Optimal Unemployment Insurance, Human Capital Depreciation, and Duration Dependence.

Nicola Pavoni*

April 2001

Abstract

This paper studies the effect of human capital depreciation and duration dependence on the design of an optimal unemployment insurance scheme. The problem is modeled as a dynamic principal-agent relationship, where the planner's inability to observe worker's job-search effort creates moral hazard. We assume both the gross wages and probabilities of reemployment to be non-increasing functions of the worker's unemployment duration. A *new approach* is developed, to study recursively the dynamic moral-hazard problem. We show that the associated value function is both non-concave and non-differentiable. In spite of that, we prove that the optimal scheme can be characterized using the usual first order conditions.

Our results partially confirm the ones obtained in most previous studies: benefits should decrease with unemployment duration. However, the introduction of human capital depreciation and duration dependence generates two main novel features in the optimal program. First, it creates the possibility of *endogenous lower bounds* on worker's expected discounted utility. Second, we use numerical simulations for the Spanish and US economies to study the optimality of imposing a history contingent wage tax after reemployment. In contrast with previous studies, we find the optimal level of *wage tax decreases* with the length of worker's previous unemployment spell, becoming a wage subsidy for long-term unemployed workers.

JEL Classification: C61, D63, D82, D83, J24, J31, J38, J64, J65.
Keywords: Unemployment Insurance, Human Capital Depreciation, Duration Dependence, Recursive Contracts, Moral Hazard.

*University College London, and Universidad Carlos III de Madrid. I would like to thank Antonio Cabrales, Per Krusell, Matthias Messner, Kjetil Storesletten and Fabrizio Zilibotti for many helpful comments. The paper benefits from a profitable discussion with Gianluca Violante and Philippe Aghion, during the preliminary stages of the work. I wish to thank conference and seminar participants at the Universitat Pompeu Fabra in Barcelona, the IIES in Stockholm, and at UCL. All remaining errors are mine. This research was supported in part by Marie Curie Fellowship MCFI-2000-00689. *Correspondence:* Nicola Pavoni, Department of Economics, University College London, Gower Street, London WC1E 6BT. *E-mail:* n.pavoni@ucl.ac.uk

1 Introduction

Unemployment insurance programs are an important ingredient of social welfare policies in developed economies. These programs have been widely criticized because of the adverse effects they can have on worker's incentives to search for a new job. This criticism has stimulated extensive research into optimal insurance schemes that take these perverse effects into account.

A series of papers use the dynamic moral hazard model to analyze the trade-off between (unemployment) insurance and (search) incentives. In their seminal work on unemployment insurance, Shavell and Weiss (1979) establish that, because of moral hazard, benefits must decrease throughout the unemployment spell, and approach zero in the limit. In two influential papers, Hopenhayn and Nicolini (1997a,b), extend the analysis of Shavell and Weiss increasing the number of policy instruments available to the government. Together with the sequence of benefits paid to the unemployed workers they introduce the possibility of contingent wage taxes after reemployment. They confirm the decreasing benefits result of Shavell and Weiss. Moreover, the analysis of Hopenhayn and Nicolini suggests that present schemes are considerably suboptimal.[1] Pavoni (2001) shows that most of the discrepancies between the optimal schemes proposed by Hopenhayn and Nicolini and the ones implemented throughout most developed countries arise from the assumption in the latter that the planner can, and will, inflict infinite punishments on workers.[2] Thus, to design an implementable scheme, Pavoni considers the possibility that, the planner must respect a lower bound on the expected discounted utility that the agent can have ex-post regardless of the previous history. And he finds that the optimal contract is quite close to actual unemployment compensation schemes, both qualitatively and quantitatively.

In all these models, the key feature of the environment is that the probability of finding a new job (or a wage offer above the agent's reservation value) depends *only* on the (unobservable) search effort made by the agent. However, many OECD countries propose and apply active labor market policies and wage subsidies for long term unemployed people, mainly because job opportunities change during unemployment. We thus believe human capital depreciation and hazard-rate duration dependence are important elements, that need to be included in the study of an optimal unemployment insurance designing problem. This is the main goal of the present work. In particular, we allow for both the gross wage and the probability of reemployment to be non-increasing functions of the length of the worker's unemployment spell.

Clearly, although an important element, skill obsolescence is not the only reason for which wage and reemployment probabilities may decrease during unemployment.[3] However,

[1] For example, Hopenhayn and Nicolini (1997a) argue that by switching from the existing policy to the their optimal transfer scheme, the US government could save approximately 30% of the overall spending in unemployment compensations. Hopenhayn and Nicolini (1997b) find costs reduction of the same magnitude for Spain.

[2] In particular, in Pavoni (2001) I show that the optimal contract proposed by Hopenhayn and Nicolini implies a weaker form of what is known - after Thomas and Worral (1990) - as the "immiserization result": if the worker's utility function is unbounded below then efficiency requires that worker's expected discounted utility falls, with positive probability, below any arbitrary negative level.

[3] Below, in this section, we review the literature on job displacement and duration dependence.

the focus of the present paper is on the design of an optimal scheme. And, at a first stage, we do not study how this designing problem interacts with other labor market policies. This makes our analysis valid, independently from the particular reasons of the decreasing behavior of the gross wage and hazard rates. As a consequence, in this paper we call "human capital" an aggregate variable, which affects both the worker's (gross) wage and the reemployment probability, and whose nature should be considered more broadly than mere skill or ability. This formulation allows us to write the problem in a very simple recursive form. Moreover, the simplicity of our model allows us, at a later stage (in Section 6), to extend the policy instruments available to the government, and introduce, for example, the possibility of implementing retraining programs and job-search assistance activities.

To our knowledge, the only other attempt to study a similar problem is Usami (1983). He proposes a finite-horizon model with moral hazard, where the probability of reemployment conditional on search depends on the previous employment history. Usami confirms the mentioned decreasing benefits result, and finds that the worker compensation should be non-decreasing during the employment period. Unfortunately, he formulates the problem choosing an "inconvenient" state variable, which prevents a complete analysis. Our recursive formulation results in a manageable value function, which allow us to characterize in detail the optimal scheme.

Results Our contribution is, first of all, a theoretical one. We are able to study recursively, in a quite general framework, the dynamic (hidden-action) moral hazard problem. In particular, this paper develops a new approach that permits to show that the associated value function is, in general, non-concave and non-differentiable. In spite of these (non-smoothness) problems, we find that the optimal contract can be characterized using the usual first order conditions.

From a *qualitative* point of view, we partially confirm one important result of most previous studies: benefits should decrease with unemployment duration. In fact, this behavior characterizes most existing schemes in OECD countries.[4] Moreover, we find that very simple schemes, defined by a low constant benefit payment b and a higher (again time-invariant) wage w, are suboptimal. This hold for any reasonable range of parameters, regardless of the characteristics of the human capital depreciation process. Moreover, we propose a simple necessary characteristic of any optimal unemployment insurance program, which we call it *the triangle rule*.

From a more *quantitative* point of view, the results of our analysis can be summarized as follows. First, provided that human capital depreciates sufficiently rapidly during unemployment, the optimal path for unemployment benefit payments is initially decreasing an then becomes completely flat. The idea is that, for low levels of human capital, the planner looses the incentives to induce the agent to supply the high effort level. Unemployment benefits eventually stop decreasing, and remain constant forever, since the long-term unemployed worker is fully insured. This generates an endogenous lower bound on worker's expected discounted utility, which provides an alternative way of eliminating the *immiserization result* of Thomas and Worral (1990).[5] Our results, especially the endogenous derivation of bounds

[4]See Figure 1 and Table 1.

[5]Usually, in the literature, the immiserization result is eliminated by *exogenously* imposing minimum

on worker's expected discounted utility, create an important link between the characteristics of the optimal unemployment insurance scheme and the speed of skills depreciation in the economy, which can also be used for positive analysis.

Second, in our model the planner can impose contingent wage taxes after reemployment. The results of our numerical exercises for the Spanish and US economies show that in an optimal scheme, the level of *wages taxes should decrease* in the length of worker's previous unemployment spell, and should become a *wage subsidy* for long-term unemployed workers. This contrasts with the increasing wage tax result of Hopenhayn and Nicolini (1997a). And is due to two main reasons. First, as human capital depreciates, the incentive costs increase. This implies that the difference between unemployment benefits and net wage must increase during unemployment. Second, as explained in Pavoni (2001), the (now endogenous) presence of a lower bound on worker's expected discounted utility reduces the effective time-horizon of the problem. This reduces the possibility of giving dynamic incentives, and forces the planner to design a scheme biased toward the "static" component of the incentives. That is, the planner has to increase the within-period gap between unemployment insurance benefits and net reemployment wages.

Our results have some specific policy implications, especially for long-term unemployed workers. However, a fuller analysis of these issues requires an increase in the number of labor market policies available to the planner. At the end of the paper, we present an extended (recursive) model which consider the possibility of adopting retraining programs and job-search assistance activities. Most of the above summarized results are robust to this extension. However, a deeper analysis on the interaction between the design of an optimal unemployment insurance scheme and the adoption of other active labor market policies needs further investigation.

Literature on recursive contracts The methodology we use is sometimes called recursive contracts and the references most related to our approach are: Abreu, Pearce and Stacchetti (1990), Fudenberg et al. (1990-1994), Spear and Srivastava (1988), Phelan and Townsend (1991) and Atkeson and Lucas (1992). In this paper, we formally study the shape of the value function associated with the dynamic moral hazard model. The approach we propose, partially builds on the one used by Grossman and Hart (1983), to study the static problem. In their seminal work, Spear and Srivastava (1988) discuss conditions under which the value function of the dynamic model with a continuum of outcome realizations, is concave. Our approach permits to study, formally, the case with finitely many output realizations, which is an inherent characteristic of the unemployment insurance designing problem. Our non-concavity result potentially contradicts the result of Spear and Srivastava. Finally, Phelan and Townsend (1991) allow for random payments, which imply the (weak) concavity of the value function. As in most applied studies, we focus on the deterministic payments case. Moreover, our non-concavity result seems to contrast with the analysis of Phelan and Townsend, who - through numerical simulations - find that lotteries are almost never used

bounds on expected discounted utility the worker can have ex-post, in the optimal scheme (see, for example, Pavoni, 2001; Atkeson and Lucas, 1995; Phelan, 1995).

in an optimal scheme.[6]

Literature on optimal unemployment insurance The literature on optimal unemployment insurance is relatively new, yet quite extended. However, most of the papers address questions and/or use approaches that cannot be directly related to our own. The interested reader can refer to the recent summary of Karni (1999). Atkeson and Lucas (1995) a recursive approach to characterize the optimal contract in a stationary pure adverse-selection setup with temporary (one-period) job offers. They are mainly interested in income distribution, and their approach is closely related to that in Hansen and Imrohoroglu (1992), where the goal is to quantify the welfare effects on unemployment insurance in general equilibrium. Wang and Williamson (1996, 1999) provide calibrated dynamic OLG models with moral hazard associated with search effort and job retention. Following Phelan (1994), they assume that each new labor-force entrant obtains a prespecified level of ex ante utility and obtain a non monotone unemployment benefits behavior.

