Over-education and long-term unemployment*

Bruno DECREUSE†

GREQAM, Université de la Méditerranée

Abstract: This paper develops a matching model of long-term unemployment in which the evolution of education attainment, long-term unemployment and wage inequality is consistent with the observed pattern in European countries. It argues that skill-biased technological change can generate simultaneously: an increase in education attainment, a rise in long-term unemployment, an increase in within-(education) group wage inequality, a small increase in between-group wage inequality, and a rise in the share of over-educated workers.

Keywords: Matching unemployment; Heterogeneity; Reservation productivity; Human capital

J.E.L. classification: J38, J64

* I thank Olivier Charlot, Pierre Granier and IZA Labour Economics summer school (2000) participants for their stimulating discussions. The usual disclaimer applies.

† GREQAM - 2, rue de la charité, 13002 Marseille, France. E-mail: decreuse@ehess.cnrs-mrs.fr

1. Introduction

This paper develops a matching model of long-term unemployment in which the evolution of education attainment, long-term unemployment (LTU) and wage inequality is consistent with the observed pattern in almost all European countries. Broadly speaking, European labour markets have known the following changes during the 1980's. First, a rise in unemployment, and especially in LTU[1] (see Machin and Manning, 1999). That increase is highly preoccupying, since long-term unemployed have very low probabilities to exit from unemployment. Second, a dramatic increase in the supply of educated workers (see OECD report, 1999). Third, a moderate increase in wage inequality, reflecting a large increase in within-group inequality, but only a small increase in between-group inequality[2] (see Gottschalk and Smeeding, 1997).

We argue that the culprit of these changes is skill-biased technological change (SBTC). Here, SBTC consists in an increase in the marginal return to ability due to, say, the development of new technologies. These activities require qualities which cannot be taught such as curiosity, determination, perseverance, which we gather under the term ability. Hence, SBTC favours the ablest, who in turn increase their education effort. Thus, firms become more demanding for two reasons: workers' productivity directly increases with SBTC, and workers increase their education effort. Those who do not want to fall below the employability threshold are compelled to over-educate[3], while those who cannot meet employers' requirements do not acquire education. Thus SBTC is simultaneously responsible for the rise in the returns to ability, the rise in education attainment, over-education and the increase in LTU. In addition to this, SBTC also has strong consequences

[1] A large part of the rise in LTU in global unemployment is simply due to the rise in unemployment itself. However, Machin and Manning also note, pp. 3094, that *"there is also variation in the incidence of long-term unemployment which does not seem capable of being explained simply by the overall level of unemployment."*

[2] Except for the UK, where both the overall inequality and between-group inequality grew much more.

[3] Therefore, we define over-educated workers as those who have education returns below the average but who find it optimal to train in order to avoid long-term unemployment.

on wage inequality: for a given education attainment, the over-educated have lower returns to education, which increases within-group wage inequality. Provided that over-education reaches the highest education attainment, SBTC only causes a small increase in between-group inequality. Obviously, there are market failures in the economy we have depicted. Indeed, multiple equilibria may result from the externality generated by firms' decision to set a hiring standard. In this case, SBTC can also imply qualitative changes: the economy may topple from a "high LTU/high education" equilibrium to a "low LTU/low education" equilibrium.

Our paper incorporates two main arguments.

First, employability is relative and long-term unemployed are actually unemployable. Acquiring skills through education does not necessarily protect workers against unemployment when all workers are highly skilled. Conversely, bad endowed agents can meet with success on the labour market provided other workers have lower productivity on average. In our framework, firms set hiring standards. Workers below these standards are unemployable. Hiring standards are in turn endogenous, since they depend on the average productivity of workers. Saint-Paul (1996) and Roed (1998) highlight other channels through which employability is relative. In Saint-Paul (1996), an increase in the supply of educated workers modifies the allocation of productive capital in the economy, from the unskilled to the skilled sector. Unskilled can therefore become unemployable. His study completes our approach. Roed (1998) studies the effect of egalitarian wage policies in a matching model with heterogenous agents. In this setting, the individual wage depends on the average wage. The least skilled workers are unemployable because hiring them is not profitable. Unemployability is thus a consequence of egalitarian wage policies. In our paper, unemployability is driven by the direct link between the reservation productivity and the average productivity of workers. Moreover, skills are endogenous through human capital acquisition.

Second, skills and education are imperfectly correlated. Namely, there are high educated but unskilled workers, and low educated workers can perform well on the labour market. This distinction between labour market ability and education attainment becomes very popular, and accordingly leads to the desired changes in wage inequality (see, for instance, Acemoglu, 1999, and Galor and Moav, 2000). Therefore, SBTC causes both an increase in between-group inequality (educated are in average more skilled) and in within-group inequality (the wage differential between skilled and unskilled rises in each education group). In our paper, the distinction between skills and education is endogenous. Indeed, low-skilled but educated are over-educated workers; but over-education is individually rational. In addition, those workers get lower returns from education than their co-workers with a normal education attainment. Thus, our paper also relates to the literature on over-education. In the standard human capital theory (see Becker, 1964), optimal schooling results from the equality between the marginal cost and the marginal benefit of education. By analogy, over-education emerges in the environment we depict when the marginal cost of schooling is greater than its marginal benefit.

This definition of over-education is consistent with empirical measures as falls in returns to schooling (see the book edited by Asplund and Pereira, 1999, which reviews the evidence on rates of return to human capital in fifteen European countries), or increasing occupational mismatch (see for instance Hartog, 2000). Indeed, over-educated workers perform bad on the labour market, so that, other things equal, average returns to schooling tend to decrease with over-education. Moreover, there are positive returns to over-education, as it is usually claimed. In his survey, Hartog reports that returns to over-education are about half to two-thirds of the returns to required education. In a related paper, Muysken and Ter Weel (2000) compare social to private returns to education. Private returns incorporate both employment and human capital returns, while only human capital matters from a social perspective. Over-education then rises when job-competition increases[4].

