Sorting Between and Within Industries*

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Abstract

Recent work using linked employer–employee data has shown, among other results, that person and firm heterogeneity in wages each explain about half of inter-industry wage differentials. One puzzling feature of this decomposition is the pattern of correlations: intra-industry correlation between firm and worker effects is often negative, while it is always positive at the inter-industry level. No extant theory can explain this combination of results. We investigate a tractable parametric

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structural economic model of unobserved heterogeneity among workers and firms. The model is estimated using aggregate moments derived from least squares estimates of the underlying person and firm effects.
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1 Motivation

The persistence of inter-industry and firm-size wage differences has been widely documented in the empirical literature.[1] Using linked longitudinal employer–employee data (as documented by Abowd and Kramarz (1999a,b)) researchers have been able to study these differences at the firm and employee levels. Following Abowd et al. (1999b), wages can be decomposed itto a worker effect (in turn, with both explained and unexplained components) and a firm effect. Abowd et al. (2001) and Abowd and Kramarz (2000) have show the relative importance of each of those effects on wages: individual and firm heterogeneity in wages each explain about half of the inter-industry wage differentials. A puzzle has surfaced from these studies: worker effects and firm effects are negatively and significantly correlated at the intra-industry level, while they are positively correlated at the inter-industry level.

Hence, it appears that workers with a large person effect are attracted to or selected by industries with high average firm effects; however, inside each industry the workers with high person effects are selected by firms with low firm effects.[2] The difficulty may lie in the abstract nature of the firm effect. Unobserved worker heterogeneity might still be observed by the workers and firms and, hence, compensated in the labor market in the same manner as observable heterogeneity among workers. Firm-level heterogeneity in wages is more difficult to explain with classical profit maximization. The estimated firm effects might, then, be due to sorting by comparative advantage in the labor market (see Roy (1951), Heckman and Sedlacek (1985, 1990), Heckman and Honoré (1990)). The sorting induced by comparative advantage converts personal heterogeneity into firm heterogeneity (literally, task-specific productivity). Hence, the observed correlations between person and firm effects might be due to a statistical artifact in the estimation: if there are not independent sources of personal and firm heterogeneity the distinction between the two types of statistical effects is arbitrary.

To shed some light on this problem, we will consider a relatively simple model with two true sources of heterogeneity: underlying productivity differences among the workers and technological variation within and between industries. We parameterize the model so that the structural relation

[1] Krueger and Summers (1987, 1988) for example.

[2] All of these statements control for observable personal characteristics.

between the distributions of heterogeneity (induced by optimal sorting of the workers and optimal adoption of technology) is directly related to the statistical moments that arise when one estimates the components of wage rate heterogeneity directly by least squares. Thus, the underlying parameters can be estimated from the observed moments of the person and firm effects between and within industries. Our model of worker heterogeneity is in the same spirit as Gibbons and Katz (1992), where true productivity is unobserved by the firm but a sorting equilibrium fully reveals the differences. Firm heterogeneity arises from differences in production functions that imply differences in optimal hiring patterns and systematic wage differences, which we model using two different kinds of incentive contracts (in the spirit of Paarsch and Shearer (2000)).

We proceed as follows: in section 2 we develop the exact statistical formulas for the estimation of worker and firm effects in a multi-sector economy. In section 3 we propose a model of wage setting, incorporating both incentive wage and efficiency wage characteristics, that can be expressed completely in the statistical framework of section 2. In section 4 we develop the necessary statistical tools, relegating most of the derivations to the appendix. Section 5 presents the results and the last section concludes.

2 A Puzzle: Good Workers in Bad Firms

Consider the following equation:

$$\log w_{i,t} = x_{i,t}\beta + \theta_i + \psi_{J(i,t)} + \varepsilon_{i,t} \tag{1}$$

where $w_{i,t}$ denotes the annualized total labor cost for worker i at date t, $x_{i,t}$ denotes time-varying observed characteristics for person i, θ_i denotes a person effect, $\psi_{J(i,t)}$ denotes a firm effect, $J(i,t)$ denotes the employing firm for worker i at date t, and $\varepsilon_{i,t}$ a statistical residual that is assumed to be orthogonal to the other components of equation (1). There are no restrictions on the covariance structure of the design of the observed characteristics, the worker effects and the firm effects.

The literature has generally associated θ_i with unobserved personal characteristics and $\psi_{J(i,t)}$ with the firm-specific compensation policy (however, see Abowd and Kramarz (1999a) for a discussion). Using this perspective, a high-wage worker is one with θ_i greater than average and a high-wage firm is one with $\psi_{J(i,t)}$ greater than average. The estimated components have no direct structural interpretation; however, when we use the terms high-wage and low-wage worker/firm in reference to the effects in equation (1) because they are directly measured.

Abowd et al. (1999b) estimated an approximation to the linear model using French data, whereas Abowd et al. (1999a) estimated the same model using data source for the State of Washington. Recently, Abowd et al. (2002)

estimated the exact least squares solution for this problem using the original French and Washington data and ALM-2002 estimate this model with data from the LEHD program at the US Census Bureau. In this article we will use results from AKLR-2002, which are based on updated French data and American data from the LEHD Program at the US Census (citations...).

Table 1 shows the decomposition the components of equation (1) for France and the United States. The row labeled "total" show the indicated statistic computed using all of the available observations. The row labeled "between" shows the same statistic but computed between 2-digit industrial sectors (95 for France, 81 for the US), weighting each sector by the number of observations in that industry. The row labeled "within" shows the statistics computed within each sector then averaged using the same sector-level weights as the"between" row. Two strinking features of this table are: the strong positive correlation of worker and firm effects across sectors (expected if workers sort themselves across industries), and the strong negative correlation within sectors (for France) of these two effects. The average within-sector correlation is zero in the US; however, this masks considerable sectoral heterogeneity. In France, the within sector correlation between the person and firm effects from equation (1) is consistently negative. In the US, the average and (weighted) median are both approximately zero; howver, as Figure 1 shows, there are many sectors in US where the correlation within-sector is negative.

A positive correlation between estimated person and firm effects at the level of industrial sectors is consistent with sorting of the workers by comparative advantage (Roy (1951), cite HeckmanSedlacek1985, Heckman and Honoré (1990)). In this interpretation, there is only one source of heterogeneity in the wage rate because the workers whose underlying productivity gives them a comparative advantage at certain technologies are sorted to the firms that use this technology. Variation in technology within sector and between sectors produces the same positive correlation. One the other hand, negative correlation between person and firm effects within sector cannot be readily reconciled with the Roy model of worker selection .

3 A Simple Theoretical Example

In this section we develop a simple model of sorting based on comparative advantage. Such models imply that the underlying person and firm effects in wage rates are perfectly correlated.

4 General Formulas for Worker and Firm Effects

In this section we show how we can derive worker and firm effect in an economy with only two types of workers and a continuum *of firms differing by size and production technology.*

Let the economy consist of two types of workers. This is not as limiting as it sounds, for it may only represent an heterogeneity inside a particular class of workers, i.e., once we consider effects net of observables.

The economy is divided in a finite number $\bar{k}$ of industries. The workers are hired by a continuum of firms, indexed by the share x of type 1 workers each firm hires, the size t of the firm, and the industry k. In industry k, there is a mass (density) $\mu_k(x,t)$ of firms of type (k,x,t).

