985
-0.80	0.20	0.997	0.997	0.989	0.993	0.999	0.999
-0.80	0.50	1.000	1.000	0.972	0.977	1.000	1.000
-0.80	0.80	1.000	1.000	0.925	0.938	1.000	1.000
-0.50	-0.80	1.000	1.000	0.562	0.595	0.172	0.307
-0.50	-0.50	1.000	1.000	0.692	0.711	**0.046**	**0.046**
-0.50	-0.20	0.913	0.925	0.727	0.742	0.318	0.324
-0.50	0.00	0.614	0.646	0.702	0.729	0.661	0.685
-0.50	0.20	0.888	0.886	0.690	0.724	0.868	0.894
-0.50	0.50	1.000	1.000	0.613	0.632	0.985	0.992
-0.50	0.80	1.000	1.000	0.430	0.450	0.999	1.000
-0.20	-0.80	1.000	1.000	0.144	0.153	0.643	0.755
-0.20	-0.50	1.000	1.000	0.175	0.183	0.209	0.231
-0.20	-0.20	0.663	0.669	0.164	0.167	**0.042**	**0.045**
-0.20	0.00	0.130	0.139	0.158	0.169	0.157	0.171
-0.20	0.20	0.696	0.660	0.186	0.203	0.453	0.499
-0.20	0.50	1.000	1.000	0.131	0.142	0.863	0.910
-0.20	0.80	1.000	1.000	0.095	0.097	0.976	0.996
0.00	-0.80	1.000	1.000	**0.043**	**0.058**	0.822	0.899
0.00	-0.50	1.000	1.000	**0.043**	**0.055**	0.501	0.509
0.00	-0.20	0.582	0.574	**0.045**	**0.059**	0.106	0.099
0.00	0.00	**0.043**	**0.053**	**0.049**	**0.058**	**0.054**	**0.059**
0.00	0.20	0.646	0.602	**0.042**	**0.047**	0.133	0.154
0.00	0.50	1.000	1.000	**0.049**	**0.051**	0.595	0.672
0.00	0.80	1.000	1.000	**0.050**	**0.053**	0.898	0.962
0.20	-0.80	1.000	1.000	0.117	0.092	0.962	0.983
0.20	-0.50	1.000	1.000	0.147	0.126	0.818	0.827
0.20	-0.20	0.605	0.593	0.174	0.142	0.402	0.382
0.20	0.00	0.130	0.110	0.148	0.125	0.131	0.111
0.20	0.20	0.686	0.649	0.171	0.140	**0.048**	**0.053**
0.20	0.50	1.000	1.000	0.134	0.116	0.283	0.348
0.20	0.80	1.000	1.000	0.093	0.082	0.798	0.909
0.50	-0.80	1.000	1.000	0.667	0.632	0.999	0.999
0.50	-0.50	1.000	1.000	0.761	0.728	0.989	0.988
0.50	-0.20	0.901	0.889	0.781	0.739	0.903	0.886
0.50	0.00	0.700	0.664	0.767	0.746	0.706	0.650
0.50	0.20	0.934	0.923	0.771	0.750	0.372	0.302
0.50	0.50	1.000	1.000	0.683	0.662	**0.044**	**0.054**
0.50	0.80	1.000	1.000	0.397	0.402	0.434	0.590
0.80	-0.80	1.000	1.000	0.994	0.995	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	0.999	0.998	0.999	0.999	0.997	0.996
0.80	0.20	1.000	1.000	1.000	1.000	0.988	0.977
0.80	0.50	1.000	1.000	0.990	0.997	0.781	0.699
0.80	0.80	1.000	1.000	0.847	0.947	**0.033**	**0.062**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 2: Monte carlo simulations for size and power of LM and LR tests of the Anselin, the random effects and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=5, σ^2_v=15)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.660	0.757	**0.039**	**0.033**
-0.80	-0.50	1.000	1.000	0.824	0.896	0.443	0.401
-0.80	-0.20	0.987	0.991	0.935	0.952	0.804	0.812
-0.80	0.00	0.896	0.923	0.950	0.963	0.940	0.953
