

# A Neoclassical Kaldor Model of Real Wage Declines

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## **Abstract**

A model linking macroeconomic equilibrium and income distribution in balanced growth equilibria is developed as a variant to the Kaldor model of factor shares. It departs from the original Kaldor model in assuming equal saving rates and a neoclassical production function. Macroeconomic equilibrium (national savings equal to investment) combines with competitive microeconomic behavior to determine the real wage and real interest rate. An increase in the ratio of national debt to labour reduces the real wage, explaining recent declines.

## **1 Introduction**

The U.S. economy in past decades has exhibited a substantial decline in the real wage relative to productivity and large changes in the real interest rate over long periods. This paper develops a model linking macroeconomic policy variables to factor prices and income distribution. Macroeconomic equilibrium (Aggregate Demand equal to Aggregate Supply, or national savings equal to investment) determines a relationship between the real interest rate and the ratio of capital to labor. Microeconomic equilibrium, arising from competitive generation of factor prices, determines a second relationship between the two variables. Together, macroeconomic and microeconomic equilibrium determine the real interest rate, the capital to labor ratio, and the real wage rate in balanced growth.

The model developed is a variant of Nicholas Kaldor's Keynesian model of income distribution (1955-1956, 1957), in which equality between savings and investment is brought about by shifts between profit and labor income instead of by fluctuations in economic activity.<sup>1</sup> In Kaldor's approach, income distribution is partly explained by macroeconomic phenomena, and shifts of factor incomes are necessary to bring about macroeconomic equilibrium. The model developed here shares with Kaldor's model the involvement of income distribution with macroeconomics and the simultaneous explanation of both distributional and macroeconomic phenomena. However, the mechanism linking macroeconomic equilibrium and income distribution is different. In Kaldor's model, full employment is assumed and an aggregate investment rate is determined exogenously by balanced growth parameters. With a greater saving rate out of profits, income shifts between profits and labor income are brought about by changes in prices relative to wage rates until the aggregate saving rate equals the required investment rate. In contrast, in the model developed here, the level of production is determined endogenously, and the saving rate is assumed to be the same for all sources of income. All variables are real, so there is no inflation to bring about changes in the wage rate relative to the price level. In Kaldor's model, there is a fixed capital

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<sup>1</sup>See discussions of Kaldor's model in G. Bertola (2000, pp. 400-498), C.E. Ferguson (1969, pp. 314-322), Mitsuhiro Iyoda (1997), Luigi Pasinetti (1962), Kurt Rothschild (1993, Chapters 17-19), Sattinger (2001, pp. liii-liv), Peter Skott (1989a, 1989b), and James Tobin (1989). Kaldor's model is one of several approaches that involve income distribution in a macroeconomic model (see Bertola, 2000, Sattinger, 1990, and Sydney Weintraub, 1958). Per Krusell and Anthony Smith (1999) consider distributional impacts among heterogeneous asset levels of eliminating business cycles.

to production ratio so that marginal products of factors are not defined and play no role in determining factor prices. In the model developed here, a neo-classical production function relates marginal products of capital and labor to the ratio of capital to labor. Use of a neoclassical production function is intended to counter the argument that factor non-substitutability generates the results.

Table 1 shows interest rate, wage and debt variables that are relevant in this paper. The data are for the U.S. in the period 1961 to 1999. The real interest rate in Column 2 is measured by the average interest rate on U.S. Treasury bonds with maturity over ten years minus inflation as measured by increases in the yearly average Consumer Price Index (CPI-U). The real wage is measured by average hourly earnings in 1982 dollars for total private employment, not seasonally adjusted. Column 4 is the real wage from Column 3 divided by productivity as measured by the Major Sector Multifactor Productivity Index for manufacturing (times 100). Column 5 shows the ratio of national debt to output, from the 2003 Economic Report of the President, measured as the ratio of national debt held by the public to Gross Domestic Product. The series can be regarded as national debt per employed worker, divided by output per employed worker. It is then national debt per employed worker controlling for productivity changes. Column 6 shows the saving rate measured in the National Income and Product Accounts by personal savings as a percentage of personal income. There are alternative ways of measuring

these variables, but the major patterns are unlikely to be affected.<sup>2</sup>

The data show substantial changes in factor prices over long periods of time, with no indication that they are returning to an earlier equilibrium level. The real interest rate lies between two and three percent in the first half of the 1960's, falls below one percent (and often goes negative) from 1973 to 1980, rises to between five to eight percent between 1982 and 1987, and falls back to a range of three to five percent from 1988 on. As the real interest rate falls, the real wage rises from below seven in 1961 to levels above eight from 1970 to 1979. Then as the real interest rate rises from below one percent to above 3 percent for a 17 year period, the real wage falls to a level below 8.<sup>3</sup> The real wage decline relative to productivity cannot be explained by increases in benefits from 1980 to 1999, the period for which data are available from the U.S. Bureau of Labor Statistics. In that period, total compensation for total private employment (including wages, salaries and benefits) divided by multifactor productivity declined by 18 percent.