We omit from what follows the k index except when it is necessary. The economy as a whole is represented by the space $\Omega = \{\Omega_1, \ldots, \Omega_{\bar{k}}\}$ with $\Omega_k = [0,1] \times \mathbb{R}^+$, the space defining each industry. There is a quantity (or number) $J = \int_\Omega d\mu$ of firms, $J_k = \int_{\Omega_k} d\mu_k$ in each industry. Firm (x,t) hires xt of type 1 workers, and $(1-x)t$ of type 2. We write $\int_\Omega xt\, d\mu(x,t) = M$ and $\int_\Omega (1-x)t\, d\mu(x,t) = N$ so that there are $M+N$ workers in the economy. Type 1 workers earn $w(x,t,k)$ in firm (x,t) in industry k while type 2 earn $u(x,t,k)$; the wage only depends on the industry, the share of type 1 workers, and the size of the firm.

As in Abowd et al. (1999b) (henceforth AKM99) we write the individual wage in each period of time as the sum of a person effect and a firm effect.

$$\ln w_{i,j(i,t),t} = \theta_i + \psi_{j(i,t)} + \varepsilon_{i,j(i),t}$$

However we consider here only the limiting (and purely theoretical) case where the number of time periods goes to infinity and the workers visit ergodically all firms and all industries. This frees us from the identification problem discussed in detail in the above paper. Moreover, the effects estimated in AKM99 would converge in probability to the theoretical values we show below.

In this case, even though the type of the worker is not directly known by the econometrician, observing the workers jump from job to job provides enough information to be able to reduce the problem to the following. Find W, U (worker effects) and $\psi(k,x,t)$ (firm effect in industry k for a firm of size t hiring a proportion x of type 1 workers) that solve

$$\min_{W,U,\psi} \int_\Omega \left[xt\,(w(x,t) - W - \psi(x,t))^2 + (1-x)t\,(u(x,t) - U - \psi(x,t))^2 \right] d\mu(x,t).$$

This is equivalent to minimizing the sum of squared residuals in ordinary least squares. The solutions to this problem are unique up to a constant. We show that:

Theorem 1. *Under integrability assumptions on w and u, the solutions are* *of the type*

$$W = \frac{\int x(1-x)tw(x,t)}{\int x(1-x)t} + \Lambda, \tag{2}$$

$$U = \frac{\int x(1-x)tu(x,t)}{\int x(1-x)t} + \Lambda, \tag{3}$$

$$\psi(x,t) = x(w(x,t) - W) + (1-x)(u(x,t) - U) - \Lambda \tag{4}$$

where the integrals are all taken on Ω with respect to the measure μ, and Λ is an arbitrary constant (we will set $\Lambda = 0$ in the rest of the paper).

Proof. Set $U = 0$ for the time being, as the solutions are only identified up to a constant. Let us write $J(W,\psi) = \int_\Omega \rho(W,\psi)(x,t)\, d\mu(x,t)$. J is defined on $\mathbb{R} \times L^2(\Omega,\mu)$, and may take the value $+\infty$. It is clear that ρ is convex in its two arguments (as sum of two squares); hence J is also convex, and can be show to be strictly convex if $w(x,t) - u(x,t)$ is not a constant. Hence it has a unique minimum under this assumption, characterized by the first order constraints

$$\int_\Omega xt(w(x,t) - W - \psi(x,t))\, d\mu(x,t) = 0 \tag{5}$$

$$\int_\Omega xt(w(x,t) - W - \psi(x,t)) + (1-x)t(u(x,t) - \psi(x,t)) = 0 \tag{6}$$

Hence $W = \int xt(w-\psi)\, d\mu / \int xt\, dB$ and $\psi = x(w-W) + (1-x)u$. Replacing ψ by this last value in equation 5 and simplifying with the fact that $\int x^2 t = M - \int x(1-x)t$ and $\int x^2 tw = \int xtw - \int x(1-x)w$ proves the theorem. □

Remark 2 (Some special cases). *If all firms hire the same number of both types of workers, the distribution μ_k for all k is $\delta_{1/2,2}$, a Dirac distribution of firms of equal size and same number of type 1 workers. This leads to $W = \sum_k w_k/\bar{k}$, $U = \sum_k u_k/\bar{k}$, $\psi_k = \frac{1}{2}(w_k - \bar{w} + u_k - \bar{u})$, where w_k is the wage in industry k.*
The case where the distribution μ has all mass in t at $t = 2$ and $w(x,t) = u(x,t) + \delta$ leads to $W = U + \bar{\delta}$ and $\psi(x,t) = u(x) - U$.

We define the firm-employee effects covariances $cov(\hat{\theta},\hat{\psi})$ as

$$\frac{\int (xtW + (1-x)tU)\psi(x,t)\, d\mu}{\int t\, d\mu} - \frac{\int (xtW + (1-x)tU)\, d\mu}{\int t\, d\mu} \frac{\int \psi(x,t)\, d\mu}{\int t\, d\mu} \tag{7}$$

For total covariance the integrals are taken over Ω, for within-sector covariance over Ω_k for all k, and the between-sector covariance is the discrete covariance between the means (i.e. integrals) on the Ω_k. Total covariance is as usual the sum of between-sector and within-sector covariances.

5 An Economic Model

In this section we build a very simple model of production that can allow for positive or negative correlations.

5.1 Setup and equilibrium

We consider an economy with a continuum of firms indexed by j in an arbitrary space. Each firm must fill two types of jobs (1 and 2) to be able to produce a unique good of price p. Firms differ by their production function, which exogenously fixes the size of the firm and the amount of a fixed production factor. Thus they will hire different quantities of workers in each type of jobs.

$$F_j(K_j, N_{1j}, N_{2j}) = \min\left(aK_jN_{1j}, b_k(K_j)\, N_{2j}\right),$$

where K_j is the exogenously fixed production factor specific to each firm (we could think of it as some sort of capital, or as some history-dependent production structure), $b_k(K_j)$ is the (possibly sector dependent) productivity of jobs of type 2. The size of each firm is fixed to N_j.

The production in each job is separable from the production of all other jobs. The firm offers two types of contracts corresponding to the two techniques of production: incentive and fixed wages.

In the first type of contract workers are paid an incentive wage w_1. In the second type of contract workers are paid a fixed wage w_2. The first type of contract—incentive pay—is used to compensate workers in type 1 jobs; the second type of contract—fixed pay—is used to compensate workers in type 2 jobs.

In the economy, there are two types of workers as in the previous section, endowed with a cost of effort κ_1 and $\kappa_2 > \kappa_1$. This type is known to the worker but not to the firm. Firms post a number of contracts for each type of job; workers apply to one job in one firm. We will look for the equilibria of the underlying game that have the following properties:

- type 1 workers apply to type 1 jobs;
- type 2 workers apply to type 2 jobs;
- firms maximize profits given workers' behavior.

The utility of a worker of type i in a firm j with wage w depends on the amount of extra effort e she exerts above the minimum amount of effort $\bar{e}$ enforceable by the firm:

$$U_i(w, e) = w - \bar{\kappa}\bar{e} - \frac{1}{2}\kappa_i e^2$$

where $\bar{\kappa}$ is the utility cost of work for minimum effort.[3] Workers are risk neutral, and have an outside opportunity worth $\overline{u}$ in utility units. Workers in a fixed wage job will only produce the amount corresponding to the minimum effort level $\bar{e}$, and thus will be willing to work for any wage greater than $\overline{u} + \bar{\kappa}\bar{e}$.