-0.80	0.20	0.956	0.961	0.935	0.947	0.974	0.981
-0.80	0.50	1.000	1.000	0.875	0.902	0.993	0.999
-0.80	0.80	1.000	1.000	0.804	0.838	0.993	0.999
-0.50	-0.80	1.000	1.000	0.301	0.320	0.093	0.175
-0.50	-0.50	1.000	1.000	0.422	0.431	**0.047**	**0.038**
-0.50	-0.20	0.853	0.878	0.496	0.532	0.248	0.262
-0.50	0.00	0.389	0.425	0.489	0.502	0.448	0.484
-0.50	0.20	0.767	0.756	0.504	0.548	0.684	0.743
-0.50	0.50	1.000	1.000	0.378	0.419	0.865	0.920
-0.50	0.80	1.000	1.000	0.306	0.328	0.923	0.989
-0.20	-0.80	1.000	1.000	0.097	0.098	0.316	0.455
-0.20	-0.50	1.000	1.000	0.119	0.112	0.120	0.131
-0.20	-0.20	0.641	0.668	0.108	0.123	**0.044**	**0.042**
-0.20	0.00	0.100	0.111	0.126	0.129	0.123	0.125
-0.20	0.20	0.638	0.605	0.129	0.148	0.291	0.324
-0.20	0.50	1.000	1.000	0.084	0.097	0.588	0.674
-0.20	0.80	1.000	1.000	0.066	0.080	0.733	0.909
0.00	-0.80	1.000	1.000	**0.049**	**0.057**	0.457	0.659
0.00	-0.50	1.000	1.000	**0.046**	**0.058**	0.265	0.304
0.00	-0.20	0.570	0.586	**0.050**	**0.053**	0.076	0.071
0.00	0.00	**0.050**	**0.055**	**0.048**	**0.052**	**0.053**	**0.049**
0.00	0.20	0.627	0.596	**0.039**	**0.039**	0.096	0.119
0.00	0.50	1.000	1.000	**0.050**	**0.043**	0.310	0.413
0.00	0.80	1.000	1.000	**0.050**	**0.045**	0.521	0.753
0.20	-0.80	1.000	1.000	0.073	0.069	0.755	0.866
0.20	-0.50	1.000	1.000	0.104	0.081	0.585	0.613
0.20	-0.20	0.552	0.564	0.091	0.083	0.269	0.257
0.20	0.00	0.084	0.070	0.108	0.082	0.107	0.091
0.20	0.20	0.691	0.660	0.109	0.097	**0.041**	**0.045**
0.20	0.50	1.000	1.000	0.075	0.068	0.199	0.245
0.20	0.80	1.000	1.000	0.071	0.072	0.435	0.629
0.50	-0.80	1.000	1.000	0.468	0.438	0.971	0.989
0.50	-0.50	1.000	1.000	0.565	0.520	0.929	0.936
0.50	-0.20	0.772	0.765	0.586	0.571	0.790	0.754
0.50	0.00	0.505	0.482	0.579	0.557	0.535	0.492
0.50	0.20	0.886	0.873	0.541	0.524	0.252	0.197
0.50	0.50	1.000	1.000	0.325	0.351	**0.039**	**0.050**
0.50	0.80	1.000	1.000	0.182	0.193	0.236	0.322
0.80	-0.80	1.000	1.000	0.984	0.987	1.000	1.000
0.80	-0.50	1.000	1.000	0.993	0.993	1.000	1.000
0.80	-0.20	0.993	0.993	0.992	0.991	0.998	0.997
0.80	0.00	0.988	0.987	0.993	0.993	0.989	0.984
0.80	0.20	0.999	0.999	0.990	0.993	0.959	0.930
0.80	0.50	1.000	1.000	0.846	0.960	0.630	0.525
0.80	0.80	1.000	1.000	0.430	0.644	**0.034**	**0.059**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Table 3: Monte carlo simulations for size and power of LM and LR tests of the Anselin, the random effects and the Kapoor-Kelejian-Prucha models; share of rejections in 2000 replications (N=50, T=5, σ^2_μ=15, σ^2_ν=5)

		Random effects model		Anselin model		Kelejian-Prucha model	
		H_0^A: ρ_1=0, ρ_2=0		H_0^B: ρ_1=0		H_0^C: ρ_1=ρ_2	