Agell and Lommerud (1997) propose an analysis of the schooling behaviour similar

[4]In Muysken and Ter Weel (2000), low-skilled workers are crowded out because education is an input in the search activity.

to ours. They consider a simple two period model with heterogenous agents and formal education. Firms have to incur an exogenous training cost and wages are proportional to human capital; this defines a reservation productivity. Agents endowed with a medium talent are therefore constrained to get an education level above the natural one. However, the reservation productivity is exogenous in their study. Hence, there is no link between the average education attainment and exclusion of the labour market. Moreover, wages are perfectly correlated to education in their paper, i.e. the higher the education attainment, the higher the wage.

Our paper is also closely related to the growing literature on LTU. Numerous papers focus on the links between the duration of unemployment spells and the conditional probability to find a job. Blanchard and Diamond (1994) for instance investigate the consequences of a small preference for the workers with the least unemployment duration (the ranking hypothesis). Other authors point to human capital losses during unemployment. Acemoglu (1995), Coles and Masters (1998), Ljungqvist and Sargent (1998) show how human capital destruction can imply demoralization in search activity, or increasing reluctance from employers to hire the long-term unemployed. Conversely, we here emphasize the role of ex-ante heterogeneity. LTU is then concentrated among the low-skilled workers[5]. Empirically, duration-dependence is still under debate, while there is no doubt that heterogeneity – both observed and unobserved – play a crucial role (see Machin and Manning, 1999).

The discussion is organized as follows. Section 2 describes a matching model with heterogenous workers and formal education. In Section 3, we analyse the equilibrium with exogenous expectations. The links between the reservation productivity, over-education and wage inequality are addressed in Section 4. Section 5 focuses on the equilibrium and the effects of SBTC. We end with some concluding remarks.

[5]High-skilled workers can also be long-term unemployed in our model. But these workers are unlucky individuals who can still find a job with an unchanged probability.

2. The model

We build a continuous time matching model à la Pissarides (1990). The model has two main specificities. First, there are constant marginal returns in the matching technology. Hence, matching probabilities are given. The value of a vacancy is then endogenously determined. Second, agents are ex-ante heterogenous across each cohort. Both assumptions will give rise to the exclusion of the least productive workers, the so-called long-term unemployed[6].

We neglect matching frictions. We could incorporate them by assuming as Blanchard and Diamond (1989, 1994) that the total number of jobs is given. Although some of our results may be altered, the basic mechanisms we enlight would still be present.

The economy comprises a large number of firms. Each firm is endowed with a single job, which can be vacant or filled. The exogenous flow probability to meet a worker is $\eta > 0$.

At each instant, $\delta > 0$ individuals are born unemployed; δ also stands for the death rate, so that the size of the population is normalized at unity. Agents are risk-neutral and discount time at rate $\rho > 0$. Immediately after birth, they choose an education level and start to prospect on the job market. The population thus comprises U unemployed, E long-term unemployed and $L \equiv 1 - U - E$ employed.

Each agent is characterized by a single parameter a which we call ability or type. It is distributed on $[0, 1]$ according to the cumulative distribution Φ. We shall denote by ϕ the density function if it exists. This distribution reflects inequality in the distribution of wealth, or in the distribution of human capital. Ability here refers to attributes which cannot be acquired through education, but are correlated with both educational and labour market performances.

Agents choose the level of their schooling effort e before their entry on the labour market. We assume unemployable agents quit the labour market. Let $W^e(a, e)$ and

[6] LTU here refers to exclusion from a primary sector where human capital is valuable. The reservation return of other opportunities is simply set to zero.

$W^u(a,e)$ be the values of being employed and unemployed respectively for a type a individual endowed with a schooling effort e. Let $r \equiv \rho + \delta$, $w(a,e)$, μ be the global discount rate, wage and exit rates. Assume for simplicity that there are no unemployment benefits. The values $W^u(a,e)$ and $W^e(a,e)$ satisfy the two arbitrage equations:

$$rW^u(a,e) = \mu\left[W^e(a,e) - W^u(a,e)\right] \tag{2.1}$$

$$rW^e(a,e) = w(a,e) \tag{2.2}$$

Symmetrically, let V^v and $V^e(a,e)$ denote the values of a vacant job and of a filled job. Those values satisfy the two arbitrage equations:

$$\rho V^v = p\eta\left\{E\left[V^e(a,e) \mid V^e(a,e) \geq V^v\right] - V^v\right\} \tag{2.3}$$

$$rV^e(a,e) = f(a,e) - w(a,e) \tag{2.4}$$

Vacant jobs may become filled with probability $\eta > 0$. Filled jobs bring a revenue to the firm of $f(a,e) - w(a,e)$.

Assumption 1 Properties of the production function

> *The function $f(a,e)$ is strictly increasing in both arguments, twice differentiable, and concave in its second argument. Cross-derivatives are strictly positive, i.e.* $f''_{12}(a,e) = f''_{21}(a,e) > 0$; *it also satisfies the boundary conditions* $f(a,0) = f(0,e) = 0$, *for all* $a \geq 0$ *and all* $e \geq 0$.

Production requires both ability and education[7]. Two properties are very important for the rest of the paper. First, the marginal productivity of education is decreasing, so that marginal returns to schooling will also be decreasing. This assumption is standard, and in line with the empirical evidence (see, for instance, Psacharopoulos, 1994, for international comparisons, and Ashenfelter and Rouse, 1998, on a US sample of twins). Second, ability and education interplay positively in marginal returns. For obvious data

[7]This is an important difference with Agell and Lommerud (1997), in which production only requires education.

problems, few studies investigate the links between ability and the marginal returns to education. Nevertheless, Card (1999), p.1854, reports that the effects of IQ on the returns to education are generally positive once controlled for family effects. Hence, the ablest will also be those who tend to acquire more education[8]. In this setting, SBTC consists in an increase in the marginal productivity of ability for the ablest.