Workers in an incentive pay job optimally choose their effort level according to the piece rate α. Productivity in the job is a random variable π with mean μ_j and second moment σ_j^2, observed after the contracts have been signed but before production occurs. The optimal effort is therefore $e = \alpha\pi/\kappa_i$, expected output is $\mu_j\bar{e} + \alpha\sigma_j^2/\kappa_i$ and expected utility of a type i worker in type 1 job is

$$U_{i,1} = \alpha\mu\bar{e} + \frac{\alpha^2\sigma_j^2}{2\kappa_i} - \bar{\kappa}\bar{e}$$

Workers in a fixed pay job only work the minimum effort level $\bar{e}$ and are paid $\bar{u} + \bar{\kappa}\bar{e}$. It is possible for $\bar{e}$ to depend on both the sector and the capital of the firm.

Assumption 1. *The minimum effort $\bar{e}$ in type 1 jobs does not depend on the firm's capital K_j (but it may be industry dependent), while in type 2 jobs $\bar{e}_k(K_j)$ is increasing in the firm's capital for type 2 jobs, inside each sector k.*

Assumption 2. *The profit that the firm extracts from a type 1 job is decreasing in the piece rate α, for all values of α such that the employee receives at least her reservation utility.*

This ensures the fact that α will be chosen so that the utility of type 1 workers in type 1 jobs is the reservation utility, and hence is constant in each industry. This is not a strong constraint since the profit is a concave parabola attaining its maximum at $p/2 - \mu\bar{e}_k(K_j)\kappa_i/\sigma_j^2$.

Theorem 3. *Under assumptions 1 and 2, type 2 workers will choose the fixed wage job and will be indifferent between all firms; type 1 workers will choose incentive pay jobs and will also be indifferent between all firms. Firms maximize profit for each type job.*

Proof. By assumption 2, the firm sets α so that $U_{1,1} = \overline{u}$. But $U_{2,1} < U_{1,1}$ for a given α, since $\kappa_1 < \kappa_2$. Thus type 2 workers will not choose a type 1 job because they would receive a lower utility, while type 1 workers are indifferent between the two types of jobs (we could always set $U_{1,1} = \overline{u} + \varepsilon > \overline{u} > U_{2,1}$ so that type 1 workers strictly prefer type 1 jobs). Notice that, since $U_{1,1}$ is increasing in α, this is the minimal value accepted by the workers. Firms' profit on type 2 jobs is decreasing in the wage w_2, so

[3] We follow Paarsch and Shearer (2000) for the description of the utility function.

profit is maximized for the minimal value of w_2 accepted by the workers, i.e. $\overline{u} + \bar{\kappa}\bar{e}_k(K_j)$. □

Assumption 3. *The elasticity of productivity in the second type of job is less than 1, i.e.* $b'_k(K)K/b_k(K) < 1$*, for all* K*.*

Theorem 4. *Under assumptions 1, 2 and 3, under the constraint that* K_j *and* N_j *are exogenously fixed, the firm's choice of* N_{1j} *is decreasing in* K_j *while firm's choice of* N_{2j} *is increasing in* K_j*.*

Proof. Due to the Leontieff specification of the output function, maximizing output per worker leads to $N_{1j} = \frac{b_k(K_j)N_j}{aK_j+b_k(K_j)}$ and $N_{2j} = \frac{aK_jN_j}{aK_j+b_k(K_j)}$. Differentiating with respect to K_j gives $\frac{\partial N_{2j}}{\partial K_j} = \frac{N_j a b_k(K_j)}{(aK_j+b_k(K_j))^2}\left(1 - \frac{b'_k(K_j)K_j}{b_k(K_j)}\right)$, which is negative by assumption 3. The derivative of N_{1j} with respect to K_j is the opposite of the derivative of N_{2j} above, which concludes our proof. □

Notice that $\frac{N_{1j}}{N_j}$ in each industry is independent of N_j, and is decreasing in K_j. Therefore we can invert this relationship and write K_j as a function of $x \equiv \frac{N_{1j}}{N_j}$ (as in section 2), and use $t \equiv N_j$

Main results

- Wage for type 1 workers is constant in each sector, potentially different across sectors.
- Wage for type 2 workers is decreasing in the share of good workers, by assumption 3.
- Firms can be indexed by the pair consisting of the share of type 1 workers and the size of the firm; if we assume these to be drawn from a bivariate distribution, we may use the results of section 2.

5.2 Covariances induced by a simpler model

We interpret the previous results in the terms used in section 2, with a simplifying assumption on the distribution of (x, t).

We assume in this subsection that given x there is only one size of firm in each industry. Thus we can use x as the index of firms in this economy, and write $t = \tau_k(x)$. If τ is increasing, then inside each industry N_{1x} (type 1 workers in firm x) increases with x, whereas N_{2x} decreases.

Lemma 5. *Inside each industry, the total person effect for each firm (*$xtW + (1-x)tU$ *in the notations of section 2) will be increasing in* x*.*

The shape of the firm effect is ambiguous. Under the assumptions 1 through 3, the firm effect will be decreasing inside each sector if $(1-x)\tau(x)(u(x)-U)$ is decreasing (in the notations of the first section). This holds if $(1-x)\tau(x)$ is not too decreasing.

Lemma 6. *Inside each sector, the firm effect can be decreasing in x.*

These lemmas show that the covariance between firm and person effects inside each sector can be negative (the firm effect is decreasing in x while the person effect is increasing in x). Thus within-industry correlations can be negative in this model, provided some assumptions on capital, minimum effort and size of the firm hold.

The between-industry correlation is governed by the mean in each sector of the person and the firm effect. If sectors are ranked by mean size of firms, then it is possible for the mean person effect to increase with the mean size of the firm, while the mean firm effect is increasing, for example if $b_k(\bar{K}_k)$ is increasing with k.

6 Estimating the Model

This section presents a parameterization of the wages and the distribution of firms in the economy.

We draw on the implied structure of wages from section 3 and on the firm and worker effects from section 2.

6.1 Parametric setup

In the notations of section 1, we assume the following specification:

Wage type 1 $w(x,t,k) = w_k$, the wage is constant in each industry $\quad (K)$

Wage type 2 $u(x,t,k) = u_{0k} + u_{1k}x + u_{2k}\log t + u_{3k}x\log t \quad (4K)$
We expect $u_1 < 0$ from the incentive argument and $u_2 > 0$ from the results on firm-size wage differentials.

Distribution of firm sizes is assumed to be log-normal in each sector, $f(t|k) = \text{log-normal}(\mu_k, \sigma_k^2) \quad (2K)$

Distribution of share of type 1 workers conditional on firm size is a Beta distribution with parameters depending on the size of the firm, sector varying; $f(x|t,k) = \text{Beta}(p_k(t), q_k(t))$ with $p_k(t) = p_{0k} + p_{1k}t$, $q_k(t) = q_{0k} + q_{1k}t \quad (4K)$

Sector sizes in number of firms, J_k for all k The size of each sector in terms of workers is given by $J_k \iint t f(t) f(x|t)\, dx dt$. $\quad (K)$

The total number of unknown parameters is therefore $12K$. Identifiable parameters: $12K-1$, since only $J_k/\sum_{k'} J_{k'}$ is identified.

The measure μ used in section 2 would therefore be $\mu_k(x,t) = J_k f(t|k) f(x|t,k)$. Formulas for the log-normal and the beta densities are given in appendix A.1.

We will use the following moments to fit the parameters:

Intra-industry variances and covariances of firm and person effects ($3K$)

Intra-industry variances and covariances by size of firm ($3KT$) if we partition firms in T categories according to size.

Total moments available: $3(T+1)K$. Therefore we must have $T \geq 3$ to be able to identify the parameters.

6.2 Correlation formulas

Given this parametric structure, we can construct the function $\Psi : \mathbb{R}^{12K} \to \mathbb{R}^{3(T+1)K}$ that, given the parameters $(w_k, u_{jk}, J_k, \mu_k, \sigma_k, p_j, q_j) \in \mathbb{R}^{12K}$, calculates the covariances and variances of the underlying model. By asymptotic least squares we will therefore be able to find the parameters that best fit the observed moments.