These changes are not an outcome of business cycle models describing fluctuations around a long run equilibrium. The long run changes in the real interest rate and wage rate could potentially be explained by the episode of inflation in the 1960's and 1970's but only by abandoning views that the

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<sup>2</sup>See Alan Kreuger, 1999, for discussion of measurement problems.

<sup>3</sup>R. Blundell and T. MacCurdy (1999, pp. 1583-1586) analyze real wage rates over time by educational level and gender. In the U.S., real wages declined for men but not for women over long periods. Real wages did not decline in the United Kingdom or Germany over the same periods. Lawrence F. Katz and David H. Autor (1999, p. 1476) also show real wage declines from 1971 to 1995 based on Current Population Survey data.

effects of inflation on factor prices end less than a decade after stabilization of monetary growth. Productivity also does not explain the long run and substantial changes. Increases in productivity should raise both the real interest rate and the real wage. However, the real wage declines relative to multifactor productivity, as shown in the ratio in Column 4, from 1978 on. Use of output per hour instead of multifactor productivity would result in even steeper declines in the ratio of wages to productivity. Imperfections in the measurement of productivity cannot be the explanation for the observed long run behavior of factor prices since real interest rates and real wages move in opposite directions. Other explanations are possible (e.g., capital-skill complementarity) and cannot be ruled out by these data.

This paper proposes an explanation based on the link between the macroeconomic sector and microeconomic determination of factor prices. The stylized facts concerning long run relationships to be addressed by the model developed here are as follows. Over the period 1961 to 1999, the real interest rate declines, increases substantially and then declines to a level greater than at the start of the period. The real wage relative to productivity is roughly inversely related to the real interest rate, rising and then declining. These patterns occur contemporaneously with a decline in the ratio of national debt to output from 1961 to the 1970's, followed by increases. While the short run effect of budget deficits on the interest rate are well known, this paper focuses on the ratio of national debt to output in balanced growth and its effects on both the real interest rate and the real wage rate. In the model that

will be developed, balanced growth of a greater national debt absorbs more savings, raising the interest rate at each capital to labor ratio. In balanced growth equilibrium, greater national debt per worker then yields a higher real interest rate and a lower real wage rate.

The model developed here is referred to as the neoclassical Kaldor model to distinguish it from the original Kaldor model with unequal saving rates. The economy is essentially Non-Ricardian since new entrants do not necessarily have the same asset levels as individuals who entered earlier. There are two forms of income: labor income from wages (plus transfers from the government) and interest from ownership of either capital or national debt. Microeconomic and macroeconomic policies are incorporated in the form of tax rates on labor and interest income, a tax rate on output (a value added tax), and the ratio of national (government) debt to employment. For simplification, only a closed economy is considered, and there is no inflation or technological change. The population grows at a constant rate.

Balanced growth equilibrium will be determined from microeconomic and macroeconomic equilibrium conditions relating the (real) interest rate to the capital to labor ratio. Section 2 derives in a straightforward way the condition for microeconomic equilibrium. Section 3 develops conditions for macroeconomic equilibrium. The saving behavior of households is a standard solution to an optimal control problem. From the government balanced growth budget constraint and the expression for taxes, it is possible to solve for the level of transfers at which the government budget constraint is satisfied. The con-

dition for macroeconomic equilibrium can then be determined from setting Aggregate Demand equal to Aggregate Supply, after substituting the expressions for taxes and transfers. Section 4 describes balanced growth equilibrium, which arises when both microeconomic and macroeconomic equilibrium conditions are satisfied. Sections 5 and 6 derive the effects of macroeconomic and microeconomic policies, respectively. Section 7 considers alternative saving behavior that would eliminate effects of macroeconomic policies on factor prices. It also relates the results to current macroeconomic analysis. Section 8 considers whether the current model meets previous criticisms of Kaldor's model. It also describes extensions to the model.

## 2 Microeconomic Equilibrium

The microeconomic equilibrium condition arises from a neoclassical production function relating output to amounts of capital and labor. Let  $f[K, L]$  be the rate of output generated using  $K$  units of capital and  $L$  units of labor. Assume  $f[K, L]$  is homogeneous of degree one, has continuous first and second partial derivatives, and has a diminishing marginal rate of technical substitution in the substitution region. Let  $f_L = \partial f / \partial L$  and  $f_K = \partial f / \partial K$  be the marginal products. Assume the price of output is normalized to be 1. Let  $t_p$  be the tax rate on output (equivalent to a value added tax), the same for all firms and all levels of output. Let  $r$  and  $w$  be the real interest and wage rates, respectively, and assume  $r$  and  $w$  are taken as given by firms.



