# Study of the Austrian Labour Market Dynamics Through a Model of Search Equilibrium

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#### Abstract

In this work we provide a theoretical overview of a search equilibrium model with continuous productivity dispersion and perform its estimation for the Austrian labour market data. We describe empirically the dynamics of equilibrium outcomes of the labour market. Special emphasis is made on the analysis of changes in labour mobility and on the dependence of expected job durations on offered wages. Moreover, we investigate the influence of excessive labour mobility and fluctuations in the monopsony power of employers on the profitability of firms. Facing the problem of top-coded wage data, we additionally suggest an appropriate adjustment of the existing estimation methodology. Finally, we extend the econometric model for inclusion of the observed heterogeneity of agents.

The obtained results demonstrate satisfactory explanatory power of the model.

#### 1 Introduction

Over the last five years models of search equilibrium acquired enhancing interest among labour market researchers. One reason for that is a rapid improvement in the theoretical development of this class of models.

Among the most significant achievements of the last years one can point out the following works. First of all it is a paper of Burdett and Mortensen (1998), who enrich the model with on-the-job search and extend it for heterogeneity of agents. They also show that with heterogeneous producers equilibrium wage offer distribution attains acceptable shapes. Secondly, these are contributions of Bontemps et al. (1997), (2000), who deliver a comprehensive study of the version of the model with the continuous productivity dispersion of producers. Moreover they develop simple and robust procedure for the estimation of this kind of models. Bontemps et al.

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(1999) also make an attempt of further generalization, discussing equilibrium solution in the version with heterogeneity in both labour supply and demand sides.

Advances in the theoretical development of search equilibrium modelling triggered a wave of applied work that resulted in a number of interesting findings. Bowlus, Kiefer and Neumann (1995), (2001) explore equilibrium wage distributions in the model with heterogeneous producers. They apply their own methodology to study the speed of transition from school to work, as well as wage differentials of black and white college graduates in the US labour market. Koning, Ridder and van den Berg (1995) use the model to study structural and frictional unemployment and describe the effects of minimum wage policy experiments in the Dutch labour market. Ridder and van den Berg (1998) offer a survey of employment and unemployment durations in the Netherlands. Eventually, Bowlus and Grogan (2001) apply search equilibrium setting to look into the gender wage differentials in the UK. Along with this line of empirical research based mainly on the two-state equilibrium search model with discrete productivity dispersion of producers, Bontemps et al. (2000) perform structural estimation of the model with continuous productivity dispersion for the French labour market data.

Considerable progress in this research area inspired also the present work, where equilibrium search model with on-the-job search and continuous productivity dispersion of producers following Burdett and Mortensen (1998) and Bontemps et al. (2000) is estimated with the Austrian labour market data.

Working with the Austrian labour market one has to admit that its performance has been studied up to date rather comprehensively. One of the most interesting works here was done by Mayrhuber and Url (1999), who investigate the dynamics of mean employment and unemployment durations of labour suppliers. not less important paper is due to Winter-Ebmer (1998), who looks into the interdependence between unemployment durations and the length of unemployment benefit. These studies create quite a general picture of the market situation with a special concentration on the supply side. However, analytical tools in these and all the other existing papers are exclusively empirical duration models, which, despite being intuitively appealing, are not constructed on the basis of the rational economic behavior of participating agents. So the first goal of the present work is fairly general. We will try to provide another way of empirical characterization of equilibrium outcomes on the labour market, since in our case econometric methods will be rather applied to estimate the structural economic model. As the structural parameters of the search equilibrium model determine optimal behavior of agents in equilibrium, their estimates will give more consistent with relevant economic theory description of the labour market performance. Given that real-life agents behave according to the assumptions of the model, we receive as a result much richer information about the market. Furthermore, specifying structural parameters to be dependent on a selected set of explanatory variables, the search equilibrium model will completely incorporate the empirical duration model. Again estimated effect of any given variable, will now be the effect predicted by the theory.

 $<sup>^{1}</sup>$ Bowlus and Grogan (2001), who modify the model to embed non-participation, are the only exception.

Richness of the available data set also allows estimating the model in two different points of time. This will enable us to detect changes in equilibrium solution and possibly argue about contemporary trends in the employment (unemployment) dynamics, profitability of active firms and market power of the parties that compete in wage setting.

Apart from such general goals search equilibrium setting can be used to learn about certain facts, which empirical duration models simply cannot grasp. As one possible application we can consider a study of the influence, which changes in the labour supply side have on the profitability of employers. Reviewing the existing literature on the Austrian labour market we can discover some questions of this kind. For instance Mayrhuber and Url (1999) find that labour mobility in the Austrian market became surprisingly high. They conjecture that it may affect the efficiency of entrepreneurs driving down their equilibrium profits. The argument is that with excessively high labour mobility companies are not able to create a pool of human capital sufficient for effective operation. The time individuals stay on the job is too short for the firm to compensate the costs, forgone to train them for this particular job. As a result firms become worse off.

To tell whether excessive labour mobility harms entrepreneur or not one has to deal with the labour demand side, which is unfortunately underrepresented in the standard duration models. To the contrary, employers are a part of any search equilibrium model and equilibrium profit is a part of its solution. Therefore, within the search equilibrium framework it may be possible to tell whether too short employment durations really contribute to inefficiency or not. Taking a more general look, the operational efficiency of the firm may vary with the employers productivity, degree of market power etc. Influence of these phenomena can also be tracked by the model.

To the best of my knowledge operational efficiency issue has not been treated in the literature on equilibrium search models up to now. So the second goal of the present work is to try to fill in this gap.

Eventually, the paper is motivated by the fact that equilibrium search models become an emerging tool in policy modelling. Thus, the analysis of the overall fit of these models to the data may be important, since it tells up to which extent they can match the reality. The higher is the fit, the more meaningful are the predictions of the model so the bigger is its value for the policy maker.

The paper is organized as follows. The second section provides a brief theoretical overview of the model. The third section describes the data and points out some of its particular features, which will require an appropriate estimation techniques. In the fourth section we describe the estimation methodology. The fifth section presents the results of the work and their discussion. In the conclusion we summarize the main findings.

### 2 The Model

As suggested above, here is a brief description of the model. The model incorporates both labour supply side (workers) and labour demand side (employers) who meet on the market. Workers are searching for jobs and employers are offering job

opportunities. Both types of agents are assumed to be rational. Workers maximize their utility of being employed and employers maximize their profits.

Workers utility U(w) is an increasing function of wage<sup>2</sup> earned. assumed to be homogeneous with respect to their opportunity cost of employment (b) and thus have the same reservation wage (R). There are two states in which workers can be, namely, "employment" and "unemployment", and workers search whenever employed or unemployed. These two states change each other randomly, therefore individuals working history can be described by a Poisson process. The transition from current job to a better paid job is also qualified as a change of state, so there are three Poisson arrival rates that govern all transitions in the working history. We define the arrival rate of an acceptable job offer to unemployed worker as  $\lambda_0$ , arrival rate of a job offer to employed worker as  $\lambda_1$  and arrival rate of a layoff as  $\delta$ . Search process of individual can be formalized as a repeated drawing of job offers from some probability distribution F(w) and acceptance or rejection of the offer after each draw. It is important to notice that rejected wage offers are unobserved. Available earnings data are just current salaries of employed individuals and so are necessarily accepted wages. Therefore instead of offer distribution F(w) only earned wage distribution G(w) can be observed.

Searching workers face an optimal stopping problem. If a searching agent is currently unemployed, Mortensen and Neumann (1988) show that the solution for this problem is a reservation wage

$$R = b + (\kappa_0 - \kappa_1) \int_R^{\overline{w}} \frac{\bar{F}(x)}{1 + r/\delta + \kappa_1 \bar{F}(x)} dx \tag{1}$$

where  $\kappa_0 = \lambda_0/\delta$ ,  $\kappa_1 = \lambda_1/\delta$ ,  $\bar{F}(x) = 1 - F(x)$ ,  $supp(F) = [\underline{w}, \overline{w}]$  and r is an interest rate. If an agent is employed, the solution is to accept any wage greater than that currently earned. This constitutes workers utility maximizing strategy of behavior on the market. Additionally Mortensen and Neumann (1988) suggest there exists an unambiguous relation between offer arrival rates and search intensity of an individual. Therefore without loss of generality they associate  $\lambda_0$  and  $\lambda_1$  that satisfy (1) with agents optimal search intensity<sup>3</sup>.