We will show first that all the double integrals appearing in section 2 can be simplified to simple integrals, and that all covariances can be written as simple functions of a finite number of simple integrals.

Definition 7. *We write* $\mathbb{S}(g(X,T)|k)$ *the integral* $\iint_{\Omega_k} g(x,t) f(t) f(x|t)\, dxdt$ *for arbitrary integrable* g.

Theorem 8. *Any integral of the type* $\mathbb{S}(X^i g(T)|k)$ *can be calculated as a simple integral; we have*

$$\mathbb{S}(X^i g(T)|k) \equiv \iint_{\Omega_k} x^i g(t) f(t) f(x|t)\, dxdt = \int_{\mathbb{R}^+} g(t) \frac{B(p(t)+i, q(t))}{B(p(t), q(t))} f(t|k)\, dt. \tag{8}$$

Proof.

$$\begin{aligned}
\iint_{\Omega_k} x^i g(t) f(t) f(x|t)\, dxdt &= \int_{\mathbb{R}} g(t) f(t|k) \int_0^1 \frac{x^{p_k(t)+i-1}(1-x)^{q_k(t)-1}}{B(p_k(t), q_k(t))}\, dx\, dt \\
&= \int_{\mathbb{R}^+} g(t) f(t|k) \frac{B(p_k(t)+i, q_k(t))}{B(p_k(t), q_k(t))}\, dt \\
&= \int_{\mathbb{R}^+} g(t) \frac{B(p_k(t)+i, q_k(t))}{B(p_k(t), q_k(t))} f(t|k)\, dt
\end{aligned}$$

□

Since $B(p+1,q) = \frac{p}{p+q}B(p,q)$ and $B(p,q) = B(q,p)$ the ratios of the beta functions has an explicit expression $\frac{B(p+i,q)}{B(p,q)} = \frac{p(p+1)\cdots(p+i-1)}{(p+q)\cdots(p+q+i-1)}$.

Setting $y = \log t$ the integral is fairly easy to calculate numerically, since $e^y f(e^y)$ is gaussian, $p(e^y) = p_0 + p_1 e^y$ and $q(e^y) = q_0 + q_1 e^y$.

$$\mathbb{S}(X^i T^j (\log T)^h | k) = \int_{\mathbb{R}} e^{jy} (\log T)^h R(y,i) \phi\left(\frac{y-\mu_k}{\sigma_k}\right) dy \tag{9}$$

with $R(y,i)$ a rational fraction in e^y, $R(y,i) \equiv \frac{p(p+1)\cdots(p+i-1)}{(p+q)\cdots(p+q+i-1)}$.

6.2.1 The person effects

We show that $\iint x(1-x)t(\cdot)f(x,t)\,dxdt$, with $(\cdot)$ equal to $w(x,t)$, $u(x,t)$ and 1, are written as sums of $\mathbb{S}(X^i T^j|k)$ terms.

We have

$$\iint x(1-x)tw(x,t)f(x,t)\,dxdt = \sum_k J_k \iint x(1-x)tw(x,t,k)f(x|t)f(t)\,dxdt$$

$$= \sum_k w_k J_k \left[\int_{\mathbb{R}^+} tf(t,k) \int_0^1 x(1-x)f(x|t)\,dx\,dt\right] \tag{10}$$

$$= \sum_k w_k J_k \mathbb{S}\left[X(1-X)T|k\right] \tag{11}$$

$$= \sum_k w_k J_k \left[\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)\right] \tag{12}$$

and

$$\iint x(1-x)tf(x,t)\,dxdt = \sum_k J_k \mathbb{S}\left[X(1-X)T|k\right] \tag{13}$$

$$= \sum_k J_k \left[\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)\right] \tag{14}$$

Thus W is the between-sector mean of the wages of type 1 workers, weighed across sectors by $J_k\left[\mathbb{S}(XT|k) - \mathbb{S}(X^2T|k)\right]$.

For the workers of type 2 we have

$$\iint x(1-x)tu(x,t)f(x,t)\,dxdt$$

$$= \sum_k J_k \iint x(1-x)t(u_{0k} + u_{1k}x + u_{2k}\log t + u_{3k}x\log t)f(x|t)f(t)\,dxdt$$

$$= \sum_k J_k \left[\begin{array}{c} u_0\mathbb{S}(XT) + u_2\mathbb{S}(XT\log T) - u_1\mathbb{S}(X^3T) - u_3\mathbb{S}(X^3T\log T) \\ +(u_1-u_0)\mathbb{S}(X^2T) + (u_3-u_2)\mathbb{S}(X^2T\log T) \end{array}\right]$$

where we have dropped the $_k$ indices for simplicity.

6.2.2 Firm effects

Firm effects are given by the formula

$$\psi(x,t,k) = x(w(x,t,k) - W) + (1-x)(u(x,t,k) - U) \tag{15}$$

$$\begin{aligned} &= x(w_k - W) + (1-x)(u_{0k} - U + u_{1k}x + u_{2k}\log t + u_{3k}x\log t) \\ &= u_{0k} - U + (w_k + U - W + u_{1k} - u_{0k})x - u_{1k}x^2 \\ &\quad + u_{2k}\log t + (u_{3k} - u_{2k})x\log t - u_{3k}x^2\log t \end{aligned} \tag{16}$$

6.2.3 Variances and covariances

The total worker effect in a firm (x,t) is equal to

$$\theta(x,t) = t\theta_m(x,t) = xtW + (1-x)tU \tag{17}$$

Since $\iint xtf(x,t)\,dxdt = M$ and $\iint(1-x)tf(x,t)\,dxdt = N$, we have

$$\begin{aligned} \bar{\theta} = \frac{\mathbb{S}(\theta)}{\mathbb{S}(T)} &= \frac{1}{M+N}\iint \theta(x,t)f(x,t)\,dxdt \\ &= \frac{1}{M+N}(MW+NU) \end{aligned} \tag{18}$$

$$\begin{aligned} var\,\theta &= \frac{1}{M+N}\iint (xtW^2 + (1-x)tU^2)f(x,t)\,dxdt - (\bar{\theta})^2 \\ &= \frac{MN}{(M+N)^2}(W-U)^2 \end{aligned} \tag{19}$$

$$var(\theta|k) = \frac{S(XT|k)}{S(T|k)}\left(1 - \frac{S(XT|k)}{S(T|k)}\right)(W-U)^2 \tag{20}$$

The mean firm effect, as weighed by the size of each firm, is

$$\begin{aligned} \bar{\psi} &= \frac{\mathbb{S}(T\psi)}{\mathbb{S}(T)} \\ &= \left(\sum_k J_k \mathbb{S}(T|k)\right)^{-1} \cdot \\ &\quad \sum_k J_k \mathbb{S}\left(\begin{array}{c} (u_{0k} - U)T + (w_k + U - W + u_{1k} - u_{0k})XT \\ -u_{1k}X^2T + u_{2k}T\log T + (u_{3k} - u_{2k})XT\log T - u_{3k}X^2T\log T \end{array} \middle| k \right). \end{aligned}$$

The variance is equal to

$$var\,\psi = \left(\sum_k J_k \mathbb{S}(T|k)\right)^{-1} \sum_k J_k \mathbb{S}(T\psi^2|k) - (\bar{\psi})^2 \tag{21}$$

and its full formula is given in the appendix A.3.