An individual firm's rate of profits,  $\pi$ , is

$$\pi = (1 - t_p)f[K, L] - rK - wL \quad (1)$$

The firm's first order conditions for profit maximization are

$$r = (1 - t_p)f_K \quad (2)$$

$$w = (1 - t_p)f_L \quad (3)$$

The microeconomic equilibrium condition arises directly from the firm's first order conditions assuming all firms have the same production function. Since  $f$  is homogeneous of degree one,  $f_K$  and  $f_L$  are homogeneous of degree zero, i.e. they are functions of  $K/L$ . Let  $\kappa[r]$  be the capital to labor ratio that solves the first order condition 2. The microeconomic equilibrium condition is then:

$$K/L = \kappa[r] \quad (4)$$

Since the marginal product of capital is a declining function of  $K/L$ , the capital to labor ratio is a decreasing function of the interest rate in the microeconomic equilibrium condition. Given  $\kappa[r]$ , the wage  $w$  can be determined from the first order condition 3.

Individuals participate in the labor force when the after tax wage rate exceeds the opportunity cost of their time. Let  $t_w$  be the tax rate on labor income, the same for all individuals and all levels of income. Let  $\lambda[(1 - t_w)w]$

be the proportion of individuals working when the after tax wage rate is  $(1 - t_w)w$ . Suppose the population grows at a constant rate  $\rho$  and let  $N_0 e^{\rho t}$  be the population size at time  $t$ . The labor supply at time  $t$  is then

$$\lambda[(1 - t_w)w]N_0 e^{\rho t} \tag{5}$$

At a given wage rate, the labor supply grows at the rate  $\rho$ .

The sequence of determination in the microeconomic sector is that the interest rate determines the capital to labor ratio through the microeconomic condition  $\kappa[r]$ . Then the capital to labor ratio determines the wage rate from the first order condition for labor, and the wage rate determines labor supply through  $\lambda[(1 - t_w)w]N_0 e^{\rho t}$ .

### 3 Macroeconomic Equilibrium

#### 3.1 Assumptions

The macroeconomic equilibrium condition is determined in two steps. Optimal intertemporal behavior determines the relation between the saving rate and the real interest rate, and Aggregate Demand equal to Aggregate Supply determines the relation between the saving rate and the capital to labor ratio. There are two types of income in the economy. Labor income consists of labor earnings plus government transfers,  $wL + R$ , where  $R$  is government transfers to individuals. Interest income consists of interest on capital and

on national debt,  $r(K + D)$ , where  $D$  is national (government) debt.

The government taxes labor and interest income at potentially different tax rates, pays interest at rate  $r$  on the national debt, expands the national debt at the balanced growth rate  $\rho$ , and distributes the residual in the form of transfers to individuals in the economy. Since only balanced growth equilibria are compared, deficits or surpluses beyond balanced growth expansion cannot occur. Total taxes in the economy are given by

$$T = t_w(wL + R) + t_r r(K + D) + t_p f[K, L] \quad (6)$$

The government's budget constraint can be expressed as

$$T + \rho D = rD + R \quad (7)$$

where the left side is government sources of funds and the right side is uses of funds. In balanced growth,  $T$ ,  $D$  and  $R$  increase at the rate  $\rho$ , so the budget constraint remains satisfied at all times. Substituting  $T$  from 6 into 7 and solving for  $R$  yields

$$R = \frac{1}{1 - t_w} (t_w wL + t_r r(K + D) + t_p f[K, L] + (\rho - r)D) \quad (8)$$

When this expression is used for  $R$  in Aggregate Demand, the government budget constraint is automatically satisfied.

Using  $r$  to discount future amounts, the present discounted value of na-

tional debt at time  $\tau$  in the future for a balanced growth equilibrium is  $D_0 e^{\rho\tau} e^{-r\tau}$ , where  $D_0$  is current government debt. If  $r > \rho$ , this present discounted value approaches zero, satisfying the no-Ponzi-game condition (Olivier J. Blanchard and Stanley Fischer, 1989, p. 127). Even if  $r < \rho$ , the present discounted value of the ratio of national debt to employment approaches zero.

### 3.2 The Saving Rate

In this section, intertemporal optimization generates a relation between savings and the interest rate. Although the neoclassical Kaldor model can be worked out for an arbitrary savings function, a relation consistent with intertemporal optimization will demonstrate that the results are not generated by departures from optimizing behavior.

Consider a household receiving after tax income from labor earnings and government transfers at the rate  $y_0$  and interest income  $\hat{r}A[t]$  at each point in time, where  $A[t]$  is the household's assets of capital and government debt at time  $t$  and  $\hat{r}$  is the after tax return on assets. Suppose the household has time-separable instantaneous utility of consumption given by the logarithm of consumption and has a discount rate  $\beta$ . Then the solution to the individual's infinite horizon continuous time optimal control problem is to set consumption at each point in time equal to  $\beta/\hat{r}$  times income. The saving rate is then

$$s[\hat{r}] = 1 - \frac{\beta}{\hat{r}} \tag{9}$$

The saving rate does not depend on labor income  $y_0$  or on the initial level of assets. Assuming all households have the same discount rate and after tax return on assets, the saving rate will be the same for all households, unlike Kaldor's original model. Defining wealth to include the asset value of the future income stream, the growth rate of the household's wealth is  $\hat{r} - \beta$ .