It should be noticed that in economies with legal minimum wage exceeding reservation wage of the agent an optimal strategy for any worker reduces to the rule of accepting any wage if unemployed and again wage greater than currently earned, if employed. Moreover in case of  $R \leq w_{\min}$  we state that F(R) is either zero or  $F(w_{\min})$ , which makes considerable simplification for further analysis. As this is exactly the case with Austria (see Section 3 for the discussion of Austrian minimum wage setting system), we adopt this simplification in all subsequent formulas, referring an interested reader to Burdett, Mortensen (1998) for fragments of a general case.

To formulate the employers problem we start off with two important findings. Both of them are due to Burdett and Mortensen (1998). Let U be a steady state number of unemployed workers, M – total number of supplying agents and N – steady

 $<sup>^2</sup>$ The present work uses the terms "wage", "income" and "earnings" interchangeably. In all cases income before tax is meant.

<sup>&</sup>lt;sup>3</sup>In the present work  $\lambda_0$  and  $\lambda_1$  will be frequently referred to as "search intensity parameters".

state number of active firms. Then in equilibrium the probability of encountering an unemployed agent is  $\frac{U}{M} = \frac{1}{1+\kappa_0}$ . Moreover, in equilibrium there exists an unambiguous dependence between unobserved offer and observed earnings distribution (F,G) and density (f,g) functions

$$\bar{F}(w) = \frac{1 - G(w)}{1 + \kappa_1 G(w)} \qquad f(w) = \frac{1 + \kappa_1}{[1 + \kappa_1 G(w)]^2} g(w) \tag{2}$$

These identities make it possible to derive the amount of workers l attracted in the steady state by any offered wage w

$$l(w) = \frac{M - U}{N} \frac{(1 + \kappa_1)}{\left[1 + \kappa_1 \bar{F}(w)\right]^2} = \frac{M - U}{N} \frac{\left[1 + \kappa_1 G(w)\right]^2}{1 + \kappa_1}$$
(3)

As we see this amount turns out to depend on the unknown search intensity parameters. It is also true that the number of workers attracted by a given wage is increasing in offered wage.

Employers now face a problem of maximizing their profit with respect to wage paid

$$\pi = \max_{\{w\}} (p - w)l(w) \tag{4}$$

with l given by (3). The profit function (4) is increasing in productivity p and ambiguous in wage paid. The productivity determinant p is both the labour productivity of a given employer and the labour productivity of given employee. So it is implicitly assumed that no employer discourages workers making them preforming worse than they actually can.

In the considered version of the model employers are heterogeneous with respect to their productivity. The probability distribution of productivity across active firms, say  $\Gamma(p)$ , is assumed to be continuous,  $supp(\Gamma) = [\underline{p}, \overline{p}]$ . Bontemps et al. (1997) show that under the assumption of a continuous productivity distribution there exists a unique single valued, monotone and continuous function w = K(p), which maps the support of the productivity distribution into the support of the wage offer distribution. This finding is important because it suggests that each p-type firm offers only one wage, which rules out discontinuity of the wage offer distribution, immanent to the models with discrete productivity dispersion. Existence and uniqueness of K(p) allows to express first order conditions for firms in terms of firms productivity. Indeed, taking the derivative of (4) with respect to w, and using some algebra single firm optimality conditions become

$$p = K^{-1}(w) = w + \frac{1 + \kappa_1 \bar{F}(w)}{2\kappa_1 f(w)} = w + \frac{1 + \kappa_1 G(w)}{2\kappa_1 g(w)}$$
 (5)

where p is a known firm-specific constant. Moreover Bontemps et al. (2000) show that using FOC in (5) one can derive the probability density of the productivity levels

of firms operational in equilibrium

$$\gamma(p) = \frac{2\kappa_1 f(w)^3}{\kappa_1 f(w)^2 - f'(w)[1 + \kappa_1 \bar{F}(w)]} 
= \frac{2\kappa_1 (1 + \kappa_1) g(w)^3}{3\kappa_1 g(w)^2 [1 + \kappa_1 G(w)]^2 - g'(w)[1 + \kappa_1 G(w)]^3}$$
(6)

Naturally, we can go even further and use equation (5) to find an optimal wage offer of a firm with given productivity level p, or in other words, find K(p). This is done under the assumption that in equilibrium  $F(w) = \Gamma(p)$ , i.e. in equilibrium agents that work more intensively get proportionally higher wage and this proportion is constant across different productivity levels. With this assumption an analytical solution for the optimal equilibrium wage offer is:

$$w_{opt} = K(p) = p - \left[1 + \kappa_1 \bar{\Gamma}(p)\right]^2 \int_{\underline{w}}^p \frac{dx}{\left[1 + \kappa_1 \bar{\Gamma}(x)\right]^2}$$
 (7)

Bontemps et al. (2000) show that whenever an upper bound of the support of the wage offer density is finite there exists at least one equilibrium on the market. A formal definition of market equilibrium in the economy with  $R \leq w_{\min}$  property concludes the theoretical overview.

**Definition 1** A market equilibrium in the economy with minimum wage exceeding

- reservation wage is a triple  $\{F(w), \mathcal{W}, \mathcal{K}_p\}$  such that

  1) The distribution of wage offers is  $F(w) = \int F(w|p)d\Gamma(p)$ , where  $\Gamma(p)$  is a productivity distribution of firms, active in the market
- 2)  $W = \max\{w_{\min}, w_{current}\}$  is the workers best response to firms wage-posting behavior
- 3)  $K_p = \arg\max\{\pi(p,w)|w_{\min} \le w \le p\}$  is a set of profit-maximizing wages posted in equilibrium by each p-type firm with  $\pi(p,w)$  defined in (4) and  $K_p$  defined in (7)

Burdett and Mortensen (1998) show that for any p-type firm the unique offer distribution F(w|p) that satisfies the definition above is given by

$$F(w|p) = \frac{1+\kappa_1}{\kappa_1} \left[ 1 - \sqrt{\frac{p-w}{p-w_{\min}}} \right]$$
 (8)

Before going on with the data description and estimation methodology one interesting point about the obtained solution should be made. As we see, the theoretical description above represents the equilibrium solution entirely in terms of employers productivity. So, in the most straightforward case, the researcher may use exclusively productivity data and, assuming some parametric form for  $\Gamma(p)$ , construct the likelihood function. Upon estimating the structural parameters, optimal wage offer (7) and its distribution F(w) implied by the model through (8) can be computed. To analyze the fit of the model F(w) should be compared to  $\Gamma(p)$ . Alternatively, one can use exclusively wage data and formulate the likelihood in terms of observed wages, relating them to unobserved offered wages through (2). Upon estimating the

structural parameters from this formulation, we use (5) to find the productivity in the optimum and its density  $\gamma(p)$  implied by the model (see (6)). To analyze the fit of the model  $\gamma(p)$  should be compared to f(w).

It turns out that the first way is indeed much more tedious, because using the productivity data we will repeatedly need to evaluate integral in (7). To the contrary no integration is needed if we chose the alternative approach. Therefore it is suggested (see Bontemps et al. (1997), (2000)) that the model should be estimated using wage rather then productivity data.

#### 3 The Data

The present section describes the data we use to estimate the model. The data are the fifteen year long Austrian social security records (Hauptverband der Sozialversicherungen) that represent working history of 10000 individuals who were followed through a fifteen year period staring from 1984 and finishing at 1998. All observations are taken at 30.05 of each year. Each individual in these samples has her own personal number, using which her career history can be tracked through the time. Individuals data include gender, age, income, professional affiliation, employment status, various indicators on employment/unemployment durations and employer information. With respect to employment status all individuals are divided into the four categories, namely, "employed", "unemployed", "on study" and "out of labour force". In what follows only those individuals who were either employed or unemployed on the date of sampling will be considered. The reason for this is that the model to be estimated is restricted to only these two states of the labour market – individuals who fall into the rest two groups have incentives different from the agents described by the model. Therefore they are to be left out. More specifically, students with their study still in progress normally do not search for full-time jobs and those who are out of the labour force (aggregated group of pensioners, women on extended maternity leave, people with disabilities etc.) have utility functions, which can not be fully specified by labour decisions only.