The need for studying the general (possibly non-stationary) dynamic moral hazard model is due to the two main modeling novelties of this paper. Namely, wage depreciation and duration dependence.

Literature on job displacement and duration dependence A great deal of attention has been paid in recent years to the consequences of *worker displacement*. Displacement is usually defined as the involuntary separation of workers from their jobs without cause (i.e. for economic reasons) and without future recall. Research on the effects of worker displacement has grown dramatically in recent years, especially in the in the United States (see Hamermesh, 1989; Faber, 1993, 1997; Hall, 1995; Fallick, 1996 and Kletzer, 1998 for surveys). Using a variety of methods and datasets, the finding are remarkably consistent. First, displaced workers face a large and persistent earning losses upon reemployment on the order of 10-25% compared with continuously employed workers (see - for example - Ruhm, 1987 and Bartel and Borjas, 1981).[7] Second, in addition to earning losses, displaced workers experience more unemployment than nondisplaced workers (see - for example - Hall, 1995; Ruhm, 1991; Swain and Podgursky, 1991).

Many labor economists interpret these finding as reflecting the destruction of firm-specific or industry-specific human capital associated with tenure. However, some authors pointed out that non-observable individual heterogeneity may bias estimated tenures upwards, so that previous tenure might have a positive effect on post-displacement wage rates (see, for example Kletzer, 1989). Another interpretation is simply the destruction of rents associated

[6] The technical reader, may conjecture that the "bang-bang" solution result of Abreu, Pearce and Stacchetti (1986, 1990) could be used to partially reconcile our results with the one of Phelan and Townsend. However, we believe that a deeper analysis of this issue requires a separate work.

[7] Evidence for European labor markets is contrasting and not always comparable. For example, recently Burda and Mertens (2001) use self-reported information on job-displacement and estimate an average wage growth reduction of approximately 3.6%, with a peak of 17% (for high pay jobs) for Germany. In contrast, Bender et al. (1999) use information on plant closing (more precisely, discontinued firm identifiers) to identify displacement, and estimated near zero losses both in France and Germany. Leonard and van Audenrode (1995) find results similar to the ones of Bender et al. for Belgium, and so does Ackum (1991) for Sweden.

with good matches, with no return to tenure per se (Mincer and Jovanovic, 1981; Altonji and Shakotko, 1987; Abraham and Faber, 1987; Ruhm, 1990; Altonji and Williams, 1992).

One of the distinctive features of many current European labor markets is the high proportion of workers that remain unemployed for a long period of time. This feature of the European labor market is widely regarded as a serious problem and has attracted a lot of attention both for efficiency and equity reasons. The *negative duration dependence* in the exit rate from unemployment is felt as one of the major causes for long-term unemployment.[8]

In the literature, labor economists suggest different justifications for the observed negative duration dependence in the exit probability from unemployment. A first steam of research supports a purely statistical explanation for this phenomenon. It is argued that the hazard rate estimations are affected by an unobservable heterogeneity effect, which induces a strong negative duration dependence in the estimated hazard rates. The idea is that the negative duration dependence is partially due to the fact that the sample characteristics are changing through unemployment. In particular, the pool of long-term unemployed, is dominated by the share of people whose individual hazard rates are lower than the average.[9] However, first, even after controlling for unobservable heterogeneity, a non negligible negative duration dependence still remain (see, for example, Nickell, 1979; Lynch, 1985; Bover et al., 1997; and the summary of Machin and Manning, 1999). Second, this argument may be not so relevant for our representative agent analysis. If, on one hand, it is potentially interesting to study the unemployment insurance designing problem in presence of both moral-hazard and adverse selection, on the other hand - apart from its inherent complexity of analysis - some authors suggested that a pooling equilibria can be the only possible solution to this composite informational problem.[10] In this context, the solution of the program for a representative agent with decreasing hazard rate, may approximate quite well the (pooling) solution of the problem with time changing workers population composition.

A second steam of research attempts to give a theoretical explanation for the mentioned "true" negative duration of the hazard rate, and can be divided in two categories. On one hand, we have what we call *supply side* explanations. It is empirically documented that, during unemployment, there is a negative time dependence in the arrival rate of job-opportunities (what Heckman and Borjas (1980) call "occurrence dependence"). A long-term unemployed worker finds it more difficult to know the existence of vacancies, either because the worker loses valuable social contacts,[11] or because the long-term unemployed worker suffers some sort of stigmatization by the other workers in (the supply side of) the market (Gregg and Wadsworth, 1996). The stock-flow approach to search of Coles and Smith (1994) and Gregg and Petrolongo (1997) gives another supply side explanation for the negative duration dependence phenomenon. The idea is that the observed occurrence dependence is in fact only apparent. What really happens is that, at the beginning of unemployment the worker faces and evaluates a stock of vacancies while, in latter periods of unemployment,

[8] Recently, Machin and Manning (1999) cast some doubts about this view. More specifically, they found that the rise in the incidence of long-term unemployment in Europe was not caused by an increase in the negative duration dependence of exit rates from unemployment.

[9] See Heckman (1991) for a survey, or Lancaster (1990), Chapter 7.

[10] In fact, Chiu (1998) justifies the existence of a (non-separating) public unemployment insurance system precisely because of the presence of both moral hazard and adverse selection problems.

[11] See, for example, Calvó-Armengol (2000) for a theoretical analysis of this aspects of the labor market.

only the newly open vacancies are processed. Finally, a third motivation is that the hazard rate decreases during unemployment, simply because the worker reduces his search effort level. This may be due to a sort of discouraged worker effect (that may induce long term unemployed to remain in the labor market, but actually looking almost passively for new jobs),[12] or because skills and work habits athropy during unemployment (Sinfield, 1981).

We then have a few *demand side* - or firm-hiring behavior - explanations for negative duration dependence. Recently, a lot of attention has been devoted to study the so called "stigma effect". The idea behind this approach is the following. It is assumed that firms imperfectly test workers prior to hiring them. If (some) firms hire only workers who pass the test, then there is an informational externality; unemployment duration is a signal of workers' productivity and firms tend to avoid to even test for hiring long-term unemployed workers (Vishwanath, 1989; Lockwood, 1991; Benzil, 1995; and Yoshiaki, 1997). Note, the stigma effect is due to a payoff-relevant firm's behavior. This contrasts with the ranking idea of Blanchard and Diamond (1994), where firms hire the workers with the lowest unemployment duration levels, independently from any payoff-relevant motivation.

Outline of the paper In Section 2 we present our dynamic moral-hazard model with human capital depreciation. In Section 3, we present our approach to characterize the optimal contract, and derive the "triangle rule." To clarify the results of the technical analysis, in Section 4, we compute a new closed form solution, for the case where the worker has logarithmic utility. In Section 5, we calibrate the model with the US and Spanish economies, and simulate the optimal scheme. In Section 6 we extend our analysis allowing the planner to adopt alternative labor market policies, such as retraining programs and job-search assistance activities. Section 7 concludes.

2 Model

The model is the natural extension of the one proposed by Hopenhayn and Nicolini (1997a). Consider a risk-neutral planner who must design an optimal unemployment compensation scheme for a risk-averse worker. In any given period the worker has (time invariant) preferences of the following separable form

$$u(c) - C(a)$$

where c is consumption and a is search effort. We assume that $a \in A = \{\alpha_0, \alpha_1, .., \alpha_N\}$, with $\alpha_i > \alpha_{i+1}$ and $N < \infty$. That is, an ordered, finite number of search effort levels.[13] We assume the cost function $C(\cdot)$ to be strictly increasing, and we normalize the search costs by setting $\alpha_0 = 0$, with $C(0) = 0$. Moreover, we assume $u(\cdot)$ to be strictly increasing, strictly concave and continuously differentiable with inverse u^{-1} bounded. In any period the worker can be either employed or unemployed and we assume the search effort affects the transition

[12] Think, for example, of a worker who is learning about his job search effectiveness parameter. A long period of unemployment will obviously imply a downward biased perception of his hazard rate.

[13] Hopenhayn and Nicolini (1997a) use a continuous of actions formulation. Most of our results can be extended to the case with countably many actions.

probability between employment states, according to a hazard rate function P that will be defined below. If the worker is employed, then he earns a gross wage $S(H)$ which - instead of being a constant, as in the previous models - is assumed to be a bounded, increasing function of the worker's human capital endowment H. Moreover, we assume H follows a given stochastic law of motion

$$H' = m(h, H) \text{ with } m(0, H) \leq H \leq m(1, H), \quad m(h, \cdot) \text{ continuous}, \tag{1}$$

where H' is the next period human capital level and $h \in \{0, 1\}$ is the present worker's employment state. The idea is that during unemployment ($h = 0$) H depreciates whereas during employment ($h = 1$) there can be human capital accumulation due, for example, to on-the-job training. We assume that, while employed, the worker faces an exogenous probability F of being fired. As said, if the worker is unemployed, he can engage in costly job-search, and the higher the search-effort a, the higher his probability $P(a, H) \in [0, 1)$ of being employed in the next period.[14] We further assume that - for each search-effort level a - $P(a, H)$ is increasing in H. This *duration dependence* in the reemployment probability is the second main modelling novelty we introduce in this paper. The timing of the model, in the unemployment state, is reported in Figure 2.

The crucial assumption of the model is that the planner cannot observe the worker's search effort a (we assume there are no informational problems related with H).[15] Thus, during unemployment there is a moral-hazard problem. This means that unemployment benefits are not paid only as insurance, but must also play the role of giving incentives for search.

Following the recursive contracts literature, we can characterize the contract using the following recursive formulation.[16] Consider first the unemployment state case ($h(t) = 0$), and let U be the discounted utility promised to the agent at the beginning of the period t. Given a utility level U, a human capital endowment level H and present state $h = 0$, the problem can be stated in terms of three functions $[a(U, H), b(U, H), U'(0, U, h', H)]$ determining the current action $a(t) = a(U_t, H_t)$, the current net transfer $b_t = b(U_t, H_t)$, and a promised future utility $U_{t+1} = U'(0, U_t, h(t + 1), H_t)$, $h(t + 1) \in \{0, 1\}$, which is contingent on the search process outcome.

If we define $U^u(0, U, H) \equiv U'(0, U, 0, H)$ and $U^e(0, U, H) \equiv U'(0, U, 1, H)$, then the choice of the functions $[a(U, H),\ b(U, H),\ U^e(0, U, H),\ U^u(0, U, H)]$ must satisfy the following two sets of constraints:

$$U \geq u(b(U, H)) - C(\hat{a}) + \beta [(1 - P(\hat{a}, H)) U^u(0, U, H) + P(\hat{a}, H) U^e(0, U, H)] \ \forall \hat{a} \in A \tag{2}$$

$$U = u(b(U, H)) - C(a(U, H)) + \beta [(1 - P(a(U, H), H)) U^u(0, U, H) + P(a(U, H), H) U^e(0, U, H)]. \tag{3}$$

Constraint (2) is the *incentive compatibility constraint* ensuring the agent is willing to deliver the amount of effort called for in the contract. Equation (3) requires the contract to deliver

[14] Formally, for any H, we assume $P(\cdot, H)$ to be strictly increasing in a.

[15] Note that, it suffices to assume the planner knows the initial endowment H_0 and the law m, since the realized employment states are perfectly observed.