ρ_1	ρ_2	LM	LR	LM	LR	LM	LR
-0.80	-0.80	1.000	1.000	0.985	0.994	**0.039**	**0.032**
-0.80	-0.50	1.000	1.000	0.997	0.999	0.642	0.610
-0.80	-0.20	0.999	1.000	0.998	0.999	0.964	0.965
-0.80	0.00	0.986	0.995	0.997	0.998	0.995	0.996
-0.80	0.20	0.998	1.000	0.996	0.998	1.000	1.000
-0.80	0.50	1.000	1.000	0.993	0.997	1.000	1.000
-0.80	0.80	1.000	1.000	0.969	0.975	1.000	1.000
-0.50	-0.80	1.000	1.000	0.727	0.769	0.271	0.408
-0.50	-0.50	1.000	1.000	0.815	0.836	**0.046**	**0.046**
-0.50	-0.20	0.927	0.945	0.814	0.831	0.384	0.370
-0.50	0.00	0.680	0.748	0.810	0.834	0.730	0.748
-0.50	0.20	0.935	0.942	0.811	0.820	0.937	0.952
-0.50	0.50	1.000	1.000	0.755	0.777	0.999	1.000
-0.50	0.80	1.000	1.000	0.589	0.619	1.000	1.000
-0.20	-0.80	1.000	1.000	0.174	0.198	0.788	0.885
-0.20	-0.50	1.000	1.000	0.210	0.235	0.241	0.267
-0.20	-0.20	0.671	0.704	0.231	0.249	**0.049**	**0.051**
-0.20	0.00	0.163	0.189	0.236	0.256	0.176	0.192
-0.20	0.20	0.735	0.732	0.230	0.237	0.509	0.555
-0.20	0.50	1.000	1.000	0.178	0.188	0.934	0.965
-0.20	0.80	1.000	1.000	0.136	0.142	1.000	1.000
0.00	-0.80	1.000	1.000	**0.042**	**0.053**	0.951	0.978
0.00	-0.50	1.000	1.000	**0.035**	**0.042**	0.632	0.652
0.00	-0.20	0.579	0.594	**0.039**	**0.050**	0.129	0.117
0.00	0.00	**0.040**	**0.047**	**0.036**	**0.045**	**0.041**	**0.049**
0.00	0.20	0.645	0.625	**0.039**	**0.048**	0.193	0.222
0.00	0.50	1.000	1.000	**0.048**	**0.053**	0.751	0.804
0.00	0.80	1.000	1.000	**0.049**	**0.053**	0.992	0.998
0.20	-0.80	1.000	1.000	0.178	0.153	0.995	0.998
0.20	-0.50	1.000	1.000	0.182	0.170	0.915	0.921
0.20	-0.20	0.644	0.655	0.196	0.166	0.514	0.480
0.20	0.00	0.153	0.136	0.214	0.189	0.176	0.142
0.20	0.20	0.699	0.673	0.206	0.165	**0.038**	**0.045**
0.20	0.50	1.000	1.000	0.178	0.148	0.414	0.476
0.20	0.80	1.000	1.000	0.120	0.102	0.969	0.990
0.50	-0.80	1.000	1.000	0.794	0.775	1.000	1.000
0.50	-0.50	1.000	1.000	0.850	0.832	0.997	0.997
0.50	-0.20	0.938	0.937	0.860	0.845	0.950	0.944
0.50	0.00	0.784	0.774	0.866	0.849	0.804	0.773
0.50	0.20	0.955	0.950	0.860	0.839	0.452	0.386
0.50	0.50	1.000	1.000	0.828	0.811	**0.040**	**0.056**
0.50	0.80	1.000	1.000	0.635	0.639	0.660	0.786
0.80	-0.80	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.50	1.000	1.000	1.000	1.000	1.000	1.000
0.80	-0.20	1.000	1.000	1.000	1.000	1.000	1.000
0.80	0.00	0.999	1.000	1.000	1.000	0.999	0.999
0.80	0.20	1.000	1.000	1.000	1.000	0.991	0.981
0.80	0.50	1.000	1.000	0.999	0.999	0.805	0.728
0.80	0.80	1.000	1.000	0.988	0.994	**0.032**	**0.063**

Note: Bold figures refer to the size of the test at nominal size of 5%. All other figures refer to the size adjusted power of the tests.