### 3.3 Conditions for Macroeconomic Equilibrium

Aggregate income  $Y$  is the sum of labor and interest income:

$$Y = wL + R + r(K + D) \quad (10)$$

Aggregate Demand,  $AD$ , is then the proportion of after-tax income that is not saved,  $(1 - s[\hat{r}])(Y - T)$ , plus investment needed for balanced growth,  $\rho K$ :

$$AD = (1 - s[\hat{r}])(Y - T) + \rho K \quad (11)$$

Substituting  $Y$  from 10,  $T$  from 6, and  $R$  from 8 yields:

$$AD = (1 - s[\hat{r}]((1 - t_p)f[K, L] + \rho D) + \rho K \quad (12)$$

Macroeconomic equilibrium occurs when Aggregate Supply minus Aggre-

gate Demand is zero:

$$\begin{aligned}
AS - AD &= 0 \\
&= f[K, L] - ((1 - s[\hat{r}])(1 - t_p)f[K, L] + \rho D) + \rho K \quad (13) \\
&= s[\hat{r}]((1 - t_p)f[K, L]) + t_p f[K, L] - (1 - s[\hat{r}])\rho D - \rho K \quad (14)
\end{aligned}$$

The notable feature of this expression for  $AS - AD$  is that tax variables  $t_w$  and  $t_r$  do not directly appear. This occurs because transfers  $R$  are the residual of government revenues and the saving rate  $s[\hat{r}]$  is the same for all income types. The tax variables then redistribute production among income types, all of which have the same saving rate, without affecting the difference between aggregate supply and aggregate demand.

Setting  $\hat{r} = (1 - t_r)r$ , substituting  $s[\hat{r}]$  from 9 into 14, dividing by  $L$  and rearranging yields the macroeconomic equilibrium condition between  $r$  and  $K/L$ :

$$r = \frac{\beta}{1 - t_r} \frac{(1 - t_p)f[K/L, 1] + \rho D/L}{f[K/L, 1] - \rho K/L} \quad (15)$$

If  $\rho > f_K$  or if  $D$  is sufficiently small, the numerator increases by a smaller proportion than the denominator when  $K/L$  goes up. Then  $r$  is a declining function of  $K/L$  in the macroeconomic equilibrium condition.

## 4 Balanced Growth Equilibrium

Balanced growth equilibrium holds when both the microeconomic and macroeconomic equilibrium conditions in 4 and 15 hold. Figure 1 shows the two conditions and balanced growth equilibrium assuming specific functional forms and parameter values. The production function is assumed to be Cobb-Douglas with the exponent of labor equal to  $2/3$ . The balanced growth rate  $\rho$  is calculated as the rate of growth of the U.S. labor force from 1961 to 1999, .01764, using data from the 2002 Economic Report of the President. The real interest rate is  $r = .027$ , calculated as the interest rate on long-term U.S. government securities minus the inflation rate measured using the Consumer Price Index for urban workers. Using 1999 data from the National Income and Product Accounts (NIPA),  $t_w$  and  $t_r$  are both calculated to equal personal tax and nontax receipts plus social security contributions divided by personal income. This yields  $t_w = t_r = .2329$ . The tax on output,  $t_p$ , is calculated as indirect business taxes divided by personal consumption expenditures, so  $t_p = .1146$ . For the period 1961 to 1999, the average saving rate from NIPA was .082575. Setting  $.082575 = 1 - \beta/(r(1 - t_r))$ ,  $\beta$  can be calculated as .01900. The capital to labor ratio at which the first order condition for capital holds is 36.14. The ratio of national debt to capital in 1999, from the 2002 Economic Report of the President, is .36279. The ratio of national debt to labor,  $D/L$ , is then calculated as 13.11.