[16] See, for example, Pavoni (1999).

the promised level of discounted utility to the worker, and is called the *promise keeping constraint* and plays the role of law of motion for the state variable U; $\beta \in (0,1)$ is the discount factor. The corresponding promise keeping equation in the employment state is

$$U = u\left(w(U,H)\right) - l + \beta\left[FU^u(1,U,H) + (1-F)U^e(1,U,H)\right] \tag{4}$$

where $w(U,H)$ is the net wage the worker receives after tax ($\tau = S(H) - w$) is paid, and we assume a fixed effort cost of working equal to $l \geq 0$. The starting value U_0 will be given by the time-zero participation constraint, and may depend on the initial level of human capital endowment H_0.

Given two generic levels U and H, the planner's value function in the unemployment state V is defined as follows

$$\begin{aligned} V(U,H) &= \sup_{a,b,U^u,U^e} -b + \beta\left[P(a,H)W(U^e,H') + (1-P(a,H))V(U^u,H')\right] \\ sub &: \ (1),\ (2) \text{ and } (3). \end{aligned} \tag{5}$$

We can interpret (5) as a cost minimization problem, where the government's (expected) monetary payments are discounted at the real interest rate $\beta^{-1} - 1$.[17]

The problem in the employment state is much simpler, since there are no incentive problems. The associate planner's value function W is defined as follows

$$\begin{aligned} W(U,H) &= \sup_{w,U^u,U^e} S(H) - w + \beta\left[(1-F)W(U^e,H') + FV(U^u,H')\right] \\ sub &: \ (1) \text{ and } (4). \end{aligned} \tag{6}$$

Thus, the employment state program solves a simple insurance problem.

3 Qualitative Analysis

It is well known that, in general, the hidden-action moral-hazard problem is not a concave one (Grossman and Hart, 1983, and Phelan and Townsend, 1991). This, first might complicates the analysis since we are looking for the global maximum. Second, first-order conditions may not longer be sufficient even for a local maximum. Third, more importantly, the usual envelope theorems[18] cannot be applied, and this may reduce considerably the usefulness of our recursive formulation.

Our first theoretical result partially confirms these difficulties. We show that, "in most cases," the associated value function is neither concave nor differentiable. However, we have also a positive and perhaps quite surprising result: we will show that the optimal contract can still be characterized with the usual first order conditions. Moreover, we give a simple rule to check for optimality of an unemployment scheme, which we call it *the triangle rule.*

[17] Pavoni (2001) solves the problem where the planner faces a continuum of workers. In that case, the government's costs are deterministically determined.

[18] For envelope theorems we refer to theorems that describe conditions under which the value function of a parametrized optimization problem is a differentiable function of parameter.

The idea of our approach is as follows. The complication involved by the recursive study of the dynamic moral-hazard problem comes from the incentive constraint: indeed the constraints in (2), in general, do not describe a convex feasible set. This prevents a direct approach to the study of the concavity and differentiability of the value function V. We thus first reformulate the problem to make it suitable for this analysis. In particular, we define a collection of concave and continuously differentiable functions, of which the value function V is the upper envelope, we then apply the envelope theorem of Milgrom (1999) to this problem.

Our successive step is to study the "switching points," that is, the utility levels for which the implemented action changes in some (present or future) period t. Given the characteristics of our class of functions, all switching points - which are the only problematic ones - have in common a very nice characteristic: either the point is not an optimum, or the function is differentiable at this point. Although we find that the second case is not that "usual" (at least in the class of analytic functions), the fact that the points of non-differentiability cannot be maximal allow us to disregard them while characterizing the optimal contract. In synthesis, we show that the optimal contract can be characterized using the usual first-order conditions.

3.1 The sequence of actions formulation and the existence result

To simplify the analysis, we start by assuming $F = 0$. In this case, the employment state function is given by

$$W(U, H) = \frac{S_e(H) - u^{-1}((1-\beta)U + l)}{1-\beta}$$

where $S_e(H)$ is the average discounted gross wage, and it is given by the (deterministic) gross wage sequence induced by S and the law m, during employment. From the assumptions on u and S, W is bounded, strictly increasing, strictly concave and continuously differentiable in U.

Consider the space $\mathcal{A}$ of all the sequences of actions $\mathbf{a} = \{a(n)\}_{n=0}^{\infty}$ $a(n) \in A$, implementable during unemployment, and - for a $\delta \in (0,1)$ - define the $\delta-$metric on $\mathcal{A}$ as follows

$$d_\delta(\mathbf{a}, \mathbf{a}') = \sum_{n=0}^{\infty} \delta^n \left| a(n) - a'(n) \right|.$$

For any human capital endowment $H \in \mathcal{H}$, action sequence $\mathbf{a} \in \mathcal{A}$ and utility level $U \in \mathcal{U} \subset \mathrm{I\!R}$ we can now define the function(al)

$$\begin{aligned} V(\mathbf{a}, U, H) &= \sup_{b, U^e, U^u} -b + \beta \left[P(a, H) W(U^e, H') + (1 - P(a, H)) V({}_1\mathbf{a}, U^u, H') \right] \quad (7) \\ sub &: \ (1),\ (2) \text{ and } (3); \end{aligned}$$

which is the planner's optimal payoffs associate with a *given* sequence of actions, in case the worker is unemployed at the beginning of the period. The symbol ${}_1\mathbf{a} = \{a(n)\}_{n=1}^{\infty}$ stands for the one step ahead continuation of $\mathbf{a}$.

It would be easy to show that the value function of the sequence problem satisfies (7). In the next Theorem we show that also the converse is true.

Theorem 1 *The Bellman operator implied by (7) defines a contraction in the space of continuous and bounded functions with the sup norm. Thus V exists and is unique, and*

$$\|V\|_\infty = \sup_{y \in Y = \mathcal{A} \times \mathcal{U} \times \mathcal{H}} |V(y)| < \infty.$$

Proof. See Appendix. ■

In the next Proposition, we show that each $V(\mathbf{a}, U, H)$ has characteristics similar to the ones of the employment state value function W.

Proposition 1 *Given a generic sequence of actions $\mathbf{a} \in \mathcal{A}$ and an endowment level H, together with a law m, the associate function $V(\mathbf{a}, U, H)$ is decreasing, concave and continuously differentiable in U and*

$$V'(\mathbf{a}, U, H) \equiv \frac{\partial V(\mathbf{a}, U, H)}{\partial U} = -\frac{1}{u'(b)}$$

Proof. See Appendix. ■

The idea of the proof is quite simple. Making the change in variable $u(b) = z$, it become easier to see that the problem of implementing optimally (minimizing costs) a *given* sequence of actions $\mathbf{a}$ is concave - with linear constraints - thus the associate value function $V(\mathbf{a}, U, H)$ is concave too. Given concavity, we can apply the Benveniste and Scheinkman (1979) Lemma to show differentiability.

We finish this subsection with an important existence result.

Theorem 2 *The set $\mathcal{A}$ of sequences of actions is compact and - given U and H - $V(\mathbf{a}, U, H)$ is continuous in $\mathcal{A}$. Thus, a maximum exists, and we can define*

$$V(U, H) = \max_{\mathbf{a} \in \mathcal{A}} V(\mathbf{a}, U, H).$$

Proof. See Appendix. ■

The continuous of actions case It is clear from the proof that Theorem 2 can be extended to countably many actions. We also conjecture that the above existence result can be proven for the continuous of actions case. In particular, we conjecture that in the topology induced by the $\delta-$metric, both the value function $V(\mathbf{a}, U, H)$ is (at least) upper semicontinuous and the set of sequences $\mathcal{A}$ is compact, also when, for example, $a(t) \in [0, 1]$. However, our proof strategy does not goes through this case, so a rigorous analysis is left for future research.

3.2 The shape of the value function

In this section we study the characteristics of the value function $V(U, H)$. Theorem 2 defines $V(U, H)$ as an upper envelope of the collection of well defined functions $V(\mathbf{a}, U, H)$. Our approach uses precisely this interpretation, and is based on the envelope theorem of Milgrom (1999).

Definition 1 *For each* U *and* H, *define the non-empty set* $\mathcal{A}^*(U,H) = \arg\max_{\mathbf{a}\in\mathcal{A}} V(\mathbf{a},U,H)$, *moreover we call* $\mathcal{A}^*(H) = \bigcup_U \mathcal{A}^*(U,H)$ *the set of all possible maximizers.*

Theorem 3 *Assume that (i)* $\mathcal{A}^*(H)$ *is non-empty and compact and that for any* U, H, *the function* $V(\cdot,U,H) : \mathcal{A}^*(H) \to I\!R$ *is continuous. Moreover assume that (ii)* $V'(\mathbf{a},U,H) \equiv \frac{\partial V(\mathbf{a},U,H)}{\partial U}$ *exists and is continuous in* $(\mathbf{a},U)$. *Then, for any* H, *the value function* $V(U,H)$ *has always both right and left derivative in* U, *and these are given by the formulas*

$$V'_+(U,H) = \max_{\mathbf{a}\in\mathcal{A}^*(U,H)} V'(\mathbf{a},U,H)$$
$$V'_-(U,H) = \min_{\mathbf{a}\in\mathcal{A}^*(U,H)} V'(\mathbf{a},U,H),$$

moreover $V(U,H)$ *is almost everywhere differentiable in* U, *and whenever the derivative exists then*

$$V'(U,H) = V'(\mathbf{a}^*,U,H) \quad \text{for any} \quad \mathbf{a}^* \in \mathcal{A}^*(U,H).$$

Proof. See Milgrom, 1999. ■

Theorems 1 and 2 guarantee condition (i) in Theorem 3 is satisfied. So, in order to apply Milgrom's result to our model, the main challenge is to verify condition (ii). First, notice that *(ii)* is verified by any finite periods version of our model. The reason is simple. If the time horizon is finite, then the set of all possible paths of actions $\mathcal{A}$ is finite. From Proposition 1 we know that each $V(\mathbf{a},U,H)$ has continuous derivative in U alone, but finiteness of $\mathcal{A}$ trivially implies joint continuity in $(\mathbf{a},U)$. In what follows, we are going to state sufficient conditions under which - for the infinite horizon problem - the set of possible maximizers $\mathcal{A}^*(H)$ is a finite set.

> **Assumption A1** *For each given* H, *if* $V(U,H) = V(\mathbf{a},U,H)$ *and* $V(U',H) = V(\mathbf{a}',U',H)$ *with* $U' \geq U$, *then* $a(t) \geq a'(t)$ for any t.

This assumption, basically, states a monotonicity property of our class of problems. Assumption **A1** is verified, for example, by the class of utility functions considered by Thomas and Worral (1990): the non-increasing absolute risk aversion ones (which includes CRRA and CARA utilities), under mild monotonicity conditions on the search effort cost and reemployment probability functions.

> **Assumption A2** *For each initial endowment* H_0, *there are two (possibly dependent on* H_0*) finite time horizons* $T_S(H_0)$ *and* $T_p(H_0)$ *such that*

$$\forall\ t \geq T_S(H_0) \qquad S(H_t) = \underline{S} \geq 0 \quad \text{and}$$
$$\forall\ t \geq T_p(H_0) \qquad P(a,H_t) = \underline{p}(a) \geq P(0,H_t) = \hat{\underline{p}}.$$

Assumption **A2** can be thought as a restriction on the law m, or on the functions $S(\cdot)$ and $P(a,\cdot)$, or on both. For example, the condition on S can be interpreted as a minimum wage condition.