The covariances are defined as

$$cov(\theta,\psi|k) = \frac{1}{\mathbb{S}(T|k)}\int_{\Omega_k} (\theta(x,t) - \mathbb{S}(\theta|k))(\psi(x,t) - \mathbb{S}(\psi|k))f(x|t)f(t)\,dxdt$$

but again the formula for the cross product is in the appendix.

7 Estimation

7.1 The Data

We use data from the Déclarations annuelles des donnés sociales (DADS), a 1/25th sample of the French work force with information from 1976-1996, and data from the Longitudinal Employer-Household Dynamics Program (LEHD), universe data for four large American states with information from 1990-2000.

The person and firm effects in equation (1) were estimated using the procedure described in ACK 2002, an exact least squares solution that calculates all identifiable person and firm effects by fixed-effect methods. Data from both sources were then aggregated using the same procedure. Observations were categorized into 2-digit industry groups (2-digit 1987 SIC for the US, NAP-100 for France). The first four moments of the pair $\left(\theta_i, \psi_{J(i,t)}\right)$ were calculated within and between these industry groups. The structural model was fit to the first two moments. The third and fourth moments are used for standard error calculations.

7.2 The Procedure

We are thus left with the problem of estimating $12K - 1$ parameters by fitting a function that needs the evaluation of 19 numerical integrals, not infeasible but computationally intensive. The following simplifications are made:

- the parameters of the distribution of firms' sizes are estimated on the observed distribution of sizes on a given year (1989 ?), as well as the number of firms in each sector (that is to say, μ, σ^2 and J_k are fitted);
- as a first step the integrals are evaluated by Gauss-Legendre quadrature; once convergence has been attained, the process is started again with Simpson-type integration;

We implement the minimization by standard non-linear least square routines.

7.3 The Results

In each industry there is a distribution of wages and of firm structures (u and x). To sum up this information we calculate the mean of these two variables in each size category, for all industries. This is done as follows:

$$\bar{x} = \frac{\iint txf(x,t)\ dxdt}{\iint tf(x,t)\ dxdt}$$

$$\bar{u} = \frac{\iint tu(x,t)f(x,t)\ dxdt}{\iint tf(x,t)\ dxdt}$$

Thus $\bar{x}$ is the average number of type 1 workers, and $\bar{u}$ is the potential mean wage of type 2 workers. Other measures are possible, for instance $\frac{\iint t(1-x)u(x,t)f(x,t)\ dxdt}{\iint t(1-x)f(x,t)\ dxdt}$, the actual mean wage of type 2 workers.

7.3.1 Analyzing the Results

Regression analysis, explaining $\bar{u}$ as a function of size, $\bar{x}$, industry dummies.

Different results for France and the States. Importance of weighting each industry by the number of persons.

Table 2: distribution of parameters in France

Table 3: distribution of parameters in the US

Figure 2: Fit of model for France with large industries only

Figure 3: Fit of model for France with all industries

Figure 4: Fit of model for US with large industries only

Figure 5: Fit of model for US with all industries

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A Appendix

A.1 Distributions

Distributions: log-normal (μ,σ^2)

$$f(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{t} \exp\left(-\frac{1}{2\sigma^2}(\log t - \mu)^2\right) \tag{22}$$

Beta (p,q)

$$f(x|t) = \frac{x^{p-1}(1-x)^{q-1}}{B(p,q)} = \frac{\Gamma(p+q)}{\Gamma(p)\Gamma(q)} x^{p-1}(1-x)^{q-1} \tag{23}$$

where $B(p,q)$ is the beta function and $f(x,t,k) = J_k f(x|t,k) f(t,k)$; J_k is the size (in firms) of sector k.

A.2 Exact formulas

We have the following equalities:

$$\begin{aligned}
\mathbb{S}(1|k) &= 1 \\
\mathbb{S}(T^n|k) &= \exp(n\mu_k + n^2\frac{\sigma_k^2}{2}) \\
\mathbb{S}(T\log T|k) &= (\mu_k + \sigma_k^2)\exp(\mu + \frac{\sigma_k^2}{2}) \\
\mathbb{S}(T(\log T)^2|k) &= (\mu_k^2 + \sigma_k^2 + 2\mu_k\sigma_k^2 + \sigma_k^4)\exp(\mu + \frac{\sigma_k^2}{2})
\end{aligned}$$

A.3 Variance and covariance of the firm and worker effects

$$\begin{aligned}
\mathbb{S}(T\psi^2|k) = \mathbb{S}(&T\,(U-u_0)^2 \\
&- 2TX\,(U-u_0)(U-W-u_0+u_1+w_0) \\
&+ TX^2\,({u_0}^2 + {u_1}^2 + (U-W+w_0)^2 + 2u_1(2U-W+w_0) - 2u_0(U-W+2u_1+w_0)) \\
&- 2TX^3\,u_1(U-W-u_0+u_1+w_0) \\
&+ TX^4\,u_1^2 \\
&+ 2T\log T\,(-U+u_0)u_2 \\
&+ 2TX\log T\,((-U+u_0)u_3 + u_2(2U-W-2u_0+u_1+w_0)) \\
&+ 2TX^2\log T\,(u_2(-U+W+u_0-2u_1-w_0) + u_3(2U-W-2u_0+u_1+w_0)) \\
&+ 2TX^3\log T\,(u_1(u_2-2u_3) + u_3(-U+W+u_0-w_0)) \\
&+ 2TX^4\log T\,u_1u_3 \\
&+ T\,(\log T)^2\;{u_2}^2 + 2TX\,(\log T)^2\,u_2(-u_2+u_3) \\
&+ TX^2\,(\log T)^2\;({u_2}^2 - 4u_2u_3 + {u_3}^2) \\
&+ 2TX^3\,(\log T)^2\;(u_2-u_3)u_3 \\
&+ TX^4\,(\log T)^2\,{u_3}^2|k)
\end{aligned}$$

$$
\begin{aligned}
\mathbb{S}(\theta\psi|k) = \mathbb{S}(&T\,U(-U+u_0) \\
&+ TX\,((-2U+W)u_0 + U(2U-2W+u_1+w_0)) \\
&+ TX^2\,((U-W)u_0 + (-2U+W)u_1 - (U-W)(U-W+w_0)) \\
&+ TX^3(U-W)u_1 \\
&+ T\log T\,U u_2 \\
&+ TX\log T\,((-2U+W)u_2 + U u_3) \\
&+ TX^2\log T\,((U-W)u_2 + (-2U+W)u_3) \\
&+ TX^3\log T\,(U-W)u_3|k)
\end{aligned}
$$

Table 1
Decomposition of Person and Firm Effects

		Variance of Person Effect	Variance of Firm Effect	Covariance of Person and Firm Effects	Correlation of Person and Firm Effects
France	Total	0.2490	0.2176	-0.0562	-0.2415
	Within	0.2450	0.2029	-0.0600	-0.2690
	Between	0.0040	0.0147	0.0038	0.4893
US	Total	0.6292	0.1304	0.0058	0.0203
	Within	0.6149	0.0945	0.0006	0.0025
	Between	0.0143	0.0359	0.0052	0.2293

Notes:

Table 2
Quantiles for France, weighted by total employment

	5	10	25	50	75	90	95
corr	-0.4402	-0.4402	-0.3925	-0.3133	-0.1750	-0.0668	-0.0668
mu	0.4155	0.4485	0.5816	1.1427	1.6902	1.9349	2.3382
sigma	0.7660	0.7838	0.9861	1.3004	1.5691	1.7671	1.9298
w	-2.6236	-1.4766	-0.8186	-0.0799	0.6733	1.3946	1.6826
u_0	-1.6614	-1.3741	-0.6507	0.1014	0.8391	1.5007	2.6445
u_1	-5.1953	-0.2685	2.8442	4.7725	8.0286	14.8137	14.8137
u_2	-1.4672	-0.8339	-0.1357	0.0131	0.0624	0.1802	0.8128
u_3	-2.1863	-0.8871	-0.3574	0.0215	0.7381	1.0809	1.8465
p_0	-0.5340	0.0016	0.2790	0.6465	3.0101	9.6754	9.6754
p_1	0.0008	0.0019	0.0052	0.0116	0.0379	0.3468	0.4914
q_0	-0.0321	0.1117	0.7231	2.3433	5.6771	44.6569	44.6569
q_1	0.0015	0.0065	0.0428	0.0765	0.1447	0.2257	0.6533

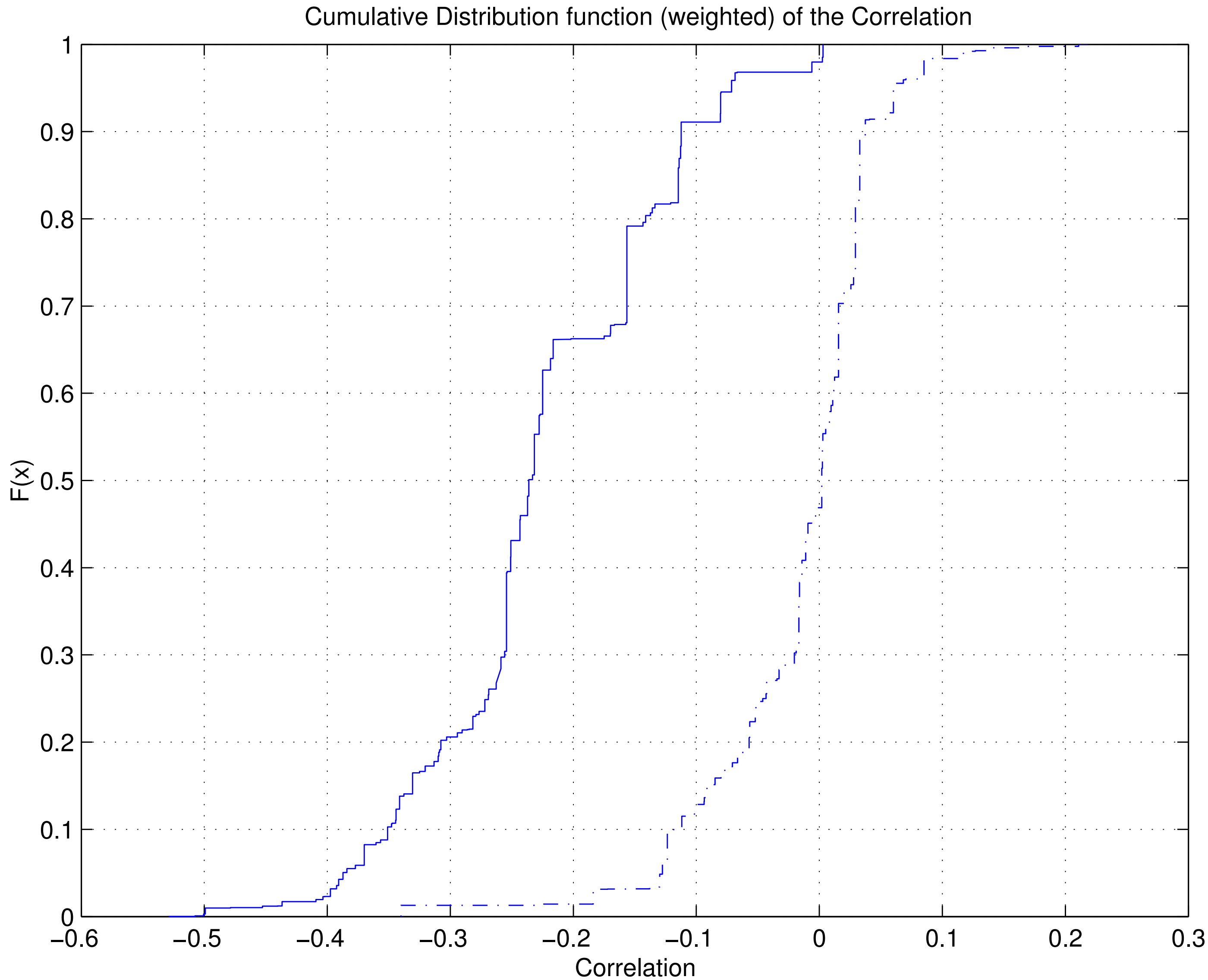
Notes: distribution of parameters for France, across 2-digit industries, 95 observations weighted by the number of total persons in the industry. Corr is the within-industry covariance of worker and firm effects, $\bar{u}$ and x are the average value of u and x respectively, μ, mean of log wage in industry; σ^2, the variance; w, wage of type 1 workers; u, wage of type 2 workers = u0+u1*x+u2*log(t)+u3*x*log(t), where x is the share of type 1 workers and t is the size of the firm; Beta(p,q), law of x conditional on t, p=p0+p1*t, q=q0+q1*t.

Table 3
Quantiles for the US, weighted by total employment

	5	10	25	50	75	90	95
corr	-0.1841	-0.1236	-0.0641	-0.0062	0.0579	0.1698	0.2108
mu	1.0649	1.1586	1.3187	1.5324	2.0654	2.4235	2.6154
sigma	1.0618	1.1414	1.2051	1.3769	1.6368	1.9890	1.9890
w	-4.9172	-4.2219	-4.1303	-2.1964	0.8552	2.7455	3.8426
u_0	-3.6534	-2.5852	-0.8232	2.1105	4.2715	4.8429	4.9670
u_1	-22.3823	-19.7591	-17.1106	-9.0151	-1.0409	4.9779	11.7983
u_2	-3.0649	-3.0649	-1.1055	-0.1154	0.7566	1.5385	2.0238
u_3	-11.9209	-8.3503	-4.1090	1.0368	2.0911	3.9060	4.1796
p_0	0.2198	0.7548	3.1761	5.4524	38.5532	103.5036	103.5036
p_1	0.0010	0.0107	0.0569	1.0724	1.6390	10.7704	11.5690
q_0	-4.6196	-0.0606	1.2338	1.7240	27.9172	60.6475	104.4037
q_1	0.0011	0.0057	0.0252	0.1902	3.6185	21.7386	54.3603

Notes: distribution of parameters for the US, across 2-digit industries, 81 observations weighted by the number of total persons in the industry. Corr is the within-industry covariance of worker and firm effects, $\bar{u}$ and x are the average value of u and x respectively, μ, mean of log wage in industry; σ^2, the variance; w, wage of type 1 workers; u, wage of type 2 workers = u0+u1*x+u2*log(t)+u3*x*log(t), where x is the share of type 1 workers and t is the size of the firm; Beta(p,q), distribution of x conditional on t, p=p0+p1t, q=q0+q1t.

Figure 1:



Note: France in solid line, the US in dashed line.