The data contain no indication of whether an employed individual works part-time or full-time. This, however, is important and important for the same reason as before. Being a part-time worker generally implies that the agent is not interested in a full-time job or a continuous job history, which is not consistent with the behavior of the agents described by the model. Therefore part-time workers should also be excluded from the sample. To do this we argue that if an agent works on a full-time basis then her income is at least as high as the legal minimum wage before tax. Thus people, with income below the minimum are left out of consideration. This simultaneously cuts off the maximum possible number of part-time workers because in Austria the legal minimum wage is never less then the reservation wage of an individual. The latter fact is always true due to the particular system of minimum wage setting. The description follows.

In Austria there is no exogenously set legal minimum wage as such. Instead, every year unions in all industries bargain with employers for minimum wages that should be paid in respective industries throughout the year. Unions, being the representatives

Table 1: "Duration Data for Austrian Labour Market"

	31.05.1988	31.05.1994
Number of Individuals	3404	3726
I. EMPLOYED	3110	3361
<u>Transitions</u>	1011	721
1) - to job	1011 757	731
<sup>2)</sup> - to unemployment <sup>3)</sup> - to other	616	707 362
<u>Durations</u>		
Uncensored		
- mean	43.189	42.324
- std. deviation	34.884	33.008
- number of observations	1146	1288
Censored		
- left	622	150
<sup>4)</sup> - right	257	1085
5) - both left and right	469	476
Censored durations total:	1348	1711
II. UNEMPLOYED	294	365
<u>Transitions</u>		
<sup>6)</sup> - to job	34	29
7) - to other	257	288
<u>Durations</u>		
Uncensored		
- mean	6.123	5.638
- std. deviation	8.358	3.226
- number of observations	34	29
Censored		
- left	-	-
8) - right	3	48
9) - both left and right	-	-
Censored durations total:	3	48

Units of measurement – Months and Individuals

COMMENTS: a) The number of *employed* individuals I. is a sum from 1) to 5), i.e. the sum of completed, left-, right- and both left and right censored durations.

b) Left censored durations are fully absorbed by the sum from 1) to 3), i.e. the sum of right-observed durations.

c) The number of *unemployed* individuals II. is a sum from 6) to 9), i.e. again the sum of completed, left-, right- and both left and right censored durations.

d) Total number of individuals is a sum of I. and II.

of workers, know exactly the reservation wage of unemployed agents. So their goal is to negotiate for the minimum wage such that  $w_{\min} \geq R$  (where R stands for reservation wage). Unions will not accept any wage offers below R. Employers in their turn do not know exactly the reservation wage of unemployed agents. Though, they can estimate it (let  $\tilde{R}$  be an estimate) with some error  $\epsilon$ . The employers goal is to pay the least wage possible. Provided their estimation is correct employers will not pay more than  $\tilde{R} + \epsilon$ . Thus every year the scope for negotiation should narrow to the interval  $[R, \tilde{R} + \epsilon]$  and the resulting minimum wage in such a setting will always be greater than or equal to the true reservation wage. The result of this bargain is fixed by a contract, which binds employers to pay at least the negotiated minimum. Bargaining just described takes place in every industry. So the minimum wage level varies from industry to industry (and in fact is not the same even within each industry). For practical purpose we compute an average minimum wage for the whole economy using the data from the Austrian Central Statistical Office.

Upon doing so we construct the samples of employed and unemployed individuals at 1988 and 1994 time points and restore employment/unemployment duration histories of these individuals. Summary statistics of duration lengths is given in Table 1.

One problem connected to this way of sampling durations should not be ignored, though. This is a so called "length-baisedness" of the sample. The problem is such that the spells with the longer length have a higher probability to be included into the sample. Ridder (1984) suggests that a solution to it is to base the inference on the joint distribution of elapsed and residual durations. In his terminology elapsed duration is the time spent by individual in a current state from the moment of entrance to it (or from the beginning of observations) till the date of sampling. Residual duration then is the time which passed from the date of sampling till exit of this individual to another state (or till the end of observations). We implement this suggestion when constructing the likelihood function (see Section 4).

There is also another problem we should be aware of. The problem is that wages in our sample are top-coded: right censoring amounts to about 10% in both samples. Again this suggests that to ensure correct inference appropriate methodology should be used. The methodology applied for the estimation of the model is described in much details in the next section.

# 4 Estimation Methodology

Structural parameters  $\{\lambda_0, \lambda_1, \delta\}$  are estimated by maximum likelihood from the econometric model, which describes in probabilistic terms the lengths of durations of employment/unemployment spells. The model however is built using restrictions on parameters that come directly from the theory outlined in the first section. In this section we present a general approach to the construction of the likelihood<sup>4</sup> and then discuss modifications that are necessary to account for the particular features of our data.

<sup>&</sup>lt;sup>4</sup>In doing so we follow Lancaster (1990). Koning, Ridder, van den Berg (1995) and Ridder, van den Berg (1998) also provide some useful explanations.

#### 4.1 The Likelihood

As the backbone process for all events in the model is Poisson, we use the property that waiting time between two events in  $\operatorname{Poi}(\theta)$  process starting from the occurrence of the preceding one is exponentially distributed with parameter  $\theta$ . However, it was pointed out in previous section that the sample is not random so this property can not be applied directly. Instead a joint distribution of elapsed  $(t_e)$  and residual  $(t_r)$  durations of a spell is analyzed. On the distribution of the elapsed duration length it is known that certain time  $t_e$  ago there was a renewal of states and since then an individual spent at least  $t_e$  in a new state (transition from job to job in present context is also qualified as renewal). On the distribution of the residual duration length our knowledge is that given a certain elapsed time  $t_e$  an individual spends in this state additional time  $t_r$   $(t_r > 0)$ . Renewal density for  $\operatorname{Poi}(\theta)$ :  $h(x) = \theta$ . Therefore appropriate densities are:

Density of Elapsed Duration:  $f(t_e) = \theta e^{-\theta t_e}$  Density of Residual Duration:  $f(t_r|t_e) = \theta e^{-\theta t_r}, t_r > 0$  Joint Density:  $f(t_e, t_r) = \theta^2 e^{-\theta (t_e + t_r)}, t_r > 0$ 

We also use the property that the hazard of  $Exp(\theta)$  is equal to  $\theta$ . For unemployed individuals hazard to exit from current unemployment is just the arrival rate of job offer so  $\theta = \lambda_0$ .<sup>5</sup> For employed individuals hazard of exit from current job is a sum of the transition intensity to a job, which pays wage greater than that currently earned (i.e. arrival of acceptable job offer) and the transition intensity to unemployment. So:  $\theta = \lambda_1 \bar{F}(w) + \delta$ . Additionally, to account for multiple destinations in the cases of transition from job to job and from job to unemployment the likelihood is multiplied with the transition probabilities, which in our example are

$$\pi_{j \to u} = \frac{\delta}{\delta + \lambda_1 \bar{F}(w)}$$
 and  $\pi_{j \to j} = \frac{\lambda_1 \bar{F}(w)}{\delta + \lambda_1 \bar{F}(w)}$ 

To account for censoring we change respective (elapsed or residual) density by exponential survivor  $(e^{-\theta t_e})$  or  $e^{-\theta t_r}$ . Eventually, whenever in result of the transition from unemployment to employment offered wage is observed, the likelihood is multiplied by the wage offer density evaluated at the observed wage. With these results and probabilities of selecting either employed or unemployed person (see Section 1) unemployed  $(\mathcal{L}_{un})$  and employed  $(\mathcal{L}_{empl})$  individuals make the following contributions to the likelihood:

$$\mathcal{L}_{un} = \frac{\delta}{\delta + \lambda_0} \lambda_0^{2 - d_r - d_l} \exp\{-\lambda_0 (t_e + t_r)\} f(w)^{1 - d_r}$$
(9)

$$\mathcal{L}_{empl} = \frac{\lambda_0 g(w)}{\delta + \lambda_0} \left[ \delta + \lambda_1 \bar{F}(w) \right]^{1-d_l} \exp\left\{ -\left[ \delta + \lambda_1 \bar{F}(w) \right] (t_e + t_r) \right\} \left[ \left[ \lambda_1 \bar{F}(w) \right]^{d_t} \delta^{1-d_t} \right]^{1-d_r}$$
(10)