As anticipated, under **A1-A2** the set of optimal action paths $\mathcal{A}^*(H)$ is a finite set. So we have the following

Proposition 2 *Under **A1-A2,** for each given H, $V'(\mathbf{a},U,H) \equiv \frac{\partial V(\mathbf{a},U,H)}{\partial U}$ exists and is jointly continuous in ($\mathbf{a}$,U).*

Proof. See Appendix. ■

A typical feature of dynamic models with information asymmetries is the following. In order to reduce the (marginal) costs of giving incentives, the planner tends to reduce the agent's expected discounted utility. This property have sometimes unpleasant consequences. Thomas and Worral (1990), in a model with adverse selection and i.i.d. shocks, show that if the worker's utility function is unbounded below (but bounded above) and displays non-increasing absolute risk aversion, then any arbitrary low level of expected discounted utility $\underline{U}$ is reached with probability one. This result is know in the literature as the *immiserization* result. Pavoni (2001) shows a similar (weaker) result for the stationary version of the dynamic moral hazard model presented in this paper.

The infinite punishments result is questionable in some circumstances. For example, it may be impossible for the planner to enforce, ex-post, such punitive plans because these would imply excessive social conflict costs. Or again, excessive punishments may induce the worker to opt out of the insurance scheme.

However, the introduction of human capital depreciation and duration dependence generates a novel feature in the optimal program. It creates the possibility of an endogenous lower bound on worker's expected discounted utility. The idea is simple. Provided that H depreciates sufficiently rapidly during unemployment, the planner eventually looses the incentives to induce the agent to supply the high effort level. Thus, utility stops decreasing because the agent is fully insured.

Proposition 3 *(Endogenous lower bound condition) If in **A2** either $\underline{S} = 0$ or $\underline{p}(a) = \underline{\hat{p}}$ or both, then after finitely many periods of unemployment the worker is required to supply the cost minimizing effort level and is fully insured, hence his expected discounted utility stops decreasing.*

Proof. When one of the two conditions is met, the optimal choice for the planner is trivially to fully insure the worker, requiring him to supply the lower cost effort. ■

Evidently, the conditions involved in Proposition 3 are only sufficient. In fact, they trivially guarantee the existence of endogenous lower bounds also for the continuous of actions case. In Section 5 we present some calibrated examples where the above conditions are not satisfied, nevertheless the optimal contract still implies an endogenous lower bound on worker's expected discounted utility.

The continuous of actions case It is immediate to see that, the main challenge for the extension to the case with a continuum of actions is again to find conditions under which condition (ii) of Theorem 3 holds. Note that Theorem 3 implies that, if for some U and H, the maximizing sequence is unique, i.e. $\mathcal{A}^*(U,H)$ is a singleton, then the value function is differentiable in U. So, when $a(t) \in [0,1]$ there is room for a collection of sufficient conditions on $P(\cdot,H)$ - perhaps similar to the ones proposed by Grossman and Hart (1983), Rogerson (1985b) or Jewitt (1988) - under which the problem is concave in a, and this could imply both the concavity and the differentiability of V. However, this is again left for future research.

3.3 Characterization of the optimal contract: the triangle rule

From the results in Section 3.2, we have that $V'_+(U,H) \geq V'_-(U,H)$, and this is not precisely a property of concave functions. In particular, when the directional derivatives differ, then $V(U,H)$ cannot be concave in any interval containing U. Indeed, in Section 4 we will present a closed form example where the value function is neither concave nor differentiable. Moreover, the next Remark clarifies that it is not that "usual" to have concave value functions

Remark 1 *Can be verified that, if we consider the class of analytic functions and we write the Taylor's expansion of two functions that cross in U and have the same derivative at the crossing point, then they must have also identical second-order derivatives in the crossing point U. Moreover, either the functions differ in their third derivative or also the fourth derivative has to be the same at U, and so on. And they must differ for some odd-order derivatives.*

Remark 1 basically says that in a dynamic moral hazard problem if, for some utility level U, it is optimal to change action in some present or future period, then the value function of the problem is "usually" neither concave nor differentiable. This is of course a quite negative result. However, we now show a simple result that will simplify enormously our optimal contract characterization task.

Lemma 1 *Assume that f is a continuous function that admits both right and left derivatives in an interior point U_0. If we have $f'_-(U_0) < f'_+(U_0)$ then U_0 cannot be a (local) maximum.*

Proof. Since f has in U_0 both left and right had derivatives, a necessary condition for U_0 to be a maximum is $f'_-(U_0) \geq 0 \geq f'_+(U_0)$ so the result is proved.[19] ■

Now we are ready to characterize the optimal contract. The main meaning of the next theorem is that, we can still use the usual first order conditions to characterize the optimal contract.

Theorem 4 *The Optimal Contract necessarily satisfies (1), (2) and (3), and*

$$
\begin{aligned}
V'(U,H) &= -\frac{1}{u'(b(U,H))} && (8)\\
W'\left(U^e(0,U,H'),H'\right) &= -\frac{1}{u'(b(U,H))} - \mu \qquad \mu \geq 0 && (9)\\
V'\left(U^u(0,U,H'),H'\right) &= -\frac{1}{u'(b(U,H))} + \frac{P(a,H)}{1-P(a,H)}\mu && (10)\\
\textit{with } \mu &= 0 \textit{ if (2) is satisfied with strict inequality for any } \hat{a} \neq a, \textit{ and}\\
V'(U,H) &= \left[P(a,H)W'\left(U^e(0,U,H),H\right) + (1-P(a,H))V'\left(U^u(0,U,H),H\right)\right]
\end{aligned}
$$

[19] To see, more in detail, why is that, write, for example, the incremental ratio for the left derivative $f'_-(U_0)$. If $\frac{f(U)-f(U_0)}{U-U_0} < 0$ for $U - U_0 < 0$, with U sufficiently closed to U_0, we must have $f(U) > f(U_0)$, which is a contradiction to U_0 be a maximum. A similar argument can be, of course, used for the right derivative.

Proof. See Appendix. ■

Note that μ is an "aggregate multiplier." That is, μ is the sum of all the (normalized) multipliers $\mu_{\hat{a}}$ - one for any possible deviation $\hat{a}$ - for which the incentive compatibility (2) is binding. Moreover, for notational simplicity, we have omitted to write that the implemented action a, is itself function of the state, so it should be red $a(U, H)$.

The implications for the optimal scheme are not yet transparent. However, rearranging the above derived first-order conditions, and using envelope theorem, we get the following

Corollary 1 *For each period t we have (i) $w_{t+1} \geq b_t \geq b_{t+1}$ and*

$$\frac{1}{u'(b_t)} = P(a(t), H_t)\frac{1}{u'(w_{t+1})} + (1 - P(a(t), H_t))\frac{1}{u'(b_{t+1})}. \tag{11}$$

Moreover, (ii) either is true that $w_{t+1} > b_t > b_{t+1}$, or $w_{t+1} = b_t = b_{t+1}$.

Proof. For a generic period t, we can rewrite $V'(U, H) = -\frac{1}{u'(b(U,H))} = -\frac{1}{u'(b_t)}$, use the envelope theorem in two successive periods, and rewrite $W'(U^e, H') = -\frac{1}{u'(w_{t+1})}$ and $V'(U^u, H') = -\frac{1}{u'(b_{t+1})}$. Now, since $\mu \geq 0$ and $P(a, H) \in [0, 1)$, both (i) and (ii) can be easily derived from (8), (9), (10) and the strict concavity of u. ■

Unfortunately, under these general conditions, we cannot say much more than this, without specifying further the problem.[20] However, notice that our results in (i) confirm the one of Shavell and Weiss (1979). Under quite general conditions, the optimal unemployment insurance scheme requires the benefits to decrease with the duration of unemployment. On the other hand, the wage tax behavior remain undeterminate. In fact, our numerical results both for US and the Spanish economy (in Section 5) show that in an optimal scheme the path of wage taxes ($\tau = S - w$) should be decreasing. This is in contrast with the result of the "fully-stationary" model of Hopenhayn and Nicolini (1997a,b), who found that τ should always be increasing in the length of previous unemployment spell.

Corollary 1 has another important implication. In many studies, unemployment insurance programs are modelled in a very simple way. Only two parameters are used to define the scheme. It is assumed to be a time invariant unemployment benefit payment b, which (usually because of job-search incentives) is strictly lower than an - again time invariant - wage payment w. We can then ask the following question. Could this simple scheme be optimal, for some combination of human capital depreciation and duration dependence? Unfortunately, result (ii) in Corollary 1 gives a clear negative answer to this question. But then: could it be at least approximately optimal? To answer to this and other related questions we now move, gradually, toward a more quantitative analysis.

To make our results ready to use for policy proposes, we can start by rearranging (11) and get

$$\frac{1}{u'(b_t)} - \frac{1}{u'(b_{t+1})} = P(a(t), H_t)\left[\frac{1}{u'(w_{t+1})} - \frac{1}{u'(b_{t+1})}\right],$$

[20]Note that Corollary 1 is not simply the martingale condition for the inverse of marginal utility (11), shown by Rogerson (1985a), who uses a variational approach to derive it for the repeated, finite horizon, case. For example, condition (11) alone would not allow us to say anything about the monotonicity of the unemployment insurance benefits behavior.

which, for the case with logarithmic utility, reduces to

$$b_t - b_{t+1} = P(a(t), H_t)\left[w_{t+1} - b_{t+1}\right]. \tag{12}$$

We name equation (12) *the triangle rule*, because of its graphical representation, shown in Figure 3. According to (12), for small $P(a(t), H_t)$ we should observe almost flat (optimal) schemes with a relatively big difference between net wage and unemployment insurance benefits. And vice versa for high hazard rates. This gives to the triangle rule a quite appealing economic interpretation. Workers facing relatively low hazard rates (P low) are motivated to search for new jobs mainly through rewards. That is, in case they find a new job they receive a high net wage w_{t+1}. In contrast, to these workers who face high probabilities of finding a new job, the optimal scheme gives search incentives mainly using punishments. In case of failure in the job-search process, workers who have many job opportunities, should see their unemployment benefit b_{t+1} to decrease by a lot.

The triangle rule can be used as a (very simple, back of the envelope) test for optimality of any unemployment insurance scheme. For that, consider an existing scheme. For each period t, using today and tomorrow's benefit payment levels and tomorrow net wage, it is always possible to draw a triangle as the one in Figure 3. The test needs also a reliable (point) estimation for the hazard rate $P(a(t), H_t)$, associated with unemployment duration of length t. Then using $P(a(t), H_t)$ one can easily check whether the two parts that form the segment $w_{t+1} - b_{t+1}$ have the right proportions, as required by (12).

Alternatively, we could consider the triangle rule as a valuable tool for the optimal unemployment insurance scheme designing problem. A hypothetical planner could indeed compute the unemployment scheme restricting himself to satisfy "at least" the triangle rule derived above.[21]

Finally, note that the triangle rule is not satisfied if we introduce other constraints into the designing problem, such as the exogenous lower bound on worker's utility we imposed in Pavoni (2001). Thus - assuming a maximizing government behavior - the triangle rule could also be used to test for the existence of exogenous minimum bounds; which is a way to discriminate between the unrestricted model of Hopenhayn and Nicolini (1997a,b) and the restricted one with utility bounds of Pavoni (2001).