Figure 2: France

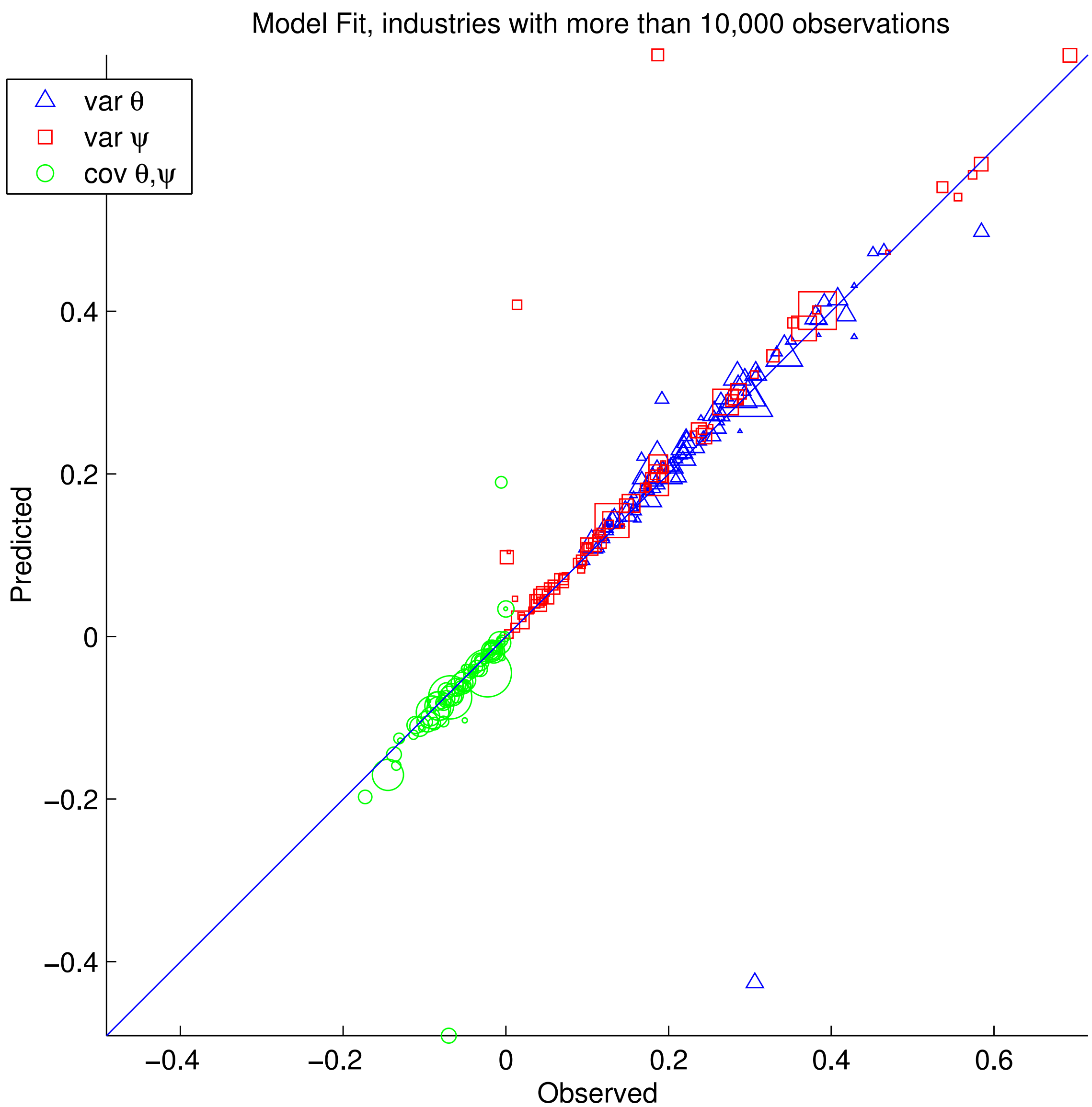
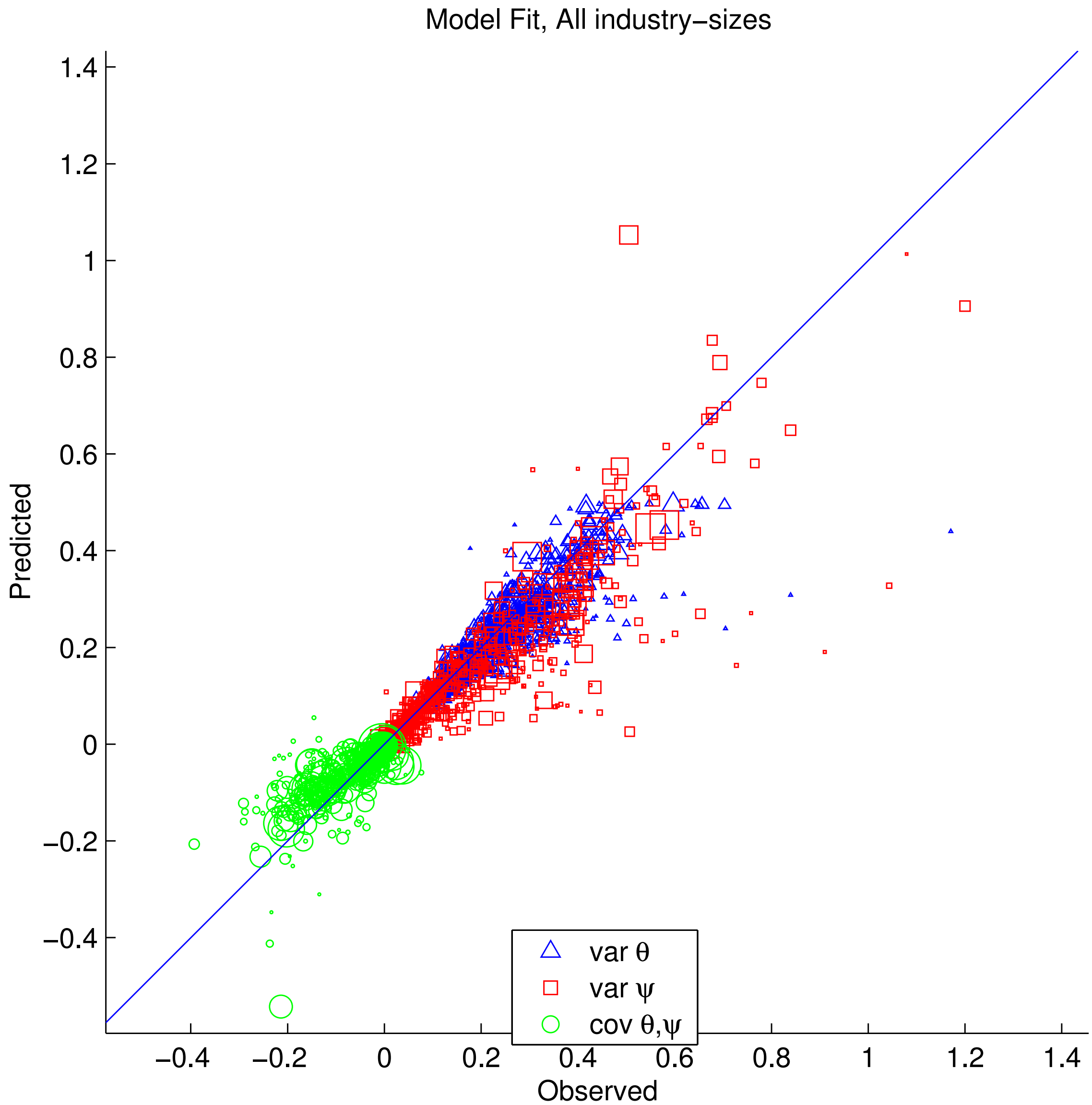


Figure 3: France



Note: Some small outliers removed.