Although dynamics will be deferred to a later paper, the mechanism that

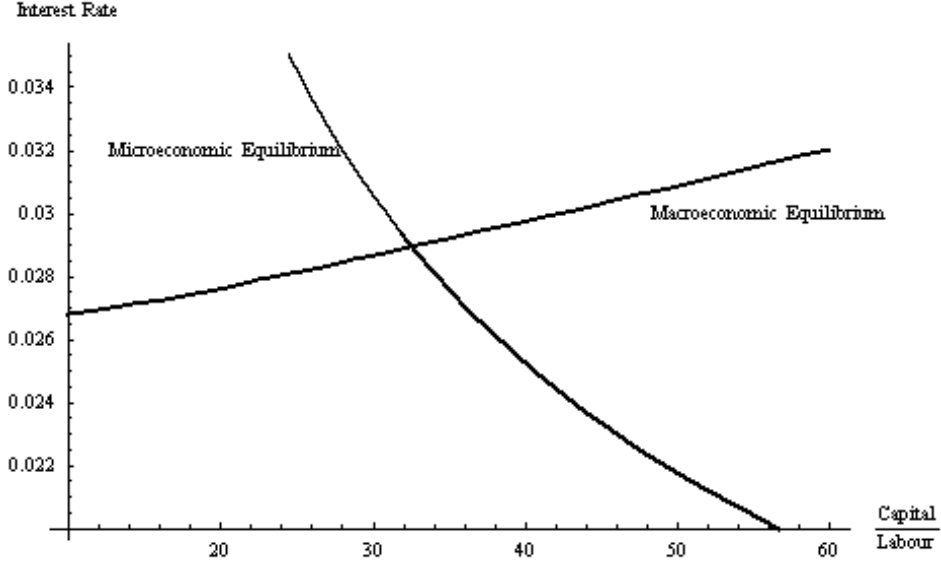


Figure 1: Balanced Growth Equilibrium

brings the economy to the balanced growth equilibrium can be briefly described. The condition that Aggregate Demand equal Aggregate Supply can be reexpressed as stating that national savings equals investment. Adjustment of the interest rate occurs through a loanable funds mechanism that equates national savings and investment. National saving is given by personal saving plus government saving, which reduces to:

$$s[r(1 - t_r)]((1 - t_p)f[K, L] + \rho D) + t_p f[K, L] - \rho D \quad (16)$$

Investment is given by  $\rho K$ . Figure 2 shows national savings and investment per worker assuming  $K/L$  adjusts according to the microeconomic equilibrium condition. At  $r_2$ , national savings exceeds investment, so the interest



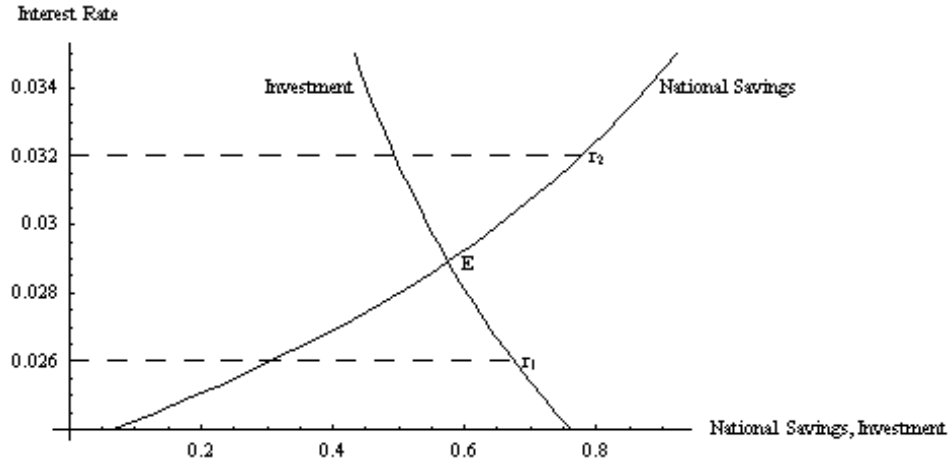


Figure 2: Interest Rate Adjustment

rate declines towards the balanced growth equilibrium at  $E$ . At  $r_1$ , investment exceeds national savings, raising the interest rate towards  $E$ .

A combination of  $r$  and  $K/L$  satisfying both the microeconomic and macroeconomic equilibrium conditions is unique since the microeconomic relation is downward sloping and the macroeconomic relation is upward sloping. It is routine to work out comparative static effects of parameter changes through their effects on the microeconomic or macroeconomic relations. Of particular interest is the result that a higher growth rate  $\rho$  shifts the macroeconomic equilibrium condition up, raising  $r$  and reducing  $K/L$ .

## 5 Macroeconomic Policy

### 5.1 National Debt

Now consider the effects of the ratio of national debt to employment,  $D/L$ , on wage and interest rates when the saving rate is determined by  $s[\hat{r}]$  in 9 and the tax on interest income is held fixed. From 15, the macroeconomic equilibrium curve is higher when  $D/L$  is higher. Then the balanced growth equilibrium has a higher interest rate and a lower capital to labor ratio. Alternatively, national savings are lower when  $D/L$  is higher, so that the intersection in Figure 2 occurs at a higher interest rate. The lower capital to labor ratio yields a lower wage from the first order condition for firm employment in 3.

### 5.2 Tax on Interest Income

A positive tax on interest income,  $t_r$ , reduces the net interest rate available to savers. A higher tax on interest income therefore reduces the saving rate at each interest rate. In Figure 1, the macroeconomic equilibrium curve is higher when  $t_r$  is higher. Then at the new balanced growth equilibrium, the capital to labor ratio is lower, the real interest rate is higher, and the wage rate is lower.