A further quantitative assessment, would regard the degree of approximation - toward the fully optimal scheme - that the use of the triangle rule would involve for generic utility function, within a reasonable range of parameters.[22] This analysis is left for future research.

[21] The planner will still have many degrees of freedom. Indeed, the triangle rule defines the optimal scheme up to affine transformations. Thus, for example, the planner may start by choosing a sequence of steps between b_t and b_{t+1}, that is a sequence of $R_t = \frac{b_t}{b_{t+1}}$, and use the triangle rule to determine the implied wage tax sequence. Finally, the scheme determination will be closed by choosing the initial b_0 either to satisfy the (aggregate) budget constraint or according to some equity preference concerns.

[22] We expect to have quite interesting results, since the logarithmic case seems to be a good approximation for the average level of risk aversion. Indeed, for example, Attanasio and Weber (1993) use UK cohort data to estimate the intertemporal elasticity of substitution. Assuming CRRA preferences, the results of Attanasio and Weber imply a constant risk aversion parameter between 1.3 and 1.5 (where 1 corresponds to the log-case). Moreover, Mehra and Prescott (1985) cite various empirical studies that provide support for a constant risk aversion parameter between 1 and 2.

4 An Analytic Solution

It is always useful to clarify the results of a theoretical analysis using some examples. In this section we present a (new) closed form solution for our dynamic moral-hazard problem with generic human capital depreciation, when the worker has logarithmic utility. To simplify the analysis we work in average terms, that is, following the game theory literature, we multiply both the worker and the planner's payoffs by $(1-\beta)$. All the assumption of Sections 2 and 3 are maintained here.

Proposition 4 *If $u(c)=\ln(c)$ and assumption **A2** is satisfied, the value functions have the following separable form*

$$V(U,H) = \mathcal{S}_n(H) - B_n(H)\exp\{U\} \quad (13)$$
$$W(U,H) = \mathcal{S}_e(H) - L\exp\{U\}, \quad (14)$$

where $L=\exp\{l\}$.

Proof. See Appendix. ■

The Proposition can be proved in two steps. First, recalling the results of Section 3 one can use the usual first order conditions to verify that the proposed functions satisfy the Bellman functional equation. Second, by Theorem 4.3 of Stokey and Lucas (1989) the proposed functions are actually the value functions of the problem. Indeed, for it one can verify that, for each given U_0 and H_0, both W and V are bounded.

Since the description of W is immediate. We will focus on the unemployment-state value function V. The expression in (13) basically says that, for each given H, the domain of $V(\cdot,H)$ is divided into $N<\infty$ different intervals $I_n(H)$, $n=1,2,....N$, with n increasing in U. Each utility interval $I_n(H)$ has associated a multiplicative coefficient $B_n(H)$, with $B_n(H)\geq B_{n+1}(H)$, and an additive term $\mathcal{S}_n(H)=\mathbf{E}\left[(1-\beta)\sum_t \beta^t S(H_t)\mid H,\, n\right]$, which corresponds to the expected discounted gross wages. Given H, the additive constants $\mathcal{S}_n(H)$ differ among the different utility intervals; since each interval $I_n(H)$ has associated a different sequence of actions, thus a different expected wage level.

The "fully-stationary" example The simplest case is obviously the one with two actions, where both the gross wage and the probability of reemployment are constant values. So, consider the case where - in the set of actions A - $N=2$. And normalize $\alpha_0=0$, $\alpha_1=1$, with $C(0)=0$ and $C(1)=v>0$. Moreover, assume both the gross wage and the hazard rate are not changing with H. So define $S(H)=S$, $P(1,H)=p\in(0,1)$ and $P(0,H)=\hat{p}<p$. Finally, as always, assume $F=0$. Pavoni (2001) solves the case where $\hat{p}=0$ and $l=v$. More generally, the value functions take the following form[23]

$$V(U) = K_n S - B_n\exp\{U\}, \quad n=0,1,$$
$$W(U) = S - L\exp\{(U\}.$$

[23] The coefficient values are as follows. $B_0=L^{\frac{\beta\hat{p}}{1-\beta(1-\hat{p})}}$, and B_1 solves

$$B_1^{\frac{1}{\beta}} = [\exp\{D\}]^{\frac{1-\beta}{\beta}}\left[pL\exp\{(1-\beta)E\}+(1-p)B_1\right], \quad (15)$$

With logarithmic utility function, the worker's utility space is the whole real line, and it will be divided in only two intervals I_0 and I_1 (actually two half-lines). If $U \in I_0$ we say we are in the full-insurance regime, since for these levels of utility the worker is required to supply zero effort ($a = 0$) and he is fully insured by the planner. If $U \in I_1$ we say we are in the incentive regime, since $a = 1$, and the switching point U^* between the two regimes is given by $U^* = \ln(S) + \ln\left(\frac{K_1 - K_0}{B_1 - B_0}\right)$. The closed form is very simple since if we start from an initial utility value such that $U_0 > U^*$ then the value function takes the above form with $n = 0$ and it will never become optimal to switch to the $n = 1$ case. The same is symmetrically true if U_0 lies below U^*, in this case, the value function takes the $n = 1$ form. If $U_0 = U^*$ then the planner is indifferent between the two regimes. Figure 4 shows a parametrized example of $V(\cdot)$, which is clearly not concave and not differentiable at U^*, confirming the result of Theorem 3. An added bonus is that the policy functions are linear functions in the space of utilities

$$\begin{aligned} u(b(U)) &= U + \ln B_n \\ u(w(U)) &= U + \ln L = U + l \\ U^u(0,U) &= \begin{cases} U & \text{if } n = 0 \\ U - \frac{1-\beta}{\beta}\left[\ln B_1 + \frac{\hat{p}v}{p-\hat{p}}\right] & \text{if } n = 1 \end{cases} \\ U^e(0,U) &= \begin{cases} U + \ln B_0 - l & \text{if } n = 0 \\ U - \frac{1-\beta}{\beta}\left[\ln B_1 - \frac{(1-\hat{p})v}{p-\hat{p}}\right] & \text{if } n = 1. \end{cases} \end{aligned}$$

The general case The main complication in computing the closed form for a non-stationary model is that, in each period t, for each implemented action $a \neq 0$, the multiplier $\mu_{\hat{a}}$ associated with the incentive compatibility constraint (2) when the deviating action is $\hat{a} \neq a$ can be either positive or zero. And the way of computing the B_n coefficients vary considerably between the two cases. In fact, for each level H, we can identify three "regimes." (i) In the full-insurance regime all the multipliers $\mu_{\hat{a}}$ are equal to zero because the action implemented is the low cost one ($a = 0$). (ii) However, when the hazard rate decreases very fast with unemployment duration, one can have $\mu_{\hat{a}} = 0$ for any $\hat{a}$ with $a \neq 0$, and we say we are in the incentive-slack regime. (iii) Finally, we have the incentive-binding regime where at least one $\mu_{\hat{a}} > 0$ and $a \neq 0$.

The policy correspondences are as follows. First, the wage and benefit payments are easy to compute, and always given by

$$\begin{aligned} u(b(U,H)) &= U + \ln B_n(H) \\ u(w(U,H)) &= U + \ln L = U + l. \end{aligned}$$

Second, the promised utilities reflect the different regimes, and the policy functions are given

with $E = \frac{v}{\beta(p-\hat{p})}$ and $D = \beta\hat{p}E = \frac{\hat{p}v}{(p-\hat{p})}$. Finally, the additive constants are given by

$$K_1 = \frac{\beta p}{1 - \beta(1-p)} \text{ and } K_0 = \frac{\beta\hat{p}}{1 - \beta(1-\hat{p})}.$$

by the following expressions

$$U^u(0,U,H) \;=\; \begin{cases} U + \ln B_n(H)\left[1 - \frac{l}{\beta(1-P(a,H))}\right] + \frac{P(a,H)l + \frac{1-\beta}{\beta}C(a)}{1-P(a,H)} \\ U - \frac{1-\beta}{\beta}\left[\ln B_n(H) + \frac{P(\hat{a},H)(C(a)-C(\hat{a}))}{P(a,H)-P(\hat{a},H)}\right] \end{cases}$$

$$U^e(0,U,H) \;=\; \begin{cases} U + \ln B_n(H) - l \\ U - \frac{1-\beta}{\beta}\left[\ln B_n(H) - \frac{(1-P(\hat{a},H))(C(a)-C(\hat{a}))}{P(a,H)-P(\hat{a},H)}\right]. \end{cases}$$

That is, for each policy, we have (only) two expressions. The expression in the second line refers to the incentive-binding regimes ($\mu_{\hat{a}} > 0$ for some $\hat{a}$), whereas the first line refers to the case where $\mu_{\hat{a}} = 0$, for any $\hat{a}$, which can be either because the incentive constraint is slack ($a \neq 0$), or because we are in the full-insurance regime ($a = 0$). For example, if we specify the first expressions by setting $a = 0$, so that we are in the full-insurance regime, we have $B_n(H) = B_0(H) = L^{\frac{\beta P(0,H)}{1-\beta(1-P(0,H))}}$, $U^u(0,U,H) = U$ and $U^e(0,U,H) = U + \ln B_0(H) - l$, which are clearly very similar to the policies associated to the "fully-stationary" example.

5 Quantitative Analysis

In our model, human capital affects both reemployment wages and the probability of finding a new job. More than giving a specific policy advise, the aim of this section is to disentangle the effects of each one of these two consequences of human capital depreciation, in the optimal program. We pursue this task using a couple of calibration exercises. The first example refers to Spain. Spanish data on wages are often poor or difficult to interpret. In contrast, most empirical studies document a clear negative duration dependence in the reemployment probability. We thus use the Spanish economy to study how the optimal unemployment insurance scheme is affected by duration dependence in the probability finding a new job *alone*. Below, we use the US economy as our second calibrated example, to study the implications of wage depreciation.