An applicant is acceptable to the firm with probability p. Productive matches end with the worker's death. It is the only source of separation. We also assume that the job disappears with the worker. Firms do not know the type of the worker they will meet. Therefore, there is a conditional expectation operator in equation (2.3). This expectation is conditional to the event that the worker is employable, i.e. the value of the filled job is greater than the value of the vacant job. The condition $V^e(a,e) \geq V^v$ thus defines a recruitment strategy.

Wages are determined by Nash bargaining:

$$w^*(a,e) = \underset{w(a,e)}{\arg\max} \left\{ V^e(a,e)^{1-\beta} \left[W^e(a,e) - W^u(a,e) \right]^{\beta} \right\} \tag{2.5}$$

where $\beta \in (0,1)$ is the bargaining power of the worker. In the next section, we solve the model under exogenous expectations.

3. Equilibrium with exogenous expectations

In this section, we show that firms' hiring behaviour consists in choosing a reservation productivity. This reservation productivity depends on the production technology, the bargaining power of the firm, and on the expectation of the value of a vacant slot. For expositional ease, we make two assumptions. First, we assume that each firm is endowed with a preliminary estimate V^v of the value of a vacancy. This expectation is common

[8] In the signalling theory (Spence, 1973), education does not provide any additional skills to workers. However, those who have higher skills also reach higher education attainments, since it is easier for them to graduate. In our paper, the ablest reach the highest education attainment but there is no uncertainty about their productivity.

to all firms. Second, we assume that $e \equiv e(a)$, so that the productivity $f(a, e(a))$ is increasing with a.

Equations (2.1) to (2.5) lead to the standard following wage

$$w^*(a, e) = \beta \frac{r + \mu}{r + \beta\mu} f(a, e) \tag{3.1}$$

It increases with the productivity, the bargaining power and the probability to meet a vacancy; it decreases with the discount rate. Using (2.1), (2.2) and (3.1), we can solve the value of search

$$W^u(a, e) = \frac{\beta\mu}{r + \beta\mu} \frac{f(a, e)}{r} \tag{3.2}$$

The value of search is equal to a fixed proportion $\beta\mu / (r + \beta\mu)$ of the total match value $f(a, e)/r$. Finally, merging (2.4) and (3.1), we obtain

$$V^e(a, e) = v^e f(a, e) \tag{3.3}$$

where $v^e = (1 - \beta)/(r + \beta\mu)$ is the firm's return per efficient unit of labour. A worker of ability a with education e is employable if $V^e(a, e) \geq V^v$, which is equivalent to $f(a, e) \geq V^v/v^e$. The ratio V^v/v^e is a reservation productivity. Below, the applicant is rejected by the firm. Above, the worker is systematically accepted by the firm. This reservation productivity, denoted by $y \equiv V^v/v^e$, increases with the expected value of a vacancy.

Since all firms are endowed with the same expectation, the condition $f(a, e) \geq y$ consists in a condition of employability: an individual whose application is rejected by one firm is actually refused in the whole economy. These workers therefore quit the labour force, and firms only meet adequate workers, i.e. $p = 1$.

The employability condition then determines a unique ability a_- below which agents are excluded of the labour market. This limit ability verifies $f(a_-, e(a_-)) = y$. Long-term unemployment is equal to the share of agents whose type is below a_-. Hence, $u_{LT} = \Phi(a_-)$. The participation rate is therefore $1 - \Phi(a_-)$ (see figure 1).

[Figure 1]

Employable agents run the risk of leaving unemployment $\mu+\delta$. The number of unemployed is then

$$U=\int_0^\infty \delta\left(1-\Phi\left(a_-\right)\right)\exp\left[-\left(\mu+\delta\right)s\right]ds=\frac{\delta}{\mu+\delta}\left(1-\Phi\left(a_-\right)\right)$$

The unemployment rate is $u=\delta/\left(\mu+\delta\right)$, i.e. the ratio of the entry rate to the global exit rate. It is independent of the share of long-term unemployed. This property reflects the assumption according to which the rate of contact μ does not depend on the individual productivity. It would be the case if search intensity was endogenous, or if there was match-specific heterogeneity. Employment is $L=\left(1-u_{LT}\right)\left(1-u\right)$.

4. Schooling and reservation productivity

This section analyses the links between ability, schooling effort and the reservation productivity. We mainly focus on the impact of the reservation productivity on over-education. An increase in the reservation productivity causes a rise in LTU, a rise in the average education attainment, a fall in between-group inequality. It also tends to increase over-education and within-group inequality.

During their studies, individuals face direct costs (as tuition or transportation), as well as psychic costs. We assume that agents choose the level of their efforts. Let $C\left(e\right)$ be the schooling cost, paid before the entry on the labour market. For ease, this cost is assumed to be independent of agents' ability.

Assumption 2 Properties of the schooling cost

The function C is strictly increasing, strictly convex and satisfies $C\left(0\right)=C'\left(0\right)=0$.

Let $y\geq 0$ be the reservation productivity. Hereafter, $e^*\left(a,.\right)$ denotes the optimal schooling effort of a type a agent. Individuals proceed in two steps. First, they consider they enter the labour force to search for a job. They then maximize the utility they get as an unemployed, net of the schooling cost. However, they are also submitted to the employability constraint: they must at least get the reservation productivity y. Formally, we have

$$Z(a) = \max_e \{W^u(a,e) - C(e)\,;\, f(a,e) \geq y\}$$

All agents do not necessarily want to acquire formal education. This is the case when $Z(a) < 0$. The following proposition analyses the optimal schooling effort in function of the reservation productivity.