These results are summarized in the following theorem.

**Theorem 1** *In the neoclassical Kaldor model, comparing alternative bal-*

*anced growth equilibria, a higher ratio of national debt to employment,  $D/L$ , or a higher tax on interest income,  $t_r$ , yields a higher interest rate, a lower capital to labor ratio, and a lower wage rate.*

## **6 Microeconomic Policy**

### **6.1 Tax on Labor Income**

The tax rate on labor income,  $t_w$ , does not enter either the microeconomic or macroeconomic equilibrium condition. Everything else the same (including  $D/L$ ), the balanced growth equilibrium values of  $r$ ,  $w$  and  $K/L$  will remain the same at higher values of  $t_w$ . However, when  $t_w$  is higher, labor supply  $L$  will be lower from 5. Then capital, production and national debt will be proportionately lower. If instead  $D$  is held fixed in comparing balanced growth equilibria, an increase in  $t_w$  reduces  $L$ , which then raises  $D/L$ . The increase in  $D/L$  would then affect the interest rate, capital to labor ratio and wage rate as indicated in Theorem 1.

### **6.2 Tax on Output**

An increase in  $t_p$  affects both the microeconomic and macroeconomic equilibrium conditions. In the microeconomic equilibrium condition in 4, an increase in  $t_p$  reduces the interest rate in proportion to  $1 - t_p$  at each capital to labor ratio. In the macroeconomic equilibrium condition in 15, an increase

in  $t_p$  reduces  $r$  less than in proportion to  $1 - t_p$  at each capital to labor ratio if  $D/L > 0$ . Thus the curve for macroeconomic equilibrium in Figure 1 shifts down more than the curve for microeconomic equilibrium. Everything else the same, the balanced growth equilibrium for a higher  $t_p$  will have a lower interest rate and a lower ratio of capital to labor. From 3, the wage will be lower both because of the lower value of  $1 - t_p$  and because of the lower marginal product of labor,  $f_L$ , at a lower capital to labor ratio. At the lower wage, labor supply will be lower, with proportionately lower capital and production.

These results are summarized as follows.

**Theorem 2** *Comparing balanced growth equilibria, a higher tax on labor income leaves the interest rate, wage rate and capital to labor ratio unaffected, everything else the same (including  $D/L$ ), but reduces the labor supply. A higher tax on output reduces the interest rate, the wage rate and the capital to labor ratio.*

## 7 Can Macroeconomic Policies Affect Real Wages?

A household's rate of income  $y_0$  and net interest rate  $(1 - t_r)r$  are affected by the alternative macroeconomic and microeconomic policies considered in the last two sections. However, the optimization problem facing the household

remains the same, and the optimal saving policy continues to be given by 9. This section considers alternatives to the saving behavior in Section 3.2 that may rule out the connection between macroeconomic policies and factor prices.

## 7.1 Ricardian Equivalence

Two major categories of micro-based macroeconomic models are Ramsey models and overlapping generation models (Blanchard and Fischer, 1989). Ramsey models typically have infinitely lived agents whereas agents are born and die in overlapping generations models. Ramsey models generally exhibit Ricardian equivalence, so that private agent responses offset government financing changes (B. Douglas Bernheim, 1987; Blanchard and Fischer, 1989, p. 114). Although individuals in the Kaldor matching model live forever and face infinite horizon optimization problems, the model falls in the category of overlapping generations because of the entry of new individuals under balanced growth, as in Phillippe Weil (1987). Agents in the economy then differ by when they enter the labor market.

For example, compare two balanced growth equilibria with different levels of national debt to labor,  $D/L$ . Through adjustments in  $w$ ,  $r$ ,  $K/L$  and  $L$ , balanced growth equilibria are determined without any change in tax rates. That is, the government budget is brought into balance through changes in factor prices and incomes rather than through changes in tax rates. Since tax rates do not change over time, households do not change their saving behavior

in anticipation of higher tax rates in the future. With a higher  $D/L$ , the interest rate is higher and the wage rate is lower, redistributing income among types. At the same time, the change in  $D/L$  redistributes future income between individuals currently in the economy and future entrants. Future entrants will typically start out with fewer assets than current workers and entrepreneurs. Then the future income of future entrants is reduced while the future income of current individuals goes up from the higher interest income and higher growth rate of assets.

Conditions under which macroeconomic policies affect real wages appear to require the same demographics as overlapping generation models. This point is supported by considering what happens when  $\rho = 0$ . Then  $D/L$  does not enter  $AS - AD$  in 14, national debt cannot affect wages, and the neoclassical Kaldor model falls under the Ramsey category since the same agents are in the economy at all times.