The Spanish Example: constant wage and decreasing hazard rate. To calibrate the model with the Spanish economy, we assume CRRA worker's preferences, i.e.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma},$$

and *constant* gross wage S. To maintain a simpler analysis, we assume $N = 2$ with $C(0) = 0$ and $C(1) = v$. Our calibration exercise consists in choosing 7 parameters: $[S, F, \beta, \sigma, U_0, v, l]$, the hazard rate paths $\{P(a, H_t)\}_{t=0}^{T}$ and the initial utility level U_0. Partially following Pavoni (2001), we normalize the wage S to 100, set $F = 0$, and assume logarithmic worker's utility $\sigma = 1$. We further interpret each period as a month, so we calibrate $\beta = 0.996$, which implies an annual discount rate of 5%. Finally, we set $v = l = 1$, which is within a reasonable range, according to the utility $u(S) = \ln(100) = 4.6$, the agent would receive from consuming the gross wage. The initial level of worker's utility U_0 is computed - as in Hopenhayn and Nicolini

(1997a,b) - backward, in accordance with the existing scheme.[24] To calibrate the hazard rate paths $\{P(a,H_t)\}_{t=0}^{T}$ we interpolate linearly the estimations of Bover et al. (1997). We set $T=30$, $P(1,H_0)=0.21$ and $P(1,H_T)=0.03$ with a time constant decrease in the hazard rate of 0.06, in each period. Furthermore, we calibrate the benchmark level of the "passive" hazard rate $P(0,H_t)$ by setting $P(0,H_t)=\hat{p}=0.01$ constant for each t. Note that, in this example, we implicitly assume that human capital affects only the effectiveness of the job-search effort a. This assumption is supported by the following empirical observation. Figure 4 in Bover et al. (1997) shows the estimated Spanish hazard rates as a function of the unemployment duration, both for the workers receiving unemployment benefits and for those not receiving any unemployment benefit. The difference between the two hazard rates

[24] In Spain, the replacement ratio is equal to 70% during the first six months of unemployment and 60% thereafter, subject of a floor of 75% of the minimum wage. Benefit duration is one-third of the last job's tenure, with a maximum of two years. The assistance system pays, for up to two years, 75% of the minimum wage to (unemployed) workers, with dependant, whose average family income is precisely below that amount. In 1998, the minimum wage was around $70,000$ pesetas ($280) (Guia Laboral 1998 y de Asuntos Sociales (1998)). The amount of the non-contributive assistance level of transfers varies across different Autonomous Communities between $30,000$ to $45,000$ pesetas ($150/180), is means-tested, and there is no fixed duration (See Lopez (1996)). The Bulletin of Labor Statistics (1999) reports as $300,000$ pesetas ($$1,200$) per month the 1998 average wage in non-agricultural activities. Following the common assumption that workers subject to severe unemployment risk face a wage that is two thirds of the average national wage, we consider the assistance level of benefits as $\frac{1}{5}=20\%$ of the gross wage S.

Given we normalized the gross wage S to 100, the current insurance system can thus be represented by a contract that has not taxes or transfers when unemployed ($w=S$), and pays a first benefit level b^1 of 70 for the first six months of unemployment, from the 7th to the 24th month the benefit level b^2 is set equal to 60, and from the 25th onward we assume the worker receives an assistance level of benefits $b^3=20$. The corresponding expected discounted utility value U_0 for an unemployed worker, can be calculated backward as follows. Give that $F=0$, when a worker finds a job his lifetime utility is

$$U_{work}=\frac{u(S)-l}{(1-\beta)}=\frac{4.6-1}{0.004}=900,$$

which represents the utility of working forever receiving the gross wage $S=100$. Moreover, note that from period T onward the worker's problem is stationary. Both the unemployment benefits and the probability of finding a job are at their minimum level. Thus, under our parametrization, the worker will be searching for a job ($a=1$), which will be found with probability $P(1,H_T)=\underline{p}$. And when a job is found the worker will be never fired ($F=0$). The value of his expected discounted utility U_T can be computed as follows

$$U_T=\frac{u(b^3)-v+\beta\underline{p}U_{work}}{1-\beta(1-\underline{p})},$$

where $b^3=20$ is the non-contributive assistance level of unemployment benefits. For any $0\le t\le T$ we can now define the value U_{T-t} recursively by

$$U_{T-t}=u(b_t)-C(a_t^*)+\beta\left[P(a_t^*,H_t)U_{work}+(1-P(a_t^*,H_t))\,U_{T-(t-1)}\right],$$

where the period t benefit level $b_t\in\{b^1,b^2,b^3\}$ is computed according to the three steps scheme described above, and a_t^* denotes the effort level chosen optimally by the worker in period t. That is, a_t^* solves

$$\max_{a_t\in\{0,1\}} u(b_t)-C(a_t)+\beta\left[P(a_t,H_t)U_{work}+(1-P(a_t,H_t))\,U_{T-(t-1)}\right].$$

decreases considerably with unemployment duration, and approaches zero after 2 years. We believe, it is reasonable to assume that the workers non receiving the benefits supply an higher effort level then the workers receiving benefits, at any duration level. These considerations induce us to interpret the decrease in the difference between the hazard rates of the two groups of workers as a decrease in the effectiveness of the search activity.

The results of our calibration exercise are reported in Figure 5. First, the interested reader can check that the scheme obeys to the triangle rule. Second, we observe that the optimal path for unemployment benefit payments (the dotted lower-level line) is initially decreasing an then becomes completely flat. The reason is that, provided human capital depreciates sufficiently rapidly during unemployment, the planner looses the incentives to induce the agent to supply the high effort level. Thus, benefits eventually stop decreasing because the agent is fully insured. The second part of the benefits' path (the flat one) is particularly important since induces an endogenous lower bound on workers' expected discounted utilities. This in a calibrated example, where the conditions of Proposition 3 are clearly not met.[25] Finally, note that the net wage w_t - represented by the upper-level solid line - is initially roughly constant and then increases in unemployment duration. Moreover, it presents an important downward jump for long term unemployed, when the worker is asked to stop searching actively and he is fully insured by the planner.

The wage tax behavior in the initial periods contrasts with the one obtained by Hopenhayn and Nicolini (1997a,b), and is quite consistent with causal observation in many OECD countries.[26] On the contrary, we never observe, in the real world, a sharp downward jump of the net wage, as the one we observe in the simulated optimal scheme after the 24th month. In this particular example, this may be due to two main reasons.[27] The first possibility is straightforward: the governments' behavior is suboptimal. Alternatively, while working, agents may have additional costs (such as transportation costs or the cost of having lunch in a restaurant during the working days, an so on). Thus, in order to compensate the worker for these additional costs, full-insurance actually implies a net wage well above the unemployment insurance benefits level.

The US Example: constant hazard rate and decreasing wage. Our second numerical example is a calibration with the US economy, and focuses on the effect of wage depreciation. We assume again two actions ($N = 2$), and a very simple relationship between human capital endowment and gross wage

$$S(H_t) = \omega H_t.$$

If the worker is unemployed, we assume a linear depreciation rule for human capital, i.e. $H_{t+1} = m(0, H_t) = (1 - \delta)H_t$, and - again for simplicity - we assume that, while employed,

[25] The gross wage S is constant, and always equal to $100 > 0$. Moreover,

$$P(1, H_T) - \hat{p} = 0.03 - 0.01 = 0.02 > 0.$$

[26] In most present schemes the wage tax is constant during unemployment.

[27] Below, we give a third motivation for this, when the gross wage is negatively affected by human capital depreciation.

worker's human capital endowment remains constant, i.e. $m(1, H_t) = H_t$. It is easy to see that in this special case, during unemployment, $\frac{S(H_{t+1})}{S(H_t)} = (1 - \delta)$, that is, the reemployment *gross wage* decreases at an exogenous and constant rate, equal to the human capital depreciation rate δ. Finally, we assume the hazard rates are independent from H, that is, $P(1, H) = p$ and $P(0, H) = \hat{p}$. With this formulation, and assuming CRRA worker's preferences, we have now eight parameter to calibrate: $(F, S, v, l, \sigma, \beta, p, \hat{p}, \delta)$, together with the initial utility level U_0. We set $F = 0$, $S(H_0) = 100$, $v = l = 1$, and $\sigma = 1$. To be consistent with the US system, we choose one week as reference period, and accordingly calibrate $\beta = 0.999$. Partially following Meyer (1990) and Hopenhayn and Nicolini (1997a), we calibrate $p = 0.1$ and the "passive" hazard rate $\hat{p} = 0.01$, and we computed U_0 as described above for Spain.[28] Finally, the human capital decreasing rate parameter δ, is calibrated following the results of Keane and Wolpin (1997, page 500). They provide (structural) econometric estimates for rates of skill depreciation during period of unemployment. They estimated an annual human capital depreciation rate for USA of between 9.6% (for blue collars) and 36.5% (for white collars). We set our weekly level of $\delta = 0.005$, which corresponds to an annual depreciation rate of 24%.

The results of the calibration exercise for US are reported in Figure 6. The optimal path for unemployment benefit payments (the dotted lower-level line) presents qualitatively the same characteristics as the Spanish case: it is initially decreasing an then - after 60 weeks - becomes completely flat. The reason is again very simple. Since p and $\hat{p}$ are constant, now the incentive costs of implementing the high effort level are constant. However, the planner's expected returns are decreasing, since the gross wage $S(H)$ decreases with unemployment duration. The result is the same: at some point, the planner looses the incentives to induce the agent to supply the high effort level and the unemployment insurance benefits stop decreasing, because the agent is fully insured. The net wage schedule w_t - represented by the upper-level solid line - is surprisingly very flat. This implies that the reemployment *wage tax* $\tau_t = S(H_t) - w_t$ *is strictly decreasing* during unemployment, since the gross wage $S(H_t)$ is decreasing at a positive rate δ. Simple back-of-the-envelope calculations show that, for long term unemployed (with duration less than 60 weeks), it is optimal to pay a *wage subsidy* after reemployment.[29]

Finally, as in the previous example, the net wage w_t presents an important downward jump after a sufficiently long period of unemployment. This occurs when the worker is asked to stop searching actively and is fully insured by the planner. However, in this example with decreasing gross wage $S(H_t)$, what really happen is that the government simply stops paying the wage subsidy.

[28] According to Meyer (1990) the unemployment insurance benefit level received in average in the sample was 66% of the average value of the pre-unemployment wage $S(H_0) = 100$.

[29] In particular, note, in Figure 6 the net wage schedule $w_t = S(H_t) - \tau_t$ (upper line) after 54 weeks is above the level of 0.9. Moreover, we know that the parameter δ is calibrated such that for $t = 54$ weeks, the gross wage $S(H_{54}) = 0.76$. This implies - after a bit more than a year of unemployment - a wage subsidy of about $\left(\frac{0.9}{0.75} - 1\right) 100 = 20\%$.

6 Extending the Policy Instruments: Retraining and Job-Search Assistance

The results in the previous section have some policy implications, especially for long-term unemployed workers. In both cases, a worker who stays unemployed for a sufficiently long period is required to stop searching for a job, and is fully insured. Now, consider the extreme case, where $P(0, H) = 0$, i.e. workers cannot find a job if they do not supply a positive level of search effort a. In this case, the long-term (fully-insured) unemployed workers are a net cost for the government, since they have no hope of finding a new job. Moreover, alternative active labor market policies may become the only valuable alternatives to rescue these people from their state. Finally, note the planner's value function V represents the monetary cost of an unemployed worker. This interpretation of the V suggest a natural way to analyze the choice of retraining programs and/or job-search activities. That is, the recursive formulation we introduced above in the paper, allows us to easily extend the problem to the case where the planner has alternative policy instruments other than unemployment benefits and reemployment wage taxes.