Here  $d_l = 1$ , if a spell is left-censored, 0 otherwise;  $d_r = 1$ , if a spell is right-censored, 0 otherwise;  $d_t = 1$  if there is a job-to-job transition, 0 otherwise. Using steady state

<sup>&</sup>lt;sup>5</sup>For convenience  $\lambda_0$ ,  $\lambda_1$  and  $\delta$  rather then  $\kappa_0$ ,  $\kappa_1$  and  $\delta$  are used while formulating the likelihood function.

identity for distributions (2), stated in Section 2, we substitute for the unknown wage offer distribution (F) and density (f) expressions with known earnings distribution (G) and density (g). As a result we obtain the likelihood in terms of structural parameters  $(\lambda_0, \lambda_1, \delta)$  and the data, given that some functional form of G(w) is chosen. As all labour suppliers act independently, joint likelihood is just a product of all individual ones.

There are two options for choosing G(w). The first one is to take it from some known parametric family and the second – estimate it nonparametrically. In view of the restrictions that are to be fulfilled by every earned wage distribution, namely the support being a compact set, flat right tail, possibility for peak on left tail of density function etc. it is more sensible and efficient for computations to follow the second option. Choice of non-parametric estimate for earnings distribution and density is also advocated by the fact that in this case exact inference can be made. Finally, nonparametric approach is informally suggested by Bontemps et al. (1997), (2000). However, it may be interesting to go firstly through the nonparametric procedure and afterwards to test for the sensitivity of the analysis to distributional assumptions on F(w). This will either justify the necessity of non-parametric approach or show its redundance. We suggest to follow this idea and in Section 5.2 we return to this issue.

All in all the estimation of the model becomes a "three-step procedure" in terminology of Bontemps et al. (2000). On the first step we compute non-parametric estimates of g(w) and G(w). On the second step we substitute values of g(w) and G(w) in the likelihood function with their estimates from the step one and maximize the likelihood with respect to  $\lambda_0, \lambda_1$  and  $\delta$ . On the third step equilibrium outcomes (4) – (7) derived from the theory are evaluated using estimates from the steps one and two.

## 4.2 Nonparametric Features of the Likelihood and the Problem of Censored Wages

It would be straightforward to estimate the model now if there were no additional complication connected with the data. As it was mentioned in Section 3, about 10% of top earnings observations are censored. Absence of information on the wages from the upper decile of earnings distribution makes non-parametric estimation of the right tail of earnings distribution unavailable. One way out of the problem, though, is to approximate of the right tail by some known distribution. Fortunately, when the problem appears at the right tail exclusively, such approximation is readily available. As Fichtenbaum and Shahidi (1988) suggest, common practice is to approximate the tail by a Pareto distribution. Thus the whole support of earnings distribution will be split into two intervals: on the first one g(w) and G(w) are to be estimated nonparametrically, on the second one g(w) and G(w) are to be of a Pareto form.

Let us start with the first interval. With presence of right censoring in the observed income natural nonparametric estimator of earnings distribution function is a Product-Limit Estimator (see Lancaster(1990) for discussion). Formally,

$$\hat{G}(w) = 1 - \prod_{i=1}^{j} \left( 1 - \frac{h_i}{n_i} \right) \tag{11}$$

where  $h_i$  is the number of wages censored by set  $\{w_i\}$  and  $n_i$  is the number of wages greater then or equal to  $w_i$ . J is the number of distinct wages in the sample and j = 1, 2, ..., J being an index of distinct wages in the sample<sup>6</sup>.

For the density estimation we use a version of a Blume-Susarla estimator suited to the case when right censoring is not random and the censoring threshold is the same constant (see Padgett (1988)). In this particular case the estimator reduces to

$$\hat{g}(w) = \left[nh^{-1}\right] \sum_{j=1}^{n} K\left(\frac{w - W_j}{h}\right) [I_j = 1]$$
(12)

Here  $I_j$  is an indicator function, which takes value 1 if  $w_j$  is less then the value of the censoring threshold and zero otherwise. K(.) is a kernel function, which satisfies conditions listed in Padgett (1988). Gaussian kernel qualifies. It may also be noticed that Vuong et al. (2000) show that whenever the distribution has a compact set as a support kernel density estimator is asymptotically downward biased towards tails. Bontemps et al. (1997) state that this bias is of a specific form, namely:  $E\left[\hat{g}(w)\right] \to \frac{g(w)}{2}$ . On these grounds Bontemps et al. (1997) suggest the following bias-corrected Gaussian kernel estimator

$$\hat{g}(w) = \left[ \frac{1}{nh} \sum_{j=1}^{n} \varphi\left(\frac{w - W_j}{h}\right) [I_j = 1] \right] \left[ \Phi\left(\frac{x - \underline{w}}{h}\right) \right]^{-1}$$
(13)

In view of right censoring and subsequent Pareto approximation of the right tail (13) presents a version of the estimator with only left tail correction making it appropriate for this particular study. Bandwidth h is chosen to be data-determined following Silverman (1986), p.45-48.

Now let's continue with the second interval. As we have mentioned before, right tail of earnings distribution is to be approximated by Pareto distribution. The length of the interval on which this approximation applies is determined by log-log plot of  $1 - \hat{G}(w)$ . Approximation is justified only on the segment where  $\ln(1 - \hat{G}(w))$  is linear against  $\ln(w)$ . This follows from a property of Paretian "survivor" – given that  $P(w) \sim Pareto(\alpha, \beta)$  it is true that  $\ln(1 - P(w)) = \beta \ln \alpha - \beta \ln(w)$  is a linear function. Linearity of  $\ln(1 - \hat{G}(w))$  in  $\ln(w)$  amounts up to top 20% of wages in both samples. By substitution of  $\ln(1 - \hat{G}(w))$  into the last equation scale and shape parameters of Pareto distribution are consistently estimated by least squares.

With knowledge of the exact form of Pareto approximate it becomes possible to specify the likelihood of individuals whose earnings fell into the approximated interval in a standard for censoring way. If the wage of a given individual is less then the censoring threshold her contributions to likelihood will be the same as (10) with only difference that g(w) and G(w) taken to substitute for unknown F(w) will be now

$$Var(\hat{G}(w)) = \left[1 - \hat{G}(w)\right]^2 \sum_{i=1}^{j} \frac{h_i/n_i}{(1 - h_i/n_i)n_i}$$

 $<sup>^6</sup>$ For the subsequent estimation of confidence bounds for the expected equilibrium employment durations we will also need to know the variance of a Product-Limit estimator. Lancaster (1990) shows it to be

the Pareto p.d.f. and c.d.f. If, to the contrary, the wage is censored then individual contribution to the likelihood is  $P(W \geq w^{cens}, t_e, t_r)$ . Depending on whether the elapsed/residual durations are censored or not joint density of elapsed, residual durations and wages under integral should be modified accordingly (the principle is the same as mentioned before). With this approach contribution of all employed workers to the likelihood is<sup>7</sup>

$$\mathcal{L}_{empl} = \left[ \prod_{\{A\}} \mathcal{L}_{empl_i}(\xi | t_{e_i}, t_{r_i}, g(w_i), G(w_i)) \right] \times \left[ \prod_{\{B\}} \mathcal{L}_{empl_i}(\xi | w_i, t_{e_i}, t_{r_i}) \right] \times \left[ \prod_{\{C\}} \left[ \int_{w^{cens}}^{\overline{w}} f(\xi | w_i, t_{e_i}, t_{r_i}) dw \right] \pi_{i,I} \right]$$
(14)

where  $\{A\}$  is a set of wages for which earnings density and distribution are calculated nonparametrically (about 80% of total sample of employed),  $\{B\} \cup \{C\}$  is a set of wages, on which Pareto approximation was applied (approximately 20%). Within this set segment  $\{B\}$  contains observed wages and segment  $\{C\}$  contains censored wages. In the last square bracket  $\pi_{i,I}$  is an appropriate probability term, which contains probability of selecting employed person and respective transition probability if necessary (transitions, if occur, are indicated by function I). In practice integral in the third component is estimated numerically using Gauss-Legendre quadrature. Integration is always one-dimensional, therefore creates no obstacles for maximization.