Figure 1: The power of the LM-test, random effects model

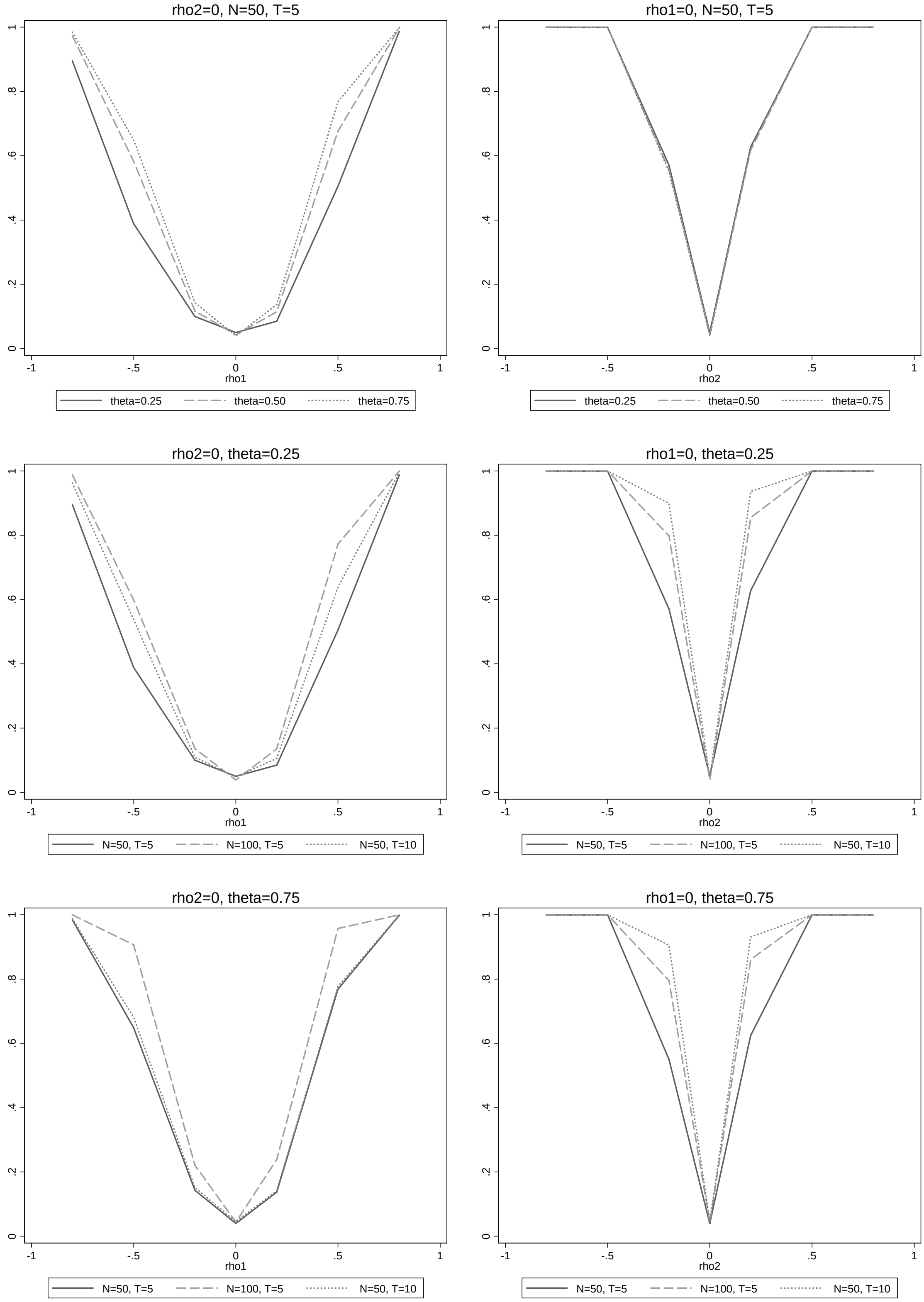