Suppose the planner can in each period t choose either to "force" the worker to follow a retraining program or to ask him to search for a new job, which can be done with or without assistance. We assume the retraining program has uncertain outcome. In particular, with probability $\theta \in (0, 1)$ the program will be successful (s) and with the complementary probability $(1 - \theta)$ the retraining program will fail (f). The retraining program consist in a fix amount $\tau > 0$ which the planner pays, in order to rebuild some of the worker's human capital endowment, depreciated during previous periods of unemployment. For simplicity, also the job-assistance activity is binary. That is, it costs a fixed monetary amount $q > 0$ to help the worker to find a job.[30]

Allowing for retraining and job-search assistance, the planner's value function V during unemployment is not longer defined as in (5). It now becomes

$$V(U, H) = \max\{V_N(U, H), V_J(U, H), V_R(U, H)\}$$

where V_N is the planner's value in case neither retraining nor job-search assistance is implemented, and is defined precisely as the unemployment state problem in Section 2

$$\begin{aligned} V_N(U, H) \quad &= \quad \sup_{a,b,U^u,U^e} -b + \beta\left[P(a, H)W(U^e, m(0, H)) + (1 - P(a, H))V(U^u, m(0, H))\right] \\ s.t. \quad &: \quad (2) \text{ and } (3), \end{aligned}$$

where W is the optimal planner's net return in case the worker finds a job, which is defined in (6). The planner's value function V_J, in case of adopting a job-search assistance activity is defined as follows

$$V_J(U, H) \quad = \quad \sup_{a,b,U^u,U^e} -b - q + \beta\left[P(a, H')W(U^e, m(0, H)) + (1 - P(a, H'))V(U^u, m(0, H))\right]$$

[30] In general, we could allow for the possibility of different retraining programs and job-search activities, with different costs and effects. It should be easy to see that our problem would remain recursive in a very natural way.

$$
\begin{aligned}
s.t. &\quad : \\
U &\geq u(b) - C(\hat{a}) + \beta \left[P(\hat{a}, H')U^e + (1 - P(\hat{a}, H'))\, U^u \right] \; \forall \hat{a} \in A \\
U &= u(b) - C(a) + \beta \left[P(a, H')U^e + (1 - P(a, H'))\, U^u \right], \text{ and} \\
H' &= J(H) > H,
\end{aligned}
$$

The job-search help, costs q, and affects positively the probability of finding a new job. We modelled it as a temporary increase of H in the probability of reemployment P, summarized by $J(H)$. The job-search assistance activity has only a transitory effect on H. Indeed, note in W and V the next-period human capital level is $m(0, H)$, updated according to the usual depreciation rule during unemployment. Finally, it should be noted the similarity of the problem which defines V_J, with the one for V_N. In both cases the worker is required to search, and the planner's inability to observe job-search effort creates moral hazard.

The planner's value when the worker is "mandated" to follows the retraining program, V_R, is a bit different. It is defined as follows

$$
\begin{aligned}
V_R(U, H) &= \sup_{b, U_s^u, U_f^u} -b - \tau + \beta \left[\theta V(U_s^u, H') + (1 - \theta) V(U_f^u, m(0, H)) \right] \\
s.t. &\quad : \\
U &= u(b) - l + \beta \left[\theta U_s^u + (1 - \theta) U_f^u \right] \\
H' &= R(H) > H.
\end{aligned}
$$

There are two main differences between the job-search assistance and the retraining program. First, the formation program has a permanent effect on worker's human capital endowment. Indeed, the function R affects permanently the level of H. Moreover, when the worker follows the formation program, he usually has a limited time for searching new jobs. So, in our model, adopting a training program implies for the planner two kind of costs: a direct cost τ, together with an opportunity cost due to the fact that during this period the worker would remain unemployed with certainty.[31]

It is possible to show that both V, V_J, V_R and V_N have the same characteristics as the value function we studied in Section 3.2. Namely, they are in general non concave and non differentiable. However, we can still use first order conditions to characterize the optimal choice.

In particular, during the search periods (with or without assistance) because of informational problems, the benefits behavior are weekly decreasing. In contrast, during retraining, since there are no informational problems, we expect to observe a different behavior, more similar to the one during the employment state (in the definition of W).

Proposition 5 *During the retraining period the unemployment benefits are constant.*

Proof. Using first order conditions in two successive periods we have easily the result. ■

Moreover we can easily conjecture the following result about the existence of endogenous lower bounds on worker's expected discounted utility:

[31] The later cost could be mitigated, for example, if the worker is assumed to have a positive probability of finding a job, also during the retraining period. For example, it is reasonable to assume that the worker can find a job with a probability corresponding to the minimizing cost action level, or of "passive search." The problem would not change much.

Conjecture 1 *(Probabilistic lower bounds) Under the conditions of Proposition 3, there is a positive probability that after finitely many periods of unemployment the worker is required to supply the cost minimizing effort level and is fully insured, hence his expected discounted utility stops decreasing.*

To summarize, the characteristics of the unemployment benefits are similar as before. Namely, we should observe decreasing benefits, which become temporarily flat during the retraining programs. Moreover, after a sufficiently long period of unemployment we may observe a permanent flat benefits' behavior. The long-term unemployed in this situation are workers for which there is no more possibility of rescue. It would be the case, for example, of relatively old displaced workers, for which the best policy is to propose *early retirement.*

7 Conclusions

Many OECD countries propose and apply active labor market policies and wage subsidies for long term unemployed people, mainly because job opportunities change during unemployment. In the present paper we extended previous studies on optimal unemployment insurance to incorporate the effects of human capital depreciation and duration dependence in this (mechanism-design) problem.

Our results partially confirm the ones obtained in most previous studies. Namely, we find that benefits should decrease with unemployment duration. In fact, this behavior characterizes qualitatively most schemes present in OECD countries. However, the introduction of human capital depreciation and duration dependence generates some novel features on the optimal program. First, if human capital depreciates sufficiently rapidly during unemployment, the planner loses the incentives to induce the agent to supply the high effort level. So unemployment benefit payments are initially decreasing an then eventually they become completely flat, since the long-term unemployed worker is fully insured by the planner. This creates an endogenous lower bound on worker's expected discounted utility, providing an alternative way of eliminating the *immiserization result* of Thomas and Worral (1990). Second, we find that the increasing wage tax result of Hopenhayn and Nicolini (1997a) is not robust to this extension. Indeed, our simulation results both for the US and Spanish economies show that it is indeed optimal to impose a wage tax after reemployment on sort-term unemployed workers. However, we find that the optimal level of *wage tax should decrease* with the length of worker's previous unemployment spell, eventually becoming a *wage subsidy* for long-term unemployed.

Our analysis has also an independent theoretical interest. In that we are able to study recursively, in a quite general framework, the dynamic moral-hazard problem, despite its well known inherent difficulties. In particular, we develop a *new approach* that allow us to show the moral-hazard problem's value function is, in general, non-concave and non-differentiable. In spite of these (non-smoothness) problems, we show the optimal contract can be characterized using the usual first order conditions. The technique we developed in this paper uses the Milgrom's (1999) envelope theorem, and can be easily extended both to a more general class of moral-hazard problems and to other problems with similar characteristics.

The core of our characterization of the contract is a simple necessary characteristic of any optimal unemployment insurance scheme, which we call it *the triangle rule*. We believe that the simplicity of the triangle rule deserves an accurate analysis. In the short run, we plan to study the degree of optimality of an unemployment scheme computed using the triangle rule, when the worker's utility function is different from the logarithmic case, within a range of parameters consistent with empirical estimations.

In the last section we extended our analysis allowing the planner to implement some mandatory training programs. This is equivalent to allow the agent to decide directly his human capital accumulation strategy, assuming that the human capital accumulation decisions are perfectly observed by the planner. We left for future research the case where there are imperfections in this monitoring process, since this - together with the monitoring problems associated with the job-search effort level - creates a two-dimensional moral-hazard problem, which could considerably complicate the analysis.

References

[1] Abraham, K.G. and H. Faber (1987), "Job Duration, Seniority, and Earnings," *American Economic Review*, 77(3), 278-297.

[2] Abreu, D., D. Pierce, and E. Stacchetti (1986), "Optimal Cartel Equilibria with Imperfect Monitoring," *Journal of Economic Theory,* 39: 251-269.

[3] Abreu, D., D. Pearce and E. Stacchetti (1990), "Towards a Theory of Discounted Repeated Games with Imperfect Monitoring," *Econometrica*, 58, 5, September: 1041-1064.

[4] Ackum, S. (1991) "Youth Unemployment, Labor market Programs and Subsequent Earnings," *Scandinavian Journal of Economics*, 93(4), 531-543.

[5] Aliprantis, C.D., and K.C. Border (1994), *Infinite Dimensional Analysis*, Springer-Verlag, Berlin.

[6] Altonji, J.G. and R.A. Shakotko (1987), "Do Wages Rise with Seniority?" *Review of Economic Studies,* 54: 431-459.

[7] Altonji, J.G. and N. Williams (1992), "The effect of Labor Market Experience, Job Seniority, and Job Mobility on Wage Growth," *NBER Working Paper No. 4133.*

[8] Atkeson, A. and R.E. Lucas (1992), "On Efficient Distribution with Private Information," *Review of Economic Studies,* 59: 427-453.

[9] Atkeson, A. and R.E. Lucas (1995), "Efficiency and Equity in a Simple Model of Efficient Unemployment Insurance," *Journal of Economic Theory,* 66: 64-88.

[10] Attanasio, O.P. and G. Weber, 91993), "Consumption Growth, the Interest rate and Aggregation," *Review of Economic Studies*, 60: 631-649.

[11] Bartel, A.P. and G.J. Borjas (1981), "Wage Growth and Job Turnover: an Empirical Analysis," In Rosen, S.S. (Ed.), *Studies in Labor Markets.* University of Chicago Press for National Bureau of Economic Research: 65-90.

[12] Bender, S., C. Dustmann, D. Margolis, and C. Meghir (1999), "Worker Displacement in France and Germany," *IFS Working Paper No. W99/14.*

[13] Benveniste and Scheinkman (1979), "On the Differentiability of the Value Function in Dynamic Models of Economics," *Econometrica* 47: 727-732.

[14] Benzil, C. (1995), "Unemployment Duration, Stigma and Re-employment Earnings," *Canadian Journal of Economics*, 28(3), August.

[15] Bover, O., M. Arellano and S. Bentolila (1997), "Unemployment Duration, Benefit Duration and the Business Cycle," *Working Paper 9717* October, CEMFI, Madrid, Spain.

[16] Burda, M.C. and A. Mertens (2001), "Estimating wage Losses of Displaced Workers in Germany," Labour Economics 8: 15-41.

[17] Calvó-Armengol, A. (2000), "Job Contact Networks," *mimeo,* Universitat Pompeu Fabra, Barcelona Spain.

[18] Chiu, W.H. (1998), "Endogenous Adverse Selection and Unemployment Insurance," Journal of Political Economy, 106(4), August: 806-827.

[19] Fallick, B.C. (1996), "A Review of Recent Empirical Literature on Displaced Workers," *Industrial and Labor Relations Review*, 50(1): 5-16.

[20] Farber, H.S. (1993), "The Incidence and Costs of Job Loss: 1982-91," *Brooking Papers*: Microeconomics, 73-132.

[21] Farber, H.S. (1997), "The Changing Face of Job Loss in the United States, 1982-91," *Brooking Papers*: Microeconomics, 55-142.

[22] Fudenberg, D., O. Holmström and P. Milgrom (1990), "Short-Term Contracts and Long-Term Agency Relationships," *Journal of Economic Theory,* 51(1): 1-31.

[23] Fudenberg, D., D. Levine and E. Maskin (1994), "The Folk Theorem with Imperfect Public Information", *Econometrica,* 62(5), September: 997-1039.