Proposition 1 OPTIMAL EDUCATION

Let $\overline{e}(a)$ be the unique solution of $C'(\overline{e}(a)) = \frac{\beta\mu}{r+\beta\mu}\frac{f(a,\overline{e}(a))}{r}$. Let also $\underline{e}(a,y)$ be the unique solution of $f(a,\underline{e}(a,y)) = y$. Then,

$$e^*(a,y) = \begin{cases} \overline{e}(a) \text{ if } f(a,\overline{e}(a)) \geq y \\ \underline{e}(a,y) \text{ if } f(a,\overline{e}(a)) < y \text{ and } \frac{\beta\mu}{r+\beta\mu} y \geq C(\underline{e}(a,y)) \\ 0 \text{ elsewhere} \end{cases}$$

Three distinct educative behaviours emerge according to the level of y, what figure 2 depicts.

[Figure 2]

When the reservation productivity is y_0, the employability constraint does not bind and the optimal effort results from the standard equality between the marginal benefit $\frac{dW^u(a,e)}{de}$ and the marginal cost $C'(e)$. It is denoted by $\overline{e}(a)$. Since it does not depend on the reservation productivity, we call this effort *natural* education. Natural education increases with the wage bargained, thus with the rate of contact μ and the bargaining power β. It also increases with the individual ability. The educational system thus tends to magnify inequality in abilities: the more gifted individuals are, the higher their education attainment. When the reservation productivity is y_1, the agent cannot perform natural education: it does not provide the worker sufficient skills. Formally, the employability constraint is binding. The worker is therefore obliged to rise his level of education until his productivity reaches the reservation productivity y_1. This schooling effort is denoted by $\underline{e}(a,y)$. We call it *constrained* education[9]. Constrained education rises with the reservation productivity. This is typically a case of over-education since the marginal cost of schooling is greater than its marginal benefit. Lastly, when the reservation productivity is y_2, the agent does not acquire skills through formal education. Indeed, he has no chance to obtain the reservation productivity: the schooling cost of the corresponding effort is greater than its utility.

4.1. Ability, schooling efforts and reservation productivity

The above analysis shows that schooling efforts vary with the reservation productivity. We now explore the links between the schooling attainment and labour market ability. This study leads to the emergence of three distinct population groups.

Proposition 2 Endogenous segmentation of the population

Let $y \leq f(1, \overline{e}(1))$, $a_-(y)$ such that $C(\underline{e}(a_-, y)) = \frac{\beta\mu}{r+\beta\mu}\frac{y}{r}$ and $a^+(y)$ such that $f(a^+, \overline{e}(a^+)) = y$. Then,

$$e^*(a,y) = \begin{cases} \overline{e}(a) \ \ \text{if} \ \ a \geq a^+ \\ \underline{e}(a,y) \ \ \text{if} \ \ a \in [a_-, a^+[\\ 0 \ \ \text{elsewhere} \end{cases}$$

Agents whose ability is below $\underline{a}$ are excluded from the economic activity. Since their skills are not marketable, their education effort is simply zero. Agents whose ability $a \in [a_-, a^+[$ are constrained in their schooling effort: they get the reservation productivity. Finally, agents whose ability is above a^+ are not constrained. The population can be divided into three groups: $\Phi(a_-)$ long-term unemployed, $\Phi(a^+) - \Phi(a_-)$ over-educated agents, and $1 - \Phi(a^+)$ individuals who achieve their unconstrained effort. Figure 3 depicts the education effort as a function of ability.

[9]Constrained education usually comes from imperfect financial markets. It then implies under-education, since marginal benefits are greater than marginal costs. By contrast, we assume perfect financial markets, and constrained education emerges when the employability constraint binds.

[Figure 3]

The schooling effort is nil on $[0, a_{-}[$. It jumps to $\underline{e}(a_{-}, y)$ when the ability is a_{-}. Then, it is strictly decreasing on $[a_{-}, a^{+}[$ and strictly increasing above a^{+}. Note that $e^{*}(1, .)$ is not necessarily greater than $e^{*}(a_{-}, .)$. Students who achieve long durations of education correspond to the most gifted individuals, but also agents facing difficulty to reach employability. Obviously, the performances on the labour market of these two kinds of individuals are very contrasted. As long as there is LTU, within-group wage inequality tends to increase with the education attainment. Two cases may arise. First, if $\overline{e}(1) > \underline{e}(a_{-}, y)$, the highly productive workers undertake the longest studies. Inequality is then the weakest for small education durations $e^{*}(a^{+}, .)$ and high ones (greater than $\underline{e}(a_{-}, y)$), increasing until duration $\underline{e}(a_{-}, y)$ is reached. Second, if $\overline{e}(1) < \underline{e}(a_{-}, y)$, less able students undertake the longest studies to get the reservation productivity. Within-group inequality is strictly increasing on $[e^{*}(a^{+}, .), \overline{e}(1)]$ and becomes then very low. The particularity here is that agents who undertake the longest studies have also the worst performances on the labour market.

4.2. LTU and education attainment

The links between education attainment and the reservation productivity are depicted by figure 4. An increase in the reservation productivity from y_0 to y_1 rises unambiguously a_{-} – from a_{-}^{0} to a_{-}^{1} – and a^{+} – from a_0^{+} to a_1^{+}.

[Figure 4]

LTU increases since more agents become unable to reach the reservation productivity. The average level of education attainment increases, reflecting the rise in productivity standards. Note that there is no systematic relation between the number of LTU and the number of over-educated workers. The number of LTU is $u_{LT} = \Phi(a_{-}(y))$, while the number of over-educated workers is $\epsilon(y) = \Phi(a^{+}(y)) - \Phi(a_{-}(y))$. LTU necessarily rises with the reservation productivity y. But, the net effect of the reservation productivity on $\epsilon(y)$ is ambiguous:

$$\frac{d\epsilon(y)}{dy} = \phi\left(a^{+}\right)\frac{da^{+}}{dy} - \phi(a_{-})\frac{da_{-}}{dy}$$

Over-education is reduced by the bottom – low gifted individuals become LTU –, but is enriched by the top – gifted individuals are now constrained in their schooling effort.