Contribution of all unemployed agents to the likelihood is

$$\mathcal{L}_{un} = \prod_{\{U\}} \mathcal{L}_{un_i}(\xi | t_{e_i}, t_{r_i}) \tag{15}$$

As there were no instances of transition from unemployment to a job with censored wage in this particular study, (15) does not take an account for that. Otherwise its construction would have been the same as in (14). The Total Likelihood is a product of both, i.e.

$$\mathcal{L} = \mathcal{L}_{un}(\xi) \mathcal{L}_{empl}(\xi) \tag{16}$$

The likelihood formulated in this way allows to consistently estimate parameters of interest provided that Pareto approximation is justified. To get justification more formal than mere good fit of least squares it might be possible to construct a goodness of fit test for the observed approximated part of the tail<sup>8</sup>. Asymptotic covariance matrix of parameter estimates from this procedure however is difficult to derive. In many applications (Bowlus et al. (2001), Bontemps et al. (2000) among others) the bootstrap is taken as a solution. We do the same here (excellent treatment of bootstrapping techniques is provided by Hall (1994)).

<sup>&</sup>lt;sup>7</sup>For the convenience of notation all parameters of interest are represented by vector  $\xi$ .

<sup>&</sup>lt;sup>8</sup>Like Kolmogorov-Smirnov test, for instance, but with a possibility to be applied to a part of a distribution. Unfortunately, so far I couldn't see any example of such test in the literature.

#### 4.3 The Likelihood and Observed Heterogeneity

Knowledge of estimated parameters is already sufficient to argue about the equilibrium outcomes on the labour market. At the same time having a single parameter  $\{\lambda_0, \lambda_1, \delta\}$  to approximate the whole set of individual-specific parameters  $\{\{\lambda_{0_i}\}, \{\lambda_{1_i}\}, \{\delta_i\}, i=1, \dots n\}$  is too restrictive. The reason is quite simple – even observed heterogeneity of employees in this case is neglected. We remember that one of the assumptions of the model under study was that all the workers are homogeneous with respect to opportunity cost of employment. However, they may be different with respect to gender, age, skills etc. To make correct inference about both mean employment and unemployment durations of heterogeneous agents, an appropriate econometric specification should be used. One of the solutions is to treat these and other possible observed qualities as regressors, suggesting that structural parameters of the model can be specified as

$$\lambda_0 = \exp(X\beta_1)$$
,  $\lambda_0 = \exp(X\beta_2)$ ,  $\delta = \exp(X\beta_3)$ 

This idea is borrowed from Ridder, van den Berg (1998), who introduce market segmentation to account for the observed heterogeneity. More precisely the idea is that any labour market can be split into a number of submarkets and agents that belong to different submarkets are different from each other with respect to certain separating quality. Inside every submarket, however, individuals are identical. For example there can be submarkets for low-skilled, semi-skilled and high-skilled workers. Pure submarket interpretation of heterogeneity implies that a set of regressors is just a set of appropriately constructed dummy variables and constant, which was the case in Ridder, van den Berg (1998). Though, more generally, any stationary regressor may enter this set.

The present work attempts the analysis of the variation of search intensity parameters and layoff arrival rate induced by age and gender. Thus, regressor matrix X consists of the constant, age and gender variables<sup>9</sup>. The results of estimation in presence of regressors are reported in Section 5, part 3. Notice that if X contains the constant only, the whole specification reduces to the basic model, described in Section 4.2-3. The only difference is a parameterization  $\lambda_0 = \exp(\beta_1)$  etc. In practice we estimate the basic model as a special case of the extended model with  $\{X = [const]\}$ .

Reporting estimated structural parameters we will rather show their reciprocals, since they have a well defined behavioral interpretation discussed in the first part of the following section.

#### 5 Estimation Results and Discussion

The model is estimated for data collected at two different time points – 31.05.88 and 31.05.94. This allows comparing equilibrium outcomes and discovering directions of changes in the labour market performance during the sampled period. Firstly, we

<sup>&</sup>lt;sup>9</sup>Because of computational problems it was not possible to extend the set of regressors for occupational dummies, which were also available from the original data set. However, these were exclusively the problems of starting values and elaboration.

list the theoretical tools, which will be helpful in the subsequent interpretation of estimation results and the analysis of changes in equilibrium outcomes over time. Afterwards, parameter estimates of the basic (without regressors) and extended for observed heterogeneity models are considered. Apart from economic interpretation of the obtained results we also provide formal justification of the applied estimation procedure.

#### 5.1 Descriptive Tools

To be simultaneously short and rigorous we restrict our attention to four indicators of labour market activity, namely to the mean employment duration, mean unemployment duration, monopsony power of firms in wage setting and firms equilibrium profits.

Following Mortensen and Neumann (1988) first two are just the reciprocals of search intensities, namely  $\lambda_0^{-1}$  for unemployed and  $[\lambda_1 \bar{F}(w_i)]^{-1}$  for employed individuals (here  $w_i$  stands for the individual-specific currently earned wage). This is so because both  $\lambda_0$  and  $\lambda_1 \bar{F}(w)$  are hazards of exit from unemployment to the job and from current to the next job respectively. As long as the time between these exits is exponentially distributed, expected length of this time is just a reciprocal hazard<sup>10</sup>.

The third indicator is an index of monopsony power defined as (p-w)/p. It can be interpreted as a share of profits not paid out to workers (see Bontemps et al. (2000) for exposition). The index always takes values between zero and one, with one being an indication of absolutely monopsonistic market. Thinking of equilibrium 'equal-profit conditions', which play key role in the derivation of equilibrium offer distribution F(w|p), it may seem reasonable to suggest that in the ideal situation index of monopsony power is a horizontal line. However this may not necessarily be true in reality. As a rule this index is an upward sloping curve, which implies that the firms with higher productivity have higher monopsony power in wage setting.

The last indicator of market performance considered here is an equilibrium profit of chosen firm in respective time periods. Profit equation is given by (4). Having profits in addition to the monopsony power index is necessary to tell whether firms do suffer from a reduction in mean employment duration. This can be seen by inspection of a profit ratio of a representative firm. Using (3), (4) and some algebra the profit ratio can be written down as

$$\frac{\pi^*}{\pi} = \frac{M^*/N^*}{M/N} \frac{p^* - w^*}{I_w(p - w)} \left[ \frac{\kappa_0^* (1 + \kappa_0)}{\kappa_0 (1 + \kappa_0^*)} \frac{(1 + \kappa_1^*) (1 + \kappa_1 \bar{F}(w))}{(1 + \kappa_1) (1 + \kappa_1^* \bar{F}^*(w^*))} \right]$$
(17)

where  $\pi^*$  and  $\pi$  are the profits at two different periods of time (in our case 1994 and 1988; asterisk defines values of 1994). The expression in square brackets is entirely dependent on wages observed at given date, parameters of the model and the offer

 $<sup>^{10}</sup>$ We also estimate confidence bounds for both expected durations. The case of mean unemployment duration is straightforward. The case of mean employment duration is less so because we need to evaluate variance of  $\left[\lambda_1 \bar{F}(w)\right]^{-1}$ , which is a non-linear transformation of estimated parameters and of  $\hat{G}(w)$ . Approximate variance of mean employment duration is given by the variance of a first order Taylor expansion of  $\left[\lambda_1 \bar{F}(w)\right]^{-1}$  around its mean (see Greene (2000), Ch. 3 for details).

distribution that can be estimated. Productivity in the ratio of productivity-wage differences can also be estimated from the theory using (5). It is obvious that from year to year minimum wage grows up so the support of wage offer (productivity) distribution always shifts to the right. To make our productivity-wage differences comparable the denominator is scaled by the index of agreed minimum wages  $I_w$  published annually by Central Statistical Office. Finally M/N ratio is a total labour force over the number of active firms in respective years. In the analysis that follows we will take firms, which operate on average productivity of each decile of  $\Gamma(p)$  as representatives. So we will be able to check the effect of labour mobility shifts on a variety of heterogeneous producers.