Figure 4: United States

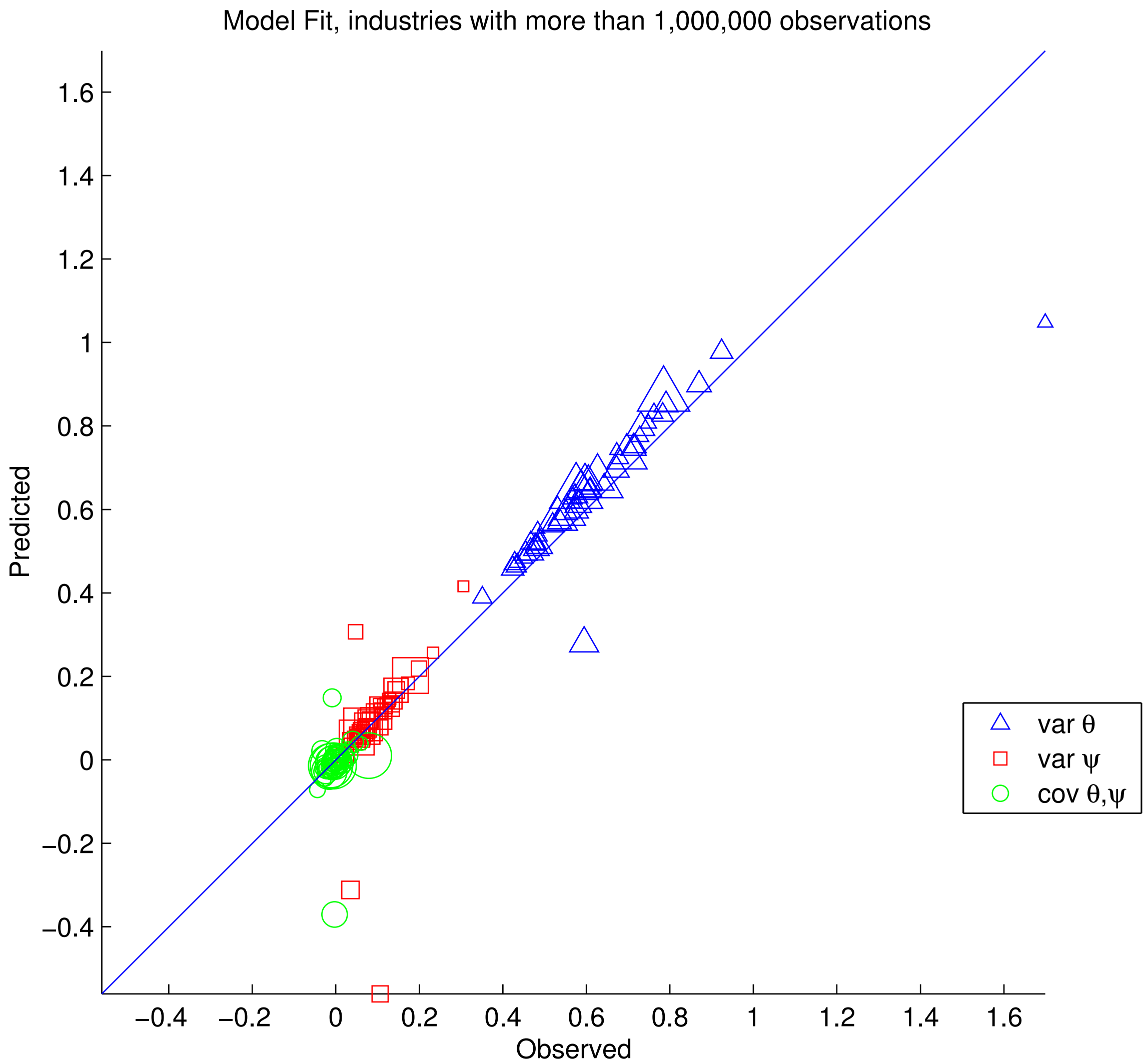
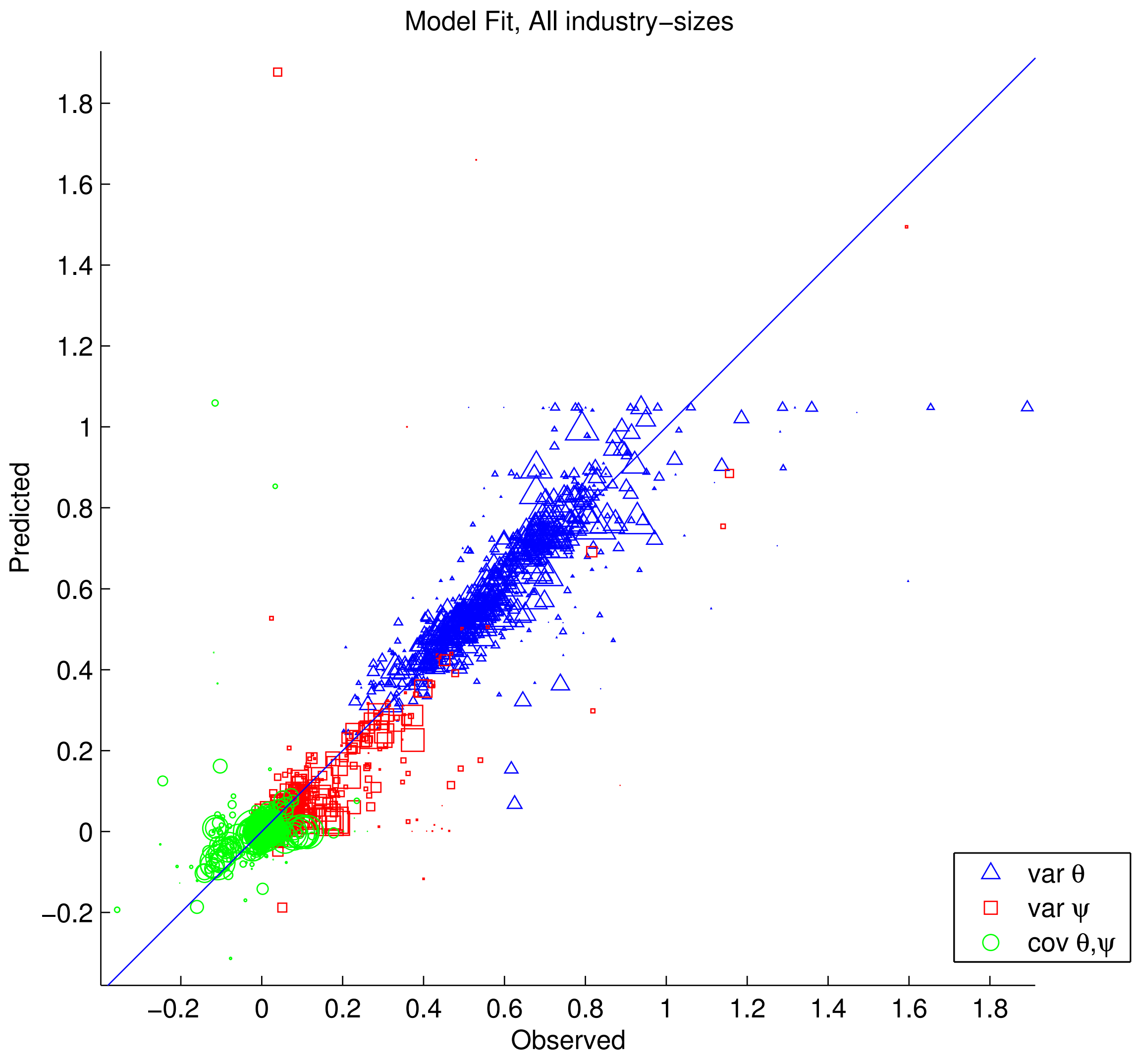


Figure 5: United States



Note: Some outliers removed.