4.3. Wage inequality

Changes in the reservation productivity tend to affect both the size and composition of wage inequality. Namely, overall inequality falls, between-group inequality tends to decrease, while within-group inequality increases. When the reservation productivity is set to zero, there is no LTU, and therefore no more over-education. Education attainment is then perfectly correlated with labour market ability: the abler the agent, the more he trains. Then between-group inequality is the highest. As the reservation productivity rises, the least skilled workers quit the labor market, and over-education appears. Hence, between-group inequality shortens. In the following section, we show that this mechanism will weaken direct effects of SBTC, so that between-group inequality increases slightly. At the extreme, when the reservation productivity tends to the productivity of the ablest workers $f(1, \overline{e}(1))$, individuals are equally productive, whatever the education attainment. Within-group inequality is low when there is no over-education. Indeed, productivity rises with education attainment. Conversely, it tends to increase with over-education, since over-educated workers get lower wages than naturally educated workers for a given education attainment. Hence, within-group inequality tends to increase with the reservation productivity.

5. Long-run consequences of SBTC

In section 3, the reservation productivity was related to identical and given expectations across firms. We then analysed optimal schooling effort for a given reservation productivity in section 4. We now close the model by requiring that expectations are equal to realizations in equilibrium. This framework will enable us to study the effects of changes in the macroeconomic environment. In particular, we shall argue that SBTC is responsible for the rise in education attainment, LTU, within-group wage inequality, as well as for the small increase in between-group wage inequality.

Dividing each side of equation (2.3) by v^e, the equilibrium reservation productivity solves

$$y = \frac{\eta}{\rho+\eta}\left[\int_{a_-}^{a^+} \frac{y}{1-\Phi(a_-)} d\Phi(a) + \int_{a^+}^{1} \frac{f(a,\overline{e}(a))}{1-\Phi(a_-)} d\Phi(a)\right] \tag{5.1}$$

The reservation productivity is equal to a fixed proportion of the average productivity across employable workers. The factor $\eta/(\rho+\eta)$ reflects the stochastic nature of the matching process. Without uncertainty, i.e. when η becomes arbitrarily large, or without discounting, i.e. when ρ becomes arbitrarily "small", the reservation productivity is equal to the average productivity. Conversely, if the average duration of search of an adequate worker is extremely long, i.e. η tends to 0, or if employers are very impatient, i.e. ρ tends to infinity, employers become less severe and the reservation productivity tends to zero.

Theorem 1 EXISTENCE OF EQUILIBRIUM

Let $\pi(y) = \int_{a_-}^{a^+} \frac{y}{1-\Phi(a_-)} d\Phi(a) + \int_{a^+}^{1} \frac{f(a,\overline{e}(a))}{1-\Phi(a_-)} d\Phi(a)$.

There exists an equilibrium. It is unique when $\pi'(y) < (\rho+\eta)/\eta$ *for all* $y \in [0, f(1,\overline{e}(1))]$

Multiple equilibria is a natural output in this framework[10]. Equilibria where the reservation productivity is the highest are also those where schooling efforts are the largest and LTU the worst. Following Cooper and John (1988), multiplicity can be related to two types of strategic complementarities. Taking the derivative of the RHS of equation (5.1) with respect to y, it comes

[10] There are other examples of multiple equilibria in the literature on education and matching unemployment. See for instance Laing, Palivos and Wang (1995), and Saint-Paul (1996). Although the mechanism is also related to coordination failures in these papers, both the externalities and the way they are conveyed differ in our work.

$$\pi'(y) = \underbrace{\frac{\Phi(a^+) - \Phi(a_-)}{1-\Phi(a_-)}}_{\substack{\text{direct effect} \\ (+)}} + \underbrace{\frac{\phi(a_-)}{[1-\Phi(a_-)]^2}\frac{da_-}{dy}\left[\int_{a^+}^{1} f(a,\overline{e}(a))\,d\Phi(a) - \left(1-\Phi\left(a^+\right)\right)y\right]}_{\substack{\text{over-education effect} \\ (+)}}$$

Both terms into brackets are positive: an increase in hiring standards translates into a higher average productivity. The decentralized choice of the reservation productivity thus conveys externalities. The first externality is direct: an increase in the reservation productivity leads to an equivalent increase in the mean productivity of workers, thus to a rise in expected profits. The second externality is based on students' responses to changes in the reservation productivity. An increase in the reservation productivity entails a rise in the average schooling effort, since more workers are forced to get a high education attainment. In either cases, the individual reservation productivity is an increasing function of others' reservation productivity choices. There is thus room for coordination failures and multiple equilibria. Those equilibria cannot be ranked easily. Indeed, firms' profits must be taken into account in this model. Hence, equilibria where the reservation productivity is the lowest are better for workers, but worse for firms.

The equilibrium is unique when the externality is sufficiently weak. For instance, this is the case when there is no heterogeneity. Indeed, assume that the ability of all workers is simply $\widehat{a}$. The reservation productivity then solves:

$$y = \frac{\eta}{\rho+\eta} f(\widehat{a}, \overline{e}(\widehat{a}))$$

Hence, heterogeneity is necessary for the emergence of multiple equilibria. However, it is not sufficient.