# 5.2 Estimation Results – The Model with Homogeneous Workers

With the analytical toolkit just described let us see and interpret the results of the estimation. Table 2 shows reciprocals of estimated structural determinants of the  $\mathrm{model}^{11}$ .

We start off with unemployment durations. From the results in Table 2 it can be found out that the mean unemployment duration changed from 5.5 to 12 months. This is the fact, which at the first glance contradicts empirical data in Table 1. However, taking a closer look at this table we can see that amount of censored unemployment durations in 1994 is much higher then in 1988. Moreover, important

Sample 1988 Sample 1994  $1/\lambda_0$ 5.47210 (0.36245) 12.28694 (0.27535) [4.968, 5.951][11.660, 12.824]  $1/\lambda_1$ 33.19244 (1.60169) 36.59468 (0.82008) [29.801, 36.682] [33.070, 40.204]  $1/\delta$ 130.75113 (3.09881) 119.59049 (2.68002) [116.890, 122.211] [127.657, 133.752] log(Likelihood) -37246.69-45091.92

Table 2 "Estimated Frictional Parameters"

(standard errors of parameters in parenthesis; bootstrap 95% confidence intervals based on 1000 replications in square brackets)

is that all censored durations in both samples are exclusively right censored ones. This tells that in 1994 comparatively to 1988 much more unemployed individuals can potentially exit to job. Unemployment duration of these will be then at least 48

 $<sup>^{11}\</sup>mathrm{Immediate}$  results of estimation are presented in Table A1 of the Appendix.

months, which will shift overall mean considerably. Therefore raw data are extremely unreliable for inference on duration of unemployment. Saying that it has, reduced as the Table 1 wold suggest, is definitely misleading. Estimation results show in fact that this duration has increased and not decreased over time.

Among others, this fact can have two possible explanations. On one hand it may become preferable to stay longer in unemployment due to a more generous benefit system. On the other hand, however, it may become more difficult to find a job due to an increased competition on the market or due to cyclicity. According to Winter-Ebmer (1998), who uses the same data for his quasi-experiments with the duration of unemployment benefit, even very big increases in the length of the latter are not able to cause proportional (and sometimes even significant) increase in unemployment duration. Furthermore, if we take the ratio of unemployed to employed individuals (from Table 1) as an indicator of economic activity, we can see that these ratios make 0.103 and 0.098 for 1988 and 1994 respectively. This does not indicate slowdown of economy in the second period if compared to the first one. Therefore we may arrive to the conclusion that over the last decade the Austrian labour market became more demanding to quality of labour suppliers.

Inference about the mean employment durations predicted by the model is less straightforward because of the presence of wage offer distribution, which has to be evaluated at the individual-specific wage. Even though it is possible to estimate equilibrium offer distribution (see Graphs 1-2 of the Appendix), inference in existing literature so far was based only on the reciprocal employed search intensity  $\lambda_1^{-1}$ . In the present work we will rather attempt to consider  $[\lambda_1 \bar{F}(w_i)]^{-1}$ . This extension should give much more comprehensive description of the expected employment durations. To make the estimated expected durations comparable across time we evaluate  $\bar{F}(w_i)$  at the wages, which are the average wages of respective deciles of the earnings distribution. Table 3 presents the lengths of expected job durations (in months) of an agent, who earns average income of each and every decile. Visual presentation of the results in Table 3 is given in Graph 3 of the Appendix.

Firstly we have to notice that those agents, whose earnings belong to the ninth and the tenth deciles of the earnings distribution have expected duration that exceeds potential job tenure (at least 76 years). This result, however, is quite natural. The interpretation is that people, who earn income of upper two deciles practically loose an incentive to search for a better job and will be happy to stay on the current job forever. Of bigger interest are the results for the rest of the workers. From both Table 3 and Graph 3 we readily obtain the verification of the reduction in mean job durations over the last decade. This is exactly the fact encountered in the empirical literature, which deals with the Austrian labour market. In our search equilibrium framework this hypothesis can be formally tested as well. Bootstrap confidence intervals for the expected durations indicate that the significant departure from the old mean starts already for those people, whose income belongs to the third decile of earnings distribution. Thus, recently about 65% of the population revised its expectations on the optimal job duration. The model suggests that such decline is due to the belief that higher wage offer became more possible and increased search intensity will therefore be rewarded. This conclusion can also be supported by the fact that in 1994 comparatively to 1988 we register a loss of producers monopsony

Table 3 "Estimated Expected Job Durations At Given Income (months)"

Average Earnings Over Deciles of Earnings Distribution	Expected Job Duration (1988)	Expected Job Duration (1994)
1	42.43017 (1.70097) [38.852, 45.924]	44.34061 (1.85277) [40.599, 47.971]
2	64.72474 (2.00067) [60.816, 68.505]	62.51731 (2.10056) [58.503, 66.371]
3	93.90406 (2.47684) [89.605, 98.098]	86.27050 (2.47232) [81.933, 90.398]
4	133.1867 (3.19751) [128.286, 138.0291]	116.1938 (2.98851) [111.404, 120.698]
5	186.7297 (4.24806) [180.952, 192.634]	155.8619 (3.71915) [150.405, 161.018]
6	266.2852 (5.87089) [259.004, 273.759]	238.1784 (5.31393) [231.376, 244.624]
7	395.2196 (8.55978) [385.520, 405.617]	337.0235 (7.28666) [328.266, 345.112]
8	648.1672 (13.9034) [633.532, 663.735]	511.3346 (10.8200) [499.527, 522.5928]
9	$1335.316 = \infty $ (28.47675)	$919.9729 = \infty $ (19.15996)
10	$3769.770 = \infty $ $(80.25259)$	$1967.350 = \infty $ (40.63293)

(standard errors of parameters in parenthesis, bootstrap 95% confidence intervals base on 1000 replications in square brackets)

power in wage setting (see Graph 5), which as well implies a potentially higher wage offer. Observing further the pattern of changes in the expected job durations we may spot that over the last decade mean job duration of the poorest agents did not

reduce, but to the contrary increased. This increase is not significant so far, but it may have an interesting interpretation. In particular it may imply strengthening of competition among labour suppliers. Those who are less competitive and earn the lowest wages foresee that it will be difficult to get a better job and so expect to stay longer in their relatively poor conditions. Evidence of such a reverse change could also be interpreted as a tendency towards state dependence. If we believe in such tendency, then the model outlines the group of those who in future may potentially be stuck with the lower income for a significantly longer time.

Let's consider now the index of firms monopsony power in wage setting. It will be more insightful to start with an exploration of changes in the estimated wage offer and productivity distributions. As it was mentioned before we notice an increasing tendency to offer higher wages. In this case it can be seen by comparing the the left tails of the offer densities (see Appendix, Graph 1) and their curvature. At the same time employers productivity distribution experienced a similar shift, i.e. there was an increase in fraction of firms with higher productivity (Graph 4). Recalling the structure of the monopsony power index it becomes clear that these two processes have an offsetting effect on each other. Relative increase in productivity means an increase in monopsony power, while relative increase in wage offer means its decline. Upon inspection of the index itself we conclude that over the last decade tendency towards higher wage offer dominates. Indication of this is readily seen in the behavior of the index (Graph 5) upper bound of which drops down from 80 to 60%. The index also becomes more flat on the first half of the productivity distribution support. This demonstrates that the reduction in the monopsony power also affects medium productivity producers, though in a smaller amount. The only exception make those employers, who operate on the lowest productivity levels. The power of these firms in wage setting surprisingly increases, which may indicate an excess of low skilled labour available in the Austrian market in late nineties.