Figure 2: The power of the LM-test, Anselin model

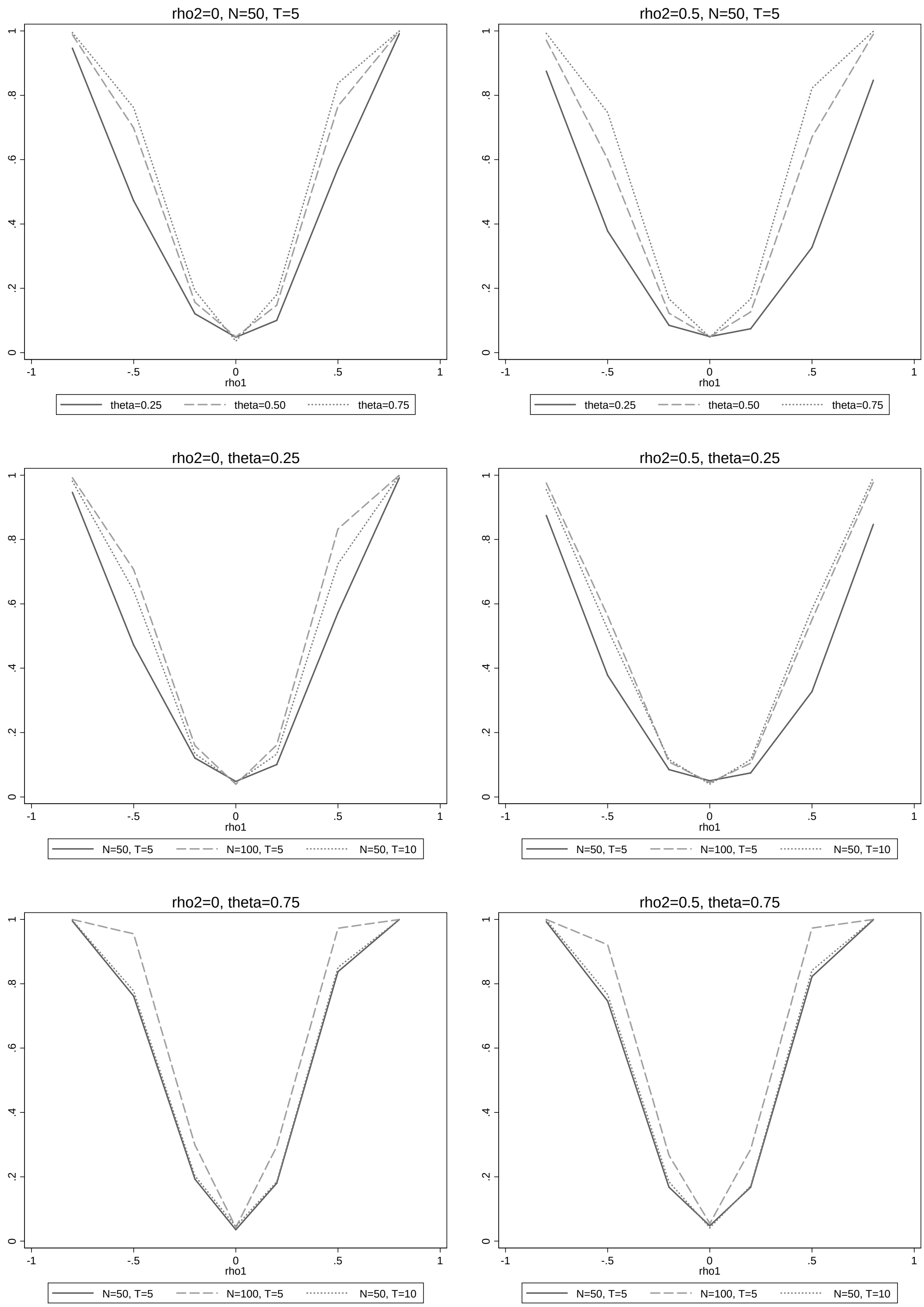

Figure 3: The power of the LM-test, KKP