Proposition 3 Example of uniqueness

Assume abilities are distributed according to a uniform law on $[0,1]$. *Let* $C(e) = ce^{\sigma}$, $c > 0$ *and* $\sigma > 1$. *Let* $f(a,e) = ae$. *For all* $a \in [0,1]$ *and all* $y \in [0, \overline{e}(1)]$,

(i) $\overline{e}(a) = \left[\frac{1}{c\sigma}\frac{\beta\mu}{r+\beta\mu}\frac{a}{r}\right]^{\frac{1}{\sigma-1}}$ *and* $\underline{e}(a,y) = y/a$

(ii) $a_-(y) = \left[\frac{r+\beta\mu}{\beta\mu}rc\right]^{1/\sigma} y^{\frac{\sigma-1}{\sigma}}$ *and* $a^+(y) = \sigma^{1/\sigma}\left[\frac{r+\beta\mu}{\beta\mu}rc\right]^{1/\sigma} y^{\frac{\sigma-1}{\sigma}}$

(iii) There exists a unique equilibrium

The optimal schooling effort $\overline{e}(a)$ admits the general properties we have seen below; it also decreases with the scale parameter c of the schooling cost function, and with the elasticity σ of this function with respect to education. It is a strictly convex function of the ability a when $\sigma \in (1, 2)$, linear when $\sigma = 2$ and strictly concave when $\sigma > 2$. This property means that wage inequality is wider when schooling costs are low, i.e. $\sigma < 2$.

In the context of proposition 3, the share of over-educated is

$$\epsilon(y) = a^{+}(y) - a_{-}(y) = \left(\sigma^{1/\sigma} - 1\right) a_{-}(y)$$

Hence, over-education amplifies when LTU increases.

This positive correlation between education, over-education and LTU is our main result. Why should the reservation productivity increase? To us, the culprit is SBTC, or, more precisely, *ability*-biased technological change. Formally, let $f(a, e) \equiv f(a, e; \gamma)$, where γ is the ability bias of technological change. Then, $f''_{a\gamma}(a, e; \gamma) > 0$. SBTC rises the return to ability of the most gifted. Firms thus become more demanding and the reservation productivity increases. That increase translates into both LTU and over-education. This preliminary effect is enhanced by the educational system. Indeed, human capital theory entails an increase in education attainment of the ablest. This strengthens the pressure on low-skilled workers, so that, once again, LTU and over-education rise. Due to the links between wage inequality and over-education, within-group inequality rises. Indeed, the direct effect of SBTC is reinforced by over-education, since there are in average more over-educated workers in each education group. Furthermore, in spite of SBTC, between-group wage inequality does not increase very much. The generalisation of over-education to all education attainments actually tends to decrease between-group wage inequality. Moreover, any structural shock can affect the conjectural scheme of firms. SBTC can thus make the economy topple from a “low LTU/low education” equilibrium to a “high LTU/high education” equilibrium.

Figure 5 illustrates the effects of SBTC on education attainments. We consider an initial situation where the reservation productivity is close to zero, so that there is no

LTU. Optimal schooling is depicted by the thin plain line. Since individuals do not need to over-educate, this curve is strictly increasing. Ability and schooling attainments are then perfectly (positively) correlated. Then SBTC occurs, causing two main changes. First, the ablest educate much more, reflecting the important rise in the returns to ability, while the natural effort of the least able increases by much less. Consequently, the curve depicting education efforts is moving upward. Second, the reservation productivity increases to $\overline{y} > 0$, so that LTU emerges. Moreover, medium-skilled workers are then over-educated, since the employability constraint is binding for them. Both LTU and over-education are illustrated by the decreasing part of the plain line curve.

[Figure 5]

Within-group wage inequality is strongly rising for education attainments between $\underline{e}\left(a^{+}, \overline{y}\right)$ and $\underline{e}\left(a_{-}, \overline{y}\right)$. Indeed, over-educated workers only get a wage proportional to the reservation productivity, while the returns to ability of the natural-educated have risen. Between-group wage inequality also tends to increase, since the ablest also benefit from a longer training period. However, over-educated workers reduce this increase, since all these workers have the same productivity. These predictions are largely consistent with the European stylized facts briefly stated in the introduction.

6. Concluding comments

Let us summarize the discussion. We built a matching model with heterogenous workers, endogenous education and constant marginal returns to search activity. Our results are the following:

1) The hiring behaviour of firms consists in choosing a reservation productivity. Applicants whose productivity is lower than the reservation productivity are systematically rejected by employers; they are long-term unemployed. Long-term unemployment (LTU) thus depends on firms' expectations and on the share of the surplus they obtain during the bargaining.

2) The average productivity across employable workers increases with the reservation productivity. The decentralized process of recruitment conveys an externality responsible for the emergence of multiple equilibria. Multiplicity hinges on strategic complementarity between employers.

3) Two very different individuals can have the same education attainment. One of them is abler and performs his natural education. The other is constrained to get the reservation productivity through formal education. He is over-educated since the marginal cost of education is greater than the marginal benefit. He also gets lower wages than the first type of individual.

4) Within-(education)group generally increases with the level of education. Skill-biased technological change (SBTC) causes an increase in LTU, in within-group inequality, and a moderate rise in between-group inequality. SBTC also exacerbates over-education.

This study neglects important links between the labour demand and the training level of the labour force. Several theoretical studies point out a positive relation between employment and education (see, for instance, Laing et al., 1995). The argument goes as follows: a rise in the education level increases the average productivity of workers, and thus the expected profits of firms. Consequently, the labour demand increases, which rises the exit rate from unemployment. Returns to education are greater, so that agents invest more in education. This mechanism can generate multiple equilibria, and public policy takes place. This literature completes our approach.

LTU originates from inequality. How to reduce it? Educational systems are obvious candidates. However, our paper highlights channels through which schooling systems actually magnify inequality. For instance, vouchers targeted towards the best students may actually worsen exclusion by rising the reservation productivity. Equivalently, subsidies to education may benefit more to the ablest. The complete study of the interactions between LTU and the education policy is left for further research.