Investigation of the behavior of the index suggests an interesting parallel between expected job duration and monopsony power of a firm. We can see that the greater is the power of an employer the longer is an expected duration of a respective employee. We also remember that in the context of this model lower productivity firms pay lower wage and single producer pays exactly one wage. These facts may imply that in order to avoid a danger of possible state dependence, discussed above, authorities may apply certain antimonopsonistic regulation to the firms that operate on the first-decile levels of the productivity distribution.

Finally, let's consider the profit ratio as formulated in (17). It is constructed again for ten representative firms that belong to ten different deciles of the support of the productivity distribution. Each firm operates on the average productivity of its own decile. The plot of this ratio is presented in the Appendix, Graph 6. It is easy to see that for all firms starting from the third decile profit decline over time is substantial. This behavior is observed along with the increase of labour mobility of the workers who earn wages of the third decile of earnings distribution, and higher. Therefore the main message the profit ratio delivers is that excessive labour mobility may indeed negatively affect producers. Moreover it demonstrates that for the producers who employ more productive, read skilled, labour the loss is higher. Naturally this relationship should not hold for the low-skilled workers since

in this case firms do not invest into the human capital of their employees. This fact is also supported by the results, since no reduction in profits was registered for low-productivity firms.

Thus search equilibrium supports the conjecture of Url and Mayrhuber (1999) about negative relationship between high labour mobility and profitability. Unfortunately, on this stage of research we can not separate the effect of the loss of monopsony power and the effect of the increase in labour mobility. To do this additional study of profit ratio (17) needed.

This completes the economic interpretation of the estimation results of the model with homogeneous workers. Though, before we start with an extension of the analysis for the observed heterogeneity let us return to the question addressed in Section 4.1. In other words let us check whether the chosen nonparametric approach to model estimation is indeed relevant.

An attempt of sensitivity analysis. The literature on the estimation aspects of search equilibrium models with continuous productivity dispersion is clearly an emerging one. Insofar estimation methodology for this type of models suggested to go through a nonparametric procedure of calculating offer and productivity distributions rather then to assume some parametric forms for them. Though neither of existing papers have ever before presented a formal justification of this procedure. It is rather assumed that the nonparametric approach performs better. Taking on the other hand parametric approach may provide more flexibility in the particular studies. For instance it may open interesting prospects to the experiments with minimum wages. As this may certainly be a temptation, we find it useful to pay more attention to a sensitivity of estimation procedure to the distributional assumptions on the wage offer (productivity). To judge whether the nonparametric procedure is more applicable we compare the estimated distributions with their parametric counterparts. In the case of wage offer distribution the most natural candidate could be a Pareto family (see Graph 1). In Table 4 we present the results of tests of wage

Table 4 "Tests of Goodness of Fit for Wage Offer Distribution"

Test	Sample 1988: $H_0: \alpha = \underline{w}, \beta = 3.589$		Sample 1994 $H_0: \ \alpha = \underline{w}, \ \beta = 3.359$	
	Test Statistic	p-Value	Test Statistic	p-Value
D-Kolmogorov Kuiper	$0.13360 \\ 0.13872$	0.00000 0.00000	$0.16298 \\ 0.16425$	0.00000 $0.00000$
	Test Statistic	5% Cr. Value	Test Statistic	5% Cr. Value
Cramer- von Mises	3.68023	0.461	5.1412	0.461

offer distribution against  $Pareto(\underline{w}, \alpha)$ , where scale parameter of Pareto distribution is fixed to the lowest wage paid and shape parameter is the one that minimizes reported statistics. The decision is based on Kolmogorov D, Kuiper and Cramer-von Mises test statistics<sup>12</sup>.

All three tests uniformly reject the hypothesis of the equivalence of assumed and calculated distributions. Impossibility to approximate the wage offer distribution by Pareto distribution implies the impossibility of approximation of the productivity distribution by any member of the Pareto family. Therefore we conclude that estimation results are indeed sensitive to the distributional assumptions on F(w) ( $\Gamma(p)$ ). So an attempt to gain by using a parametric approach will most probably lead to inconsistent results. Necessity to minimize mentioned assumptions justifies the choice of the nonparametric procedure for structural estimation of the model.

The next subsection discusses some consequences of the introduction of observed heterogeneity into the specification of the model.

# 5.3 Estimation Results – Extension for the Observed Heterogeneity

The present subsection extends estimation for the observed heterogeneity. In this part of the work we will focus on the duration dynamics of heterogeneous workers only. We suggest that workers are different with respect to their age and gender and use the specification from 4.3 to re-estimate the model. Direct estimation results presented in Table A2 of Appendix are again used to recover the expected employment/unemployment durations and expected time until match break for heterogeneous groups. These expected durations are reported in Tables 5,a and 5,b for 1988 and 1994 years respectively.

Table 5,a "Expected Durations of Heterogeneous Agents: 1988 (months)"

		Mean Unempl. Duration	Mean Empl. Duration	Mean Waiting Time until Match Break
	Age			
Males				
	30	3.016 (0.084)	142.022(2.700)	102.934 (6.465)
	45	$6.634\ (0.088)$	251.169(5.747)	$153.230\ (6.465)$
Females		,	,	,
	30	5.137 (0.134)	575.467 (2.556)	146.271 (6.466)
	45	$11.301\ (0.137)$	$1006.91 = \infty$	217.743 (6.466)
		,	(5.583)	,

(standard errors in parenthesis)

To demonstrate the age effect we evaluate expected durations for both man and women at 30 and 45 noticing that overall effect of age on any duration is positive.

 $<sup>^{12}</sup>$ See Stephens (1974) for details.

This conclusion comes from the negative sign of the age coefficient<sup>13</sup> in 1/exp(.) transformation. The results for expected employment duration presented in Tables 5,a-5,b are computed for both men and women that earn an average income.

Table 5,b "Expected Durations of Heterogeneous Agents: 1994 (months)"

		Mean Unempl. Duration	Mean Empl. Duration	Mean Waiting Time Until Match Break
	Age			
Males				
	30	4.662 (0.066)	157.181 (3.470)	77.801 (2.744)
	45	$13.564\ (0.080)$	285.713 (9.632)	150.795(2.744)
Females		, ,	, ,	` ,
	30	8.304 (0.106)	390.218 (3.192)	109.526 (2.744)
	45	24.160 (0.115)	$709.31 = \infty$	212.285(2.744)
		,	(9.206)	,

(standard errors in parenthesis)

As it may have been expected we see that within each year males are more mobile than females no matter if employed or unemployed. Moreover, the difference in the length of expected employment duration between men and women is increasing with age of both. This tells us that as time goes by Austrian women become relatively less ambitious in finding a better job comparatively to men. Moreover it could be interesting to see that that difference in the length of expected unemployment duration between men and women does not depend on age. This implies that attitude to unemployment for both genders remains the same all the time.

A striking result is obtained by looking at the dynamics of male-female job mobility. From the analysis in previous subsection we remember an overall reduction in the expected employment duration. Now observing males only we can see that in 1994 comparatively to 1988 their mean job duration did not fall at all. At the same time female expected employment duration experienced about 30% decrease. Thus there were exclusively females who contributed to the above mentioned overall decline. So we may suggest that over the last decade women in the Austrian labour market became more competitive, which positively characterizes the market performance.

The last finding is also a very good example of the necessity of introduction of the observed heterogeneity into the econometric specification. Hypothesis of such necessity can be also easily formulated and tested. Recognizing that basic model is nested in the extended one usual LR test is a natural candidate for that. Table 6 below presents the results of testing the homogeneous specification versus the heterogeneous one. For both samples the homogeneity assumption is rejected at 5% level, which in fact is quite an expected outcome.

 $<sup>^{13}</sup>$ See Table A2.

Table 6 "LRT: Homogeneous vs. Heterogeneous Agents"

	LR Test Statistic	Degr. of Freedom	p-Value
1988 :	561.71546	6	0.00000
1994:	865.14077	6	0.00000

So in the conclusion to this subsection we can affirm that the model is capable of grasping the effects of heterogeneous agents and give quite intuitive exposition of those. This opens new prospects for the analysis and may eventually turn the model into an attractive tool for the subsequent policy making.