With  $\rho > 0$ , Ricardian equivalence does not automatically hold, so that macroeconomic policies can affect the real wage rate when saving behavior is given by  $s[\hat{r}]$  in 9. The rest of this section considers alternatives to 9 that may insulate real wage rates from macroeconomic policies even in the absence of “Ricardian demographics.”

## 7.2 Savings Determined by Bequests

In overlapping generations models, bequests may nullify effects of fiscal policy that would benefit one generation at the expense of another (R. Barro,

1974; B. Douglas Bernheim, Andrei Schleifer and Lawrence H. Summers, 1985). Bequest motives can also insulate factor prices from the effects of macroeconomic policy in the neoclassical Kaldor model.

Assuming growth in the population and labor force arises from the progeny of the current population, a bequest motive could lead wealth owners to transfer wealth to progeny. If wealth owners transfer amounts that leave new individuals with the same amounts as the owners, wealth would be transferred at the rate  $\rho(D + K)$ . Suppose this bequest motive is the only reason for saving and that the ratio of wealth to income is the same for all individuals in the population. Then the saving rate would be

$$s_b = \frac{\rho(D + K)}{(1 - t_p)f[K, L] + \rho D} \quad (17)$$

National savings would be

$$s_b((1 - t_p)f[K, L] + \rho D) - \rho D = \rho(D + K) - \rho D = \rho K \quad (18)$$

so national savings would equal investment for all values of  $D/L$ . Real wage and interest rates would be determined by microeconomic equilibrium, unaffected by  $D/L$  or  $t_r$ . The tax on interest income would fall completely on interest recipients.

### 7.3 Bequests Combined with Optimal Savings

The saving rate considered in 17 requires that new members of the population receive wealth equal to current members, that wealth be distributed in exact proportion to income, and that the saving rate depend only on the ratio of bequests to income and not on the interest rate. A less restrictive alternative is to suppose that savings arise from optimal saving out of after-tax income net of bequests. This assumption does not require that wealth be proportional to income for all individuals. With bequests equal to  $\rho(D + K)$ , national savings would be

$$\begin{aligned} & s[(1 - t_r)r](Y - T - \text{Bequests}) + \text{Bequests} - \rho D \\ = & s[(1 - t_r)r]((1 - t_p)f[K, L] + \rho D) + \rho K \end{aligned} \quad (19)$$

Then  $s[(1 - t_r)r]$  must equal zero for national savings to equal investment. A zero saving rate occurs when  $(1 - t_r)r = \beta$ . With this savings behavior, the real wage and interest rates are unaffected by  $D/L$ . The after-tax interest rate would always equal the discount rate  $\beta$ , so both the real wage and interest rates would still depend on  $t_r$ .

### 7.4 Policy Rule

A second mechanism that would insulate factor prices is a policy rule to adjust the tax rate on interest income,  $t_r$ , in response to changes in the ratio of national debt to labor,  $D/L$ . Rearrange 14 to yield the saving rate at



which  $AD$  equals  $AS$  :

$$s_{pr} = \frac{-t_p f[K, L] + \rho(D + K)}{(1 - t_p) f[K, L] + \rho D} \quad (20)$$

Setting the saving rate in 20 equal to the right hand side of 9 and solving for  $t_r$  yields

$$t_r = 1 - \frac{\beta}{r} \frac{(1 - t_p) f[K, L] + \rho D}{f[K, L] - \rho K} \quad (21)$$

If the tax rate on interest income is set according to the rule in 21 (holding  $r$  fixed), then a change in  $D/L$  yields no change in the interest rate at each ratio of capital to labor, so that the macroeconomic equilibrium curve does not shift. An increase in  $D/L$  requires a reduction in  $t_r$  so that the saving rate will increase, returning equality between Aggregate Demand and Aggregate Supply.

## 7.5 Constant Saving Rate

In Kaldor's original model, the saving rates for different types of income were unequal but constant. If the saving rate does not depend on the interest rate, macroeconomic equilibrium would determine the ratio of capital to labor, and the microeconomic conditions then would determine the real wage and real interest rate. The macroeconomic equilibrium curve would be a vertical line and would shift left when national debt increased, raising  $r$  and reducing  $K/L$ . Macroeconomic policies would then continue to affect factor prices.

In summary, wage and interest rates will be unaffected by macroeconomic

policy (either national debt or the tax rate on interest income) if the saving rate is determined entirely by the bequest motive or if the government follows the policy rule in 21, setting a lower tax on interest income if national debt is higher. If savings were determined by the optimal saving rate out of after-tax income net of bequests, the real wage would be unaffected by the ratio of national debt to employment but would continue to be affected by the tax rate on interest income.