A. Proofs

Proposition 1 Let $G(a,e) = W^u(a,e) - C(e)$, where $W^u(a,e)$ is given by equation (3.2); it is strictly concave in e. The program of an agent who trains is

$$Z(a) = \max_{e} \{G(a,e);\ f(a,e) \geq y\}$$

Consider the lagrangean

$$\ell(e,\lambda) = G(a,e) + \lambda (f(a,e) - y)$$

where $\lambda \geq 0$ is the Kühn and Tucker factor. Optimality conditions can be written

$$\frac{\partial W^u(a,e)}{\partial e} = \frac{\beta\mu}{r+\beta\mu}\frac{f_2'(a,e)}{r} = C'(e) - \lambda f_2'(a,e) \tag{A.1}$$

$$\underset{\geq 0}{\lambda}(\underset{\geq 0}{f(a,e) - y}) = 0 \tag{A.2}$$

According to (A.1), the marginal benefit of education is lower than the marginal cost when the agent is constrained, i.e. when $e = \underline{e}(a,.)$. Since $C(0) = 0$, $Z(a) \geq 0$ when $e = \overline{e}(a)$. Hence, $e^*(a,.) = \overline{e}(a)$ when $y \leq f(a,\overline{e}(a))$. If $y > f(a,\overline{e}(a))$, the schooling effort of an agent who gets training is $e = \underline{e}(a,y)$. Two cases may arise: if $W^u(a,\underline{e}(a,y)) \geq C(\underline{e}(a,y))$, then $e^*(a,y) = \underline{e}(a,y)$. Elsewhere, $Z(a) < 0$ and $e^*(a,.) = 0$. ■

Proposition 2 Let $g(a) = f(a,\overline{e}(a))$. We have $g(0) = 0$, $g(1) = f(1,\overline{e}(1))$ and $g'(a) > 0$. Thus there exists a unique $a^+ \equiv a^+(y) \in [0,1]$ for all $y \in [0, f(1,\overline{e}(1))]$. Consider the function $G(a,\underline{e}(a,y)) = \frac{\beta\mu}{r+\beta\mu}\frac{y}{r} - C(\underline{e}(a,y))$. It is strictly increasing in a, with $\lim_{a\to 0} G(a,\underline{e}(a,y)) = -\infty$ and

$$\begin{aligned} G(1,\underline{e}(1,y)) &= W^u(1,\underline{e}(1,y)) - C(\underline{e}(1,y)) \geq \tfrac{\partial W^u(1,\underline{e}(1,y))}{\partial e} - C'(\underline{e}(1,y)) \\ &\geq \tfrac{\partial W^u(1,\overline{e}(1))}{\partial e} - C'(\overline{e}(1)) = 0 \end{aligned}$$

The second inequality comes from the strict concavity of G with respect to its second argument. Consequently, there exists a unique $a_- \equiv a_-(y)$ such that $G(a_-, \underline{e}(a_-, y)) = 0$ for all $y \in [0, f(1, \overline{e}(1))]$. The concavity of G implies that $0 \leq a_- < a^+ \leq 1$.

Since $g(a)$ is strictly increasing in a, $f(a, \overline{e}(a)) \geq y$ for all $a \geq a^+$ and $e^*(a) = \overline{e}(a)$. When $a < a^+$, the worker is constrained to make the effort $\underline{e}(a, y)$ if he gets training. He thus obtains the gain $Z(a) = G(a, \underline{e}(a, y))$. But, $Z(a) \geq 0$ is equivalent to $\frac{\beta\mu}{r+\beta\mu}\frac{y}{r} \geq C(\underline{e}(a, y))$, i.e. $a \leq a_-$. Therefore $e^*(a) = \underline{e}(a, y)$ if $a \in [a_-, a^+)$, and $e^*(a) = 0$ if $a < a_-$. ■

Theorem 1 Let

$$\Psi(y) = y - \frac{\eta}{\rho+\eta}\left[\int_{a_-(y)}^{a^+(y)} \frac{y}{1-\Phi(a_-(y))} d\Phi(a) + \int_{a^+(y)}^{1} \frac{f(a, \overline{e}(a))}{1-\Phi(a_-(y))} d\Phi(a)\right]$$

An equilibrium is a $y^* \geq 0$ which solves $\Psi(y^*) = 0$. Since Ψ is continuous in y, and since $\Psi(0) = -\frac{\eta}{\rho+\eta}\int_0^1 f(a, \overline{e}(a))\, d\Phi(a) < 0$ while $\Psi(f(1, \overline{e}(1))) = \frac{\rho}{\rho+\eta} f(1, \overline{e}(1))$, there exists at least an equilibrium on $(0, f(1, \overline{e}(1)))$. ■

Proposition 3 When $\phi(a) = 1$ if $a \in [0, 1]$, we have $\Phi(a) = a$. An equilibrium therefore solves

$$\begin{aligned} y &= \tfrac{\eta}{\rho+\eta}\left[\int_{a_-}^{a^+} \tfrac{y}{1-a_-} da + \int_{a^+}^{1} \tfrac{a\overline{e}(a)}{1-a_-} da\right] \\ &= \tfrac{\eta}{\rho+\eta}\left[\tfrac{a^+-a_-}{1-a_-} y + \tfrac{1}{1-a_-}\int_{a^+}^{1} a\overline{e}(a)\, da\right] \end{aligned} \tag{A.3}$$

When $C(e) = ce^\sigma$, $c > 0$ and $\sigma > 1$, we have $\overline{e}(a) = \left[\frac{1}{c\sigma}\frac{\beta\mu}{r+\beta\mu}\frac{a}{r}\right]^{\frac{1}{\sigma-1}}$, and $\underline{e}(a, y) = y/a$, for all $a \in [0, 1]$; hence (i) is proved.

Then, $a^+\overline{e}(a^+) = y$ is equivalent to $a^+(y) = \sigma^{1/\sigma}\left[\frac{r+\beta\mu}{\beta\mu} rc\right]^{1/\sigma} y^{\frac{\sigma-1}{\sigma}}$, for all $y \in [0, \overline{e}(1)]$. Moreover, $W^u(a_-, y/a_-) = C(y/a_-)$ is equivalent to $a_-(y) = \left[\frac{r+\beta\mu}{\beta\mu} rc\right]^{1/\sigma} y^{\frac{\sigma-1}{\sigma}}$, for all $y \in [0, \overline{e}(1)]$. Hence, (ii) is proved.