## 6 Concluding Remarks

The present work uses a search equilibrium model with continuous productivity dispersion to study the dynamics of the Austrian labour market. It turns out that the model does well in tracking changes in firms profitability and explanation of their three possible causes: excessive labour mobility, changes in productivity and changes in market power of the parties. This makes it superior to the empirical duration models, mostly applicable to the description of the labour supply side only. The model is also competitive in explanation of the variations in the mean employment and unemployment durations. Specifically it provides a clear-cut inference on the mean employment durations and their dependence on the offered wages. This advantage allows us to study in details labour mobility of workers and analyze its influence on the profitability of employers.

The work discovers that over the last decade competition on the labour supply side of the Austrian labour market became stronger, which increased frictions in unemployed search. It also does indicate significant changes in employed search, which was naturally expected given the results on big declines in employment durations available from the empirical literature. Moreover, further investigations that take into account observed heterogeneity of workers indicate that these changes happened mostly due to the enhanced activity in the employed female search. This is certainly a positive tendency in the labour market performance. Discovered shifts in labour mobility and relative loss of producers monopsony power explain the subsequent decline in profits, enjoyed by employers due to their more advantageous position on the market previously. Eventually, the model warns about possible state dependence tendency among low-paid workers and outlines the group of those at risk.

Satisfactory overall fit of the model to the real-life data suggests that theoretical results on any policy experiments based on this model should predict true outcomes fairly well. This makes the model an attractive tool for the future studies of labour markets.

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Table A1: "Estimation Results – Homogeneous Model";	,
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	1988		1994	
	Parameters	Std. Errors	Parameters	Std. Errors
$oldsymbol{eta_1}$	-1.69966	0.06624	-2.50854	0.04287
$eta_2$	-3.50232	0.04825	-3.59990	0.04811
$oldsymbol{eta}_3$	-4.87330	0.02370	-4.78407	0.02241
log(Likelihood)	-37246.69070		-45091	.92039

<sup>\*)</sup> Parameterization:  $\lambda_0 = \exp(\beta_1), \lambda_1 = \exp(\beta_2), \delta = \exp(\beta_3)$ 

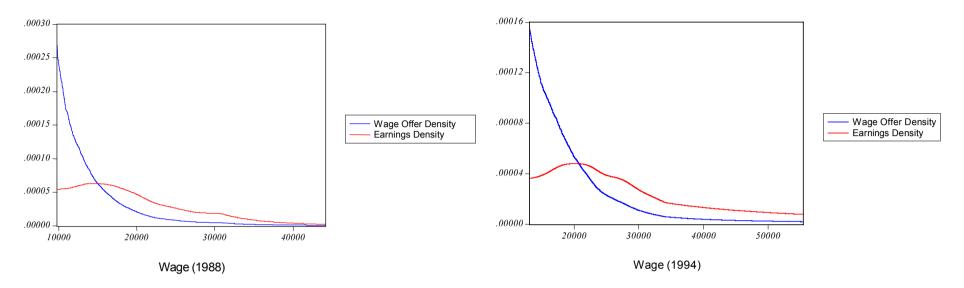
Table A2: "Estimation Results – Extension for Observed Heterogeneity\*\*"

		1988		1994	
		Parameters	Std. Errors	Parameters	Std. Errors
	CONSTANT	-0.05977	0.03024	0.01922	0.03179
$oldsymbol{eta}_1$	AGE	-0.05256	0.00292	-0.07120	0.00197
	GENDER	0.53272	0.12923	0.57728	0.09965
$oldsymbol{eta}_2$	CONSTANT	-2.83424	0.20030	-2.66590	0.18967
	AGE	-0.03801	0.00578	-0.03984	0.00519
	GENDER	1.39920	0.11067	0.90931	0.10455
	CONSTANT	-4.18976	0.09795	-3.37264	0.09410
$\beta_3$	AGE	-0.02652	0.00261	-0.04412	0.00244
	GENDER	0.35138	0.05145	0.34201	0.05457
log(Likelihood)		-36965.83297		-44659.35000	

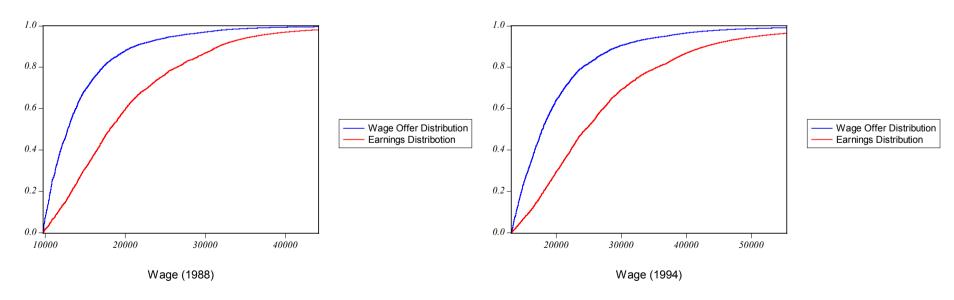
<sup>\*\*)</sup> Parameterization:  $\lambda_0 = \exp(X\beta_1)$ ,  $\lambda_1 = \exp(X\beta_2)$ ,  $\delta = \exp(X\beta_3)$ X={Constant, Age, Gender}

The likelihood was maximized in GAUSS 3.5 using Newton method. All integrals are evaluated by one-dimensional Gauss-Legendre quadrature.

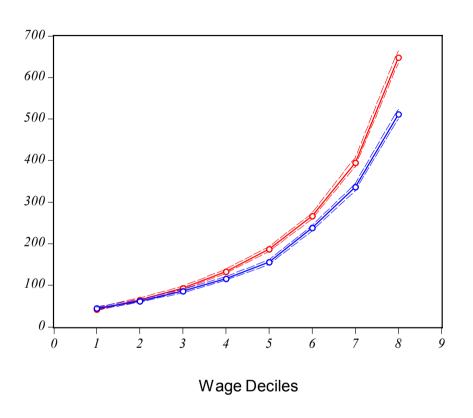
Graph 1 "Wage Offer and Earnings Densities for Two Samples"



Graph 2 "Wage Offer and Earnings Distributions for Two Samples"



Graph 3 "Expected Job Duration (months)"



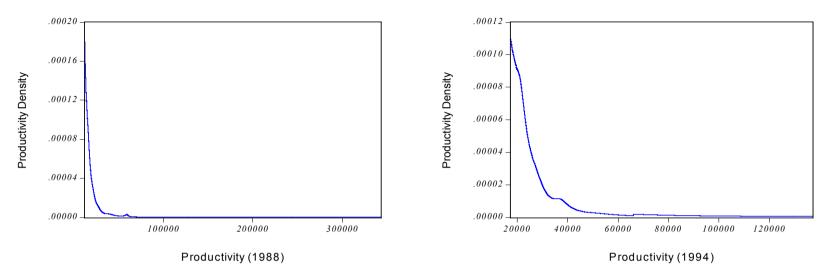
——— (confidence bounds for durations in 1988)

Expected Job Duration (1988)

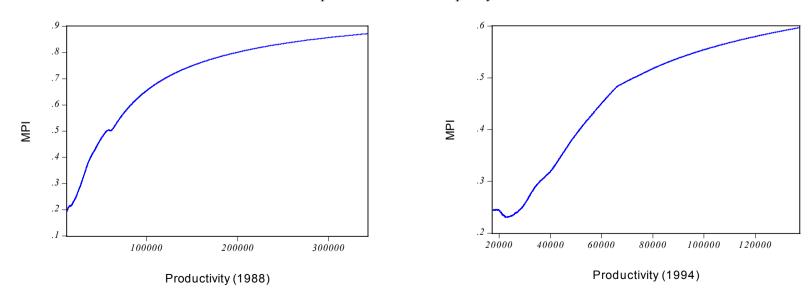
— (confidence bounds for durations in 1994)

—o— Expected Job Duration (1994)

Graph 4 "Productivity Density for Two samples"



Graph 5 "Index of Monopsony Power"



Graph 6 "Profit Ratio Plot"