model - part I

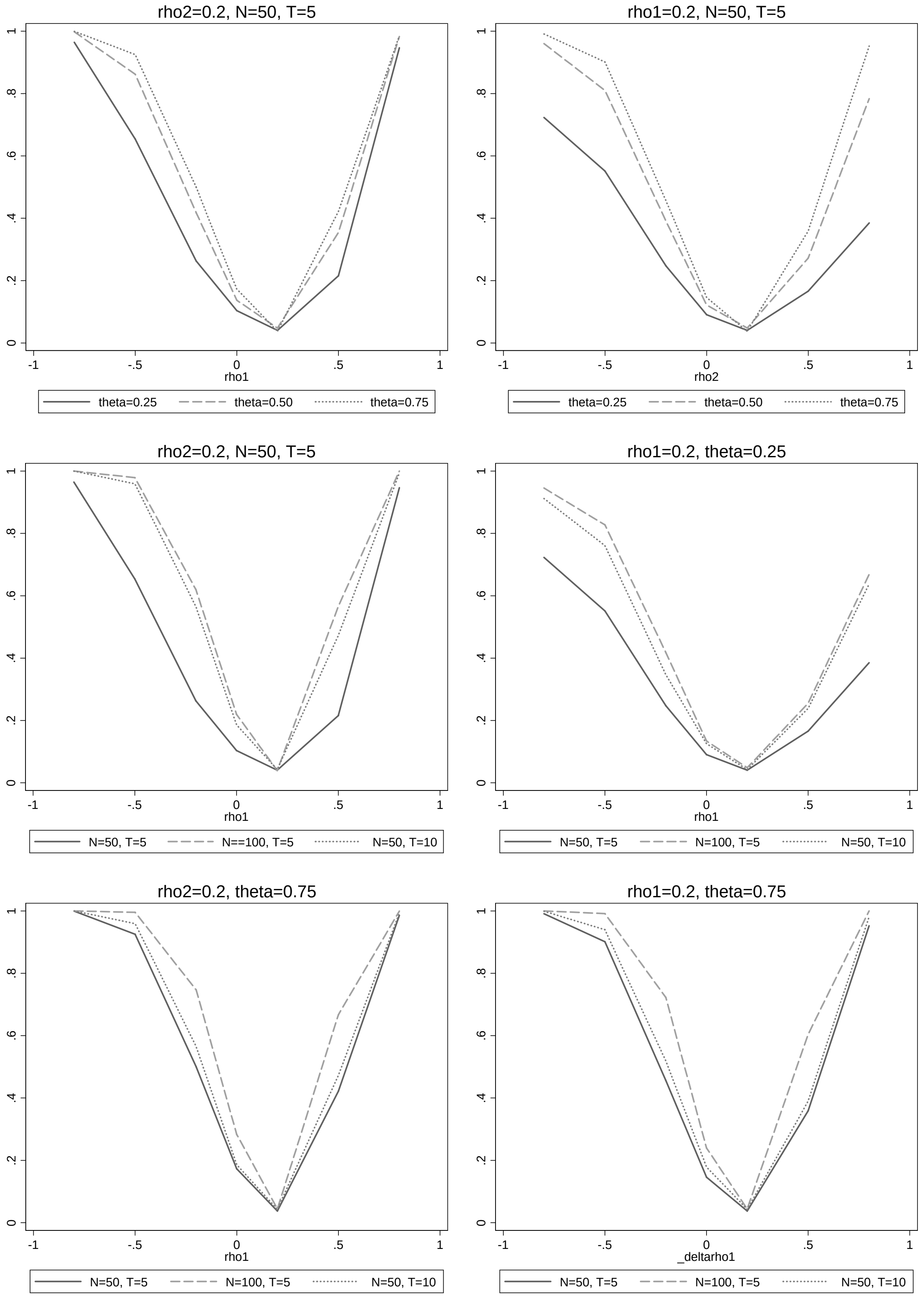

Figure 4: The power of the LM-test, KKP model - part II