Using (A.3) together with a_- and a^+, we are led to solve the roots – in the interval $[0, \overline{e}(1)]$ – of the following polynomial function

$$-\left[\frac{\rho}{\rho+\eta} + \frac{\eta}{\rho+\eta}\frac{\sigma^{\sigma^{-1}+1}}{2\sigma-1}\right] D^{1/\sigma} y^{2-\sigma^{-1}} + y - \frac{\eta}{\rho+\eta}\sigma^{\frac{1}{1-\sigma}} D^{\frac{1}{1-\sigma}} \frac{\sigma-1}{2\sigma-1}$$

where $D = \frac{r+\beta\mu}{\beta\mu} rc$. Making the change of variable $z = y^{1/\sigma}$, we have

$$P(z) = -\left[\frac{\rho}{\rho+\eta} + \frac{\eta}{\rho+\eta}\frac{\sigma^{\sigma^{-1}+1}}{2\sigma-1}\right] D^{\sigma^{-1}} z^{2\sigma-1} + z^{\sigma} - \frac{\eta}{\rho+\eta}\sigma^{\frac{1}{1-\sigma}} D^{\frac{1}{1-\sigma}} \frac{\sigma-1}{2\sigma-1}$$

An equilibrium is a $z^* \in \left[0, \overline{e}(1)^{1/\sigma}\right]$ which solves $P(z^*) = 0$. We know that $P(0) < 0$. Moreover,

$$P'(z) = z^{\sigma-1}\left[\sigma - (2\sigma-1)\left(\frac{\rho}{\rho+\eta} - \frac{\eta}{\rho+\eta}\frac{\sigma^{\frac{1+\sigma}{\sigma}}}{2\sigma-1}\right) z^{\sigma-1}\right]$$

The polynomial function P is strictly increasing on $\left(0, \left[\frac{\sigma(\rho+\eta)}{(2\sigma-1)\rho-\eta\sigma^{(1+\sigma)/\sigma}}\right]^{1/(\sigma-1)}\right)$. To establish uniqueness, it is sufficient to show that $P\left(\overline{e}(1)^{1/\sigma}\right) \geq 0$. But, $\overline{e}(1)^{1/\sigma} = \sigma^{\frac{1}{\sigma(1-\sigma)}} D^{\frac{1}{\sigma(1-\sigma)}}$. Hence,

$$P\left(\overline{e}(1)^{1/\sigma}\right) = \sigma^{\frac{1}{1-\sigma}} D^{\frac{1}{1-\sigma}} \frac{\rho}{\rho+\eta}\left(1 - \sigma^{-1/\sigma}\right) > 0$$

and (iii) is proved. ■

References

[1] Acemoglu, D.T., 1995. Public policy in a model of long-term unemployment. Economica 62, 161-178

[2] Acemoglu, D.T., 1999. Changes in unemployment and wage inequality: an alternative theory and some evidence. American Economic Review 89, 1259-1278

[3] Agell, J., Lommerud, K.E., 1997. Minimum wages and the incentives for skill formation. Journal of Public Economics 64, 25-40

[4] Ashenfelter, O., Rouse, C., 1998. Income, schooling and ability: evidence from a new sample of identical twins. Quarterly Journal of Economics 113, 253-284

[5] Asplund, R., Pereira, P.T., 1999. Returns to human capital in Europe: a literature review. ETLA, The Research Institute of the Finnish Economy, Sarja B 156 Series. Helsinki: Taloustieto Oy

[6] Becker, G.S., 1964. Human capital. Columbia University Press

[7] Blanchard, O.J., Diamond, P., 1989. The Beveridge curve. Brooking Papers on Economic Activity 1, 1-74

[8] Blanchard, O.J., Diamond, P., 1994. Ranking, unemployment duration and wages. Review of Economic Studies 61, 417-434

[9] Card, D., 1999. The causal effect of education on earnings. Handbook of Labor Economics, vol. 3, 1801-1864

[10] Coles, M., Masters, A., 1998. Retraining and long-term unemployment in a model of unlearning by not doing. European Economic Review, forthcoming

[11] Cooper, R., John, A., 1988. Coordinating coordination failures in keynesian models. Quarterly Journal of Economics 103, 441-463

[12] Galor, O., Moav, O., 2000. Ability-biased technological transition, wage inequality and economic growth. Quarterly Journal of Economics 115, 469-497

[13] Gottschalk, P., Smeeding, T., 1997. Cross national comparisons of earnings and income inequality. Journal of Economic Literature 35, 633-687

[14] Hartog, J.A., 2000. Over-education and earnings: where are we, where should we go? Economics of Education Review 19, 131-147

[15] Laing D., Palivos, T., Wang, P., 1995. Learning, matching and growth. Review of Economic Studies 62, 115-129

[16] Ljungqvist, L., Sargent, T.J., 1998. The European unemployment dilemma. Journal of Political Economy 106, 514-550

[17] Machin, S., Manning, A., 1999. The causes and consequences of longterm unemployment in Europe. Handbook of Labor Economics, vol. 3C, 3085-3134

[18] Muysken, J., Ter Weel, B.J., 2000. Over-education, job-competition and unemployment. MERIT working paper n° 32

[19] OECD, 1999. Education at a glance, OECD indicators 1998

[20] Pissarides, C., 1990. Equilibrium unemployment theory. Basil Blackwell

[21] Psacharopoulos, G., 1994. Returns to investment in education: a global update. World Development 22, 1325-1343

[22] Roed, K., 1998. Egalitarian wage policies and long-term unemployment. Scandinavian Journal of Economics 100, 611-625

[23] Saint-Paul, G., 1996. Dual labor market: a macroeconomic perspective. MIT Press

[24] Spence, M., 1973. Job market signalling. Quarterly Journal of Economics 87, 355-374