## 8 Conclusions

In the area of income distribution, the major conclusion of this paper is that the real wage and interest rate are not uniquely determined by microeconomic competitive conditions and instead can be affected by macroeconomic policies. The effect of national debt on the real wage is fairly general, as long as the saving rate is not determined by the bequest motive and the tax on interest income does not follow the insulating policy rule in 21. Comparing alternative balanced growth equilibria, a higher ratio of national debt to employment generates a higher real interest rate and a lower real wage rate. This result can explain the general patterns of the data in Table 1. When the national debt to output ratio is relatively high (in the beginning and last parts of the period), the real interest rate is higher and the real wage rate is lower than in the middle period, when the debt to employment ratio is relatively low. Empirical testing of the effect of national debt will

require disentangling the balanced growth relationships from short run open economy macroeconomic adjustments and shifts in savings behavior.

James Tobin (1989, p. 38) expressed three reservations concerning the original Kaldor model. The first concerned whether factor prices could be determined independently of their productivities. In the neoclassical Kaldor model developed here, factor prices are not independent of their productivities, but also are not uniquely determined by them. For a given capital to labor ratio, competitive behavior of workers and entrepreneurs determine factor prices consistent with neoclassical optimizing behavior. Tobin's second reservation was that the consumption function could not explain both income shares and level of output. In the neoclassical Kaldor model, derived demands for factors are combined with the consumption function (expressed in terms of the saving rate  $s[\widehat{r}]$ ) to determine income shares and level of output in balanced growth equilibrium. While the consumption function plays a role, it is not being overburdened. Tobin's third reservation was that investment was wholly exogenous in Kaldor's original model. In the neoclassical Kaldor model, both national savings and investment are endogenous. Investment can vary depending on the ratio of capital to labor. National savings can vary depending on the amount of savings absorbed by balanced growth expansion of the national debt.

This paper has only explored balanced growth links between macroeconomics and factor prices. The neoclassical Kaldor model provides a framework to analyze general equilibrium consequences of public finance policies.

Consequences of changes in tax rates on labor or interest income can be considered in a second-best context with a constrained level of government transfers, endogenous output and employment, and both microeconomic and macroeconomic adjustment. Another extension would provide an alternative to first-best Real Business Cycle models by developing the short run responses to shocks incorporating both microeconomic and macroeconomic adjustment in an economy with distortions from public finance variables.

Previous analyses of Kaldor's original model focused on differential saving rates, growth factors, and the absence of marginal products as the source of factor price effects. This paper instead emphasizes the condition imposed on competitive factor price determination by macroeconomic equilibrium.

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Table 1: U.S. Factor Prices and Debt

1	2	3	4	5	6
Year	Real r	Real w	Wage/Productivity	Debt/Output	Saving
1961	2.90	6.88	9.99	44.8	8.3
1962	2.95	7.07	9.87	43.6	8.3
1963	2.70	7.17	9.74	42.4	7.8
1964	2.85	7.33	9.68	40.1	8.8
1965	2.61	7.52	9.68	37.9	8.6
1966	1.75	7.62	9.77	35.0	8.3
1967	1.75	7.72	9.96	32.8	9.4
1968	1.06	7.89	9.87	33.3	8.4
1969	0.62	7.98	9.91	29.3	7.8
1970	0.88	8.03	10.14	28.0	9.4
1971	1.34	8.21	10.09	28.0	10.0
1972	2.43	8.53	10.11	27.3	8.9
1973	0.10	8.55	9.95	26.1	10.5
1974	-4.02	8.28	10.18	23.8	10.7
1975	-2.12	8.12	10.29	25.3	10.6
1976	0.98	8.24	10.09	27.5	9.4
1977	0.56	8.36	10.08	27.9	8.7
1978	0.29	8.40	10.05	27.4	9.0
1979	-2.56	8.17	9.88	25.6	9.2
1980	-2.69	7.78	9.57	26.1	10.2

Table 1 Continued: U.S. Factor Prices and Debt

1	2	3	4	5	6
Year	Real r	Real w	Wage/Productivity	Debt/Output	Saving
1981	2.59	7.69	9.39	25.8	10.8
1982	6.03	7.68	9.22	28.6	10.9
1983	7.64	7.79	9.14	33.1	8.8
1984	7.69	7.80	8.88	34.0	10.6
1985	7.15	7.77	8.71	36.4	9.2
1986	6.24	7.81	8.61	39.5	8.2
1987	5.03	7.73	8.27	40.7	7.3
1988	4.88	7.69	8.08	40.9	7.8
1989	3.79	7.64	8.18	40.5	7.5
1990	3.33	7.52	8.06	42.0	7.8
1991	3.96	7.45	8.06	45.3	8.3
1992	4.52	7.41	7.88	48.2	8.7
1993	3.46	7.39	7.79	49.5	7.1
1994	4.81	7.40	7.61	49.4	6.1
1995	4.14	7.39	7.45	49.2	5.6
1996	3.80	7.43	7.43	48.5	4.8
1997	4.37	7.55	7.29	46.1	4.2
1998	4.09	7.75	7.29	42.9	4.7
1999	3.93	7.86	7.18	39.8	2.4