[24] Gregg, P. and B. Petrolongo (1997), "Random or non-random matching ? Implications for the use of the UV curve as a measure of matching effectiveness," *Discussion Paper no. 348* (CEP, London School of Economics).

[25] Gregg, P. and J. Wadsworth (1996), "How effective are state Employment Agencies ? Jobcentre Use and Job Matching in Great Britain," *Oxford Bulletin of Economic and Statistics*, 58: 443-467.

[26] Grossman, S.J. and O.D. Hart (1983), "An Analysis of the Principal-Agent Problem," *Econometrica*, 51: 7-45.

[27] Hall, R.E. (1995), "Lost Jobs," *Brooking Papers on Economic Activity*, 1: 221-273.

[28] Hamermesh, D.S. (1989), "What do we Know about Worker Displacement in United States?" *Industrial Relations*, 28(1): 51-59.

[29] Hansen, G.D. and A. Imrohoroglu (1992), "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy,* 100, February: 118-142.

[30] Harrison (1976), "The Demoralizing experience of Prolonged Unemployment," *Department of Employment Gazette*, 84, 330-349.

[31] Hansen, G.D. and A. Imrohoroglu (1992) "The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard," *Journal of Political Economy,* 100, February: 118-142.

[32] Heckman, J.J. (1991), "Identifying the hand of the past: distinguishing state dependence from heterogeneity," *American Economic Review*, 81(2): 75-79.

[33] Heckman, J.J. and G.J. Borjas (1980), "Does unemployment cause future unemployment ? Definitions, questions and answers from a continuous time model of heterogeneity and state dependence," *Economica* 47 (187): 247-283.

[34] Hopenhayn, H. and J.P. Nicolini (1997a), "Optimal Unemployment Insurance," *Journal of Political Economy,* 105(2): 412-438.

[35] Hopenhayn, H. and J.P. Nicolini (1997b), "La Reforma de Seguro de Desempleo en España: ¿ Hay Algo que Aprender de la Teoría ?" *Moneda y Credito,* 204: 263-303.

[36] Jewitt, I. (1988), "Justifying the First-Order Approach to Principal-Agent Problems," *Econometrica*, 56: 1177-1190.

[37] Kalisch, D. W., T. Aman and L. A. Buchele (1998), "Social and Health Policies in OECD Countries: A Survey of Current Programmes and Recent Developments," *Labour Market and Social Policy - Occasional Papers no. 33*, 67299 and 67364. Paris: OECD.

[38] Karni, E. (1999), "Optimal Unemployment Insurance: A Survey," *Southern Economic Journal*, 66(2): 442-465.

[39] Keane, M. P. and K. I. Wolpin, (1997), "The Career Decisions of Young Men," *Journal of Political Economy,* 105(3): 473-522.

[40] Kletzer, L.G. (1989), "Return to Seniority After Permanent Job Loss," *American Economic Review,* 79(3): 536-543.

[41] Kletzer, L.G. (1998), "Job Displacement," *Journal of Economic Perspectives,* 12(1): 115-136.

[42] Lancaster, T. (1990), "The econometric analysis of transition data," *Econometric society monographs no. 17.* Cambridge University Press, Cambridge.

[43] Lockwood (1991), "Information externalities in the labor market and the duration of unemployment," *Review of Economic Studies*, 58(4): 733-753.

[44] Lopez, Lopez, M. T. (1996), "La Proteccion Social a la Familia en España y en los Demas Estados Miembros de la Union Europea," *Working Paper Fundacion BBV Documenta,* Centro de Estudios de Economia sobre el Sector Publico.

[45] Lynch, L.M. (1985), "State dependence in youth Unemployment," *Journal of Econometrics*, 28:71-84.

[46] MacGregor, A. (1978), "Unemployment Duration and re-employment Probability," *Economic Journal*, 88 693-706.

[47] Marcet A. and R. Marimon (1999), "Recursive Contracts," *mimeo,* Universitat Pompeu Fabra, Barcelona, Spain.

[48] Mehra, R. and E. C. Prescott (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics,* 15, March: 145-161.

[49] Meyer, B. D. (1990), "Unemployment Insurance and Unemployment Spells," *Econometrica*, 58 (4), July: 757-782.

[50] Milgrom, P. (1999), "Envelope Theorems," *Working Paper 99-016*, Stanford University.

[51] Mincer, J. and B. Jovanovic (1981), "Labor Mobility and Wages," In Rosen, S.S. (Ed.), *Studies in Labor Markets.* University of Chicago Press for National Bureau of Economic Research: 21-63.

[52] Nickell, S.J. (1979), "Estimating the probability of leaving Unemployment," *Econometrica*, 47:1249-1265.

[53] Pavoni, N. (1999), *Recursive Methods and Dynamic Contracts Under Complete Information: Theory and Applications,* Ph.D. dissertation, in Mathematics for Economic Decisions, January, University of Trieste, Italy.

[54] Pavoni, N. (2001), "On Optimal Unemployment Compensation," *mimeo,* UCL, London.

[55] Phelan, C. (1994), "Incentives and Aggregate Shocks," *Review of Economic Studies*, 61: 681-700.

[56] Phelan, C. (1995), "Repeated Moral Hazard and One-Sided Commitment," *Journal of Economic Theory,* 66(2): 488-506.

[57] Phelan, C. and R.M. Townsend (1991), "Computing Multi-Period, Information-Constrained Optima," *Review of Economic Studies*, 58: 853-881.

[58] OECD (1994), "*The OECD Jobs Study: Evidence and Explanations. Part II: The Adjustment Potential of the Labor Market,*" Paris: OECD.

[59] Reissert, B. and G. Schmid (1994), "Unemployment Compensations and Active Labor Market Policy: The Impact of Unemployment Benefits on Income Security, Work Incentives and Public Policy," in *Labor Market Institutions in Europe*, G. Schmid Ed., Sharpe, US.

[60] Rogerson, W. (1985a), "Repeated Moral Hazard," *Econometrica*, 53: 69-76.

[61] Rogerson, W. (1985b), "The Firs-Order Approach to Principal-Agent Problems," *Econometrica*, 53: 1357-1368.

[62] Ruhm, C.J. (1987), "The Economic Consequences of Labor Mobility," *Industrial and Labor Relations Review*, 41(1): 30-49.

[63] Ruhm, C.J. (1990), "Do Earnings Increase with Job Seniority?" *Review of Economics and Statistics,* 72, February: 143-147.

[64] Ruhm, C.J. (1991), "Are Workers Permanently Scarred by Job Displacement?" *American Economic Review*, 81(1), 319-324.

[65] Shavell, S. and L. Weiss (1979), "The Optimal Payment of Unemployment Insurance Benefits Over Time," *Journal of Political Economy,* 87, December: 1347-1362.

[66] Sinfield, A. (1981), *What Unemployment Means*, Oxford University Press: Martin Robertson Ed.

[67] Spear, S.E. and S. Srivastava (1987), "On Repeated Moral Hazard with Discounting," *Review of Economic Studies,* 53: 599-617.

[68] Stokey N.L., and R.E. Lucas, with E. C. Prescott (1989), *Recursive Methods in Economic Dynamics,* Cambridge (MA), Harvard University Press.

[69] Swaim, P.L. and M.J. Podgursky (1991), "Displacement and Unemployment," In; Addison, J.T. (Ed.), *Job Displacement.* Wayne State University Press: 136-161.

[70] Thomas, J. and T. Worral (1990), "Income Fluctuations and Asymmetric Information: An Example of Repeated Principal-Agent Problem," *Journal of Economic Theory, 51:* 367-390.

[71] Usami, Y. (1983) "Payroll-Tax Financed Unemployment Insurance with Human Capital," *Ph.D. dissertation, Massachusetts Institute of Technology.*

[72] Vishwanath, T. (1989), "Job Search, Stigma Effect, Escape Route from Unemployment," *Journal of Labor Economics*, 7(4), October.

[73] Wang, C. and S. Williamson (1996) "Unemployment Insurance with Moral Hazard in a Dynamic Economy," *Carnegie-Rochester Conference Series on Public Policy*, 44: 1-41.

[74] Wang, C. and S. Williamson (1999) "Moral Hazard, Unemployment Insurance and Experience Rating," *Working Paper A1.156*, March, University of Iowa, USA.

[75] Yoshiaki, O. (1997), "Stigma Effects of Nonemployment," *Economic Inquiry*, April.

8 Appendix

8.1 Proofs of Section 3

Proof of Theorem 1 First we simplify the notation by eliminating the H indexation. It will become clear below that the continuity of $m(0,\cdot)$, together with Lemma 9.5 of Stokey and Lucas (1989), allows us to do this simplification, at this stage. Define $s = (\mathbf{a}, U)$ with $\mathbf{a} = \{a(0), a(1), a(2).....\} = \{a(0),_1 \mathbf{a}\} \in \mathcal{A}$ and $\|s - s'\| = \|\mathbf{a} - \mathbf{a}'\|_\delta + |U - U'|$. Using the promise keeping constraint we rewrite the Bellman operator T as follows

$$(TV)(s) = \sup_{U^u, U^e;\, sub(2)} -u^{-1}\left(U - C(a(0)) - \beta\left[P(a(0))U^e + (1 - P(a(0)))U^u\right]\right) +$$

$$+\beta\left[P(a(0))W(U^e) + (1 - P(a(0)))\, V({}_1\mathbf{a}, U^u)\right]$$

First, we show that the operator T maps bounded and continuous functions into itself. From the definition of continuity, we must verify that for each given point s and for each $\varepsilon > 0$, there exists a $\gamma > 0$ such that

$$\text{if } \|s - s'\| < \gamma \quad \text{then} \quad |(TV)(s) - (TV)(s')| < \varepsilon.$$

To this extent, we rewrite the previous condition using the definition of the Bellman operator T

$$\left| \sup_{U^u, U^e;\, sub(2)} g(U, U^u, U^e, a(0)) + \beta\left(1 - P(a(0))\right) V({}_1\mathbf{a}, U^u) + \right.$$

$$\left. - \sup_{U^u, U^e;\, sub(2)} g(U', U^u, U^e, a'(0)) + \beta\left(1 - P(a'(0))\right) V({}_1\mathbf{a}', U^u) \right| < \varepsilon \tag{16}$$

where

$$g(U, U^u, U^e, a(0)) = -u^{-1}\left(U - C(a(0)) - \beta\left[P(a(0))U^e + (1 - P(a(0)))U^u\right]\right) + \beta P(a(0))W(U^e)$$

and

$$W(U^e) = \frac{S - u^{-1}((1-\beta)U^e + l)}{1 - \beta}.$$

Now consider the two cases. Case 1: Suppose that $a(0) = a'(0)$. In this case, we can assume $a(0)$ as a parameter of the problem, and apply the Maximum Theorem to the problem

$$\begin{aligned} W(U,_1\mathbf{a}) &= \sup_{U^u, U^e} g(U, U^u, U^e, a(0)) + \beta\left(1 - P(a_0)\right) V({}_1\mathbf{a}, U^u) \\ sub &: (2),\, U^u, U^e \in \Gamma(U) \end{aligned} \tag{17}$$

to show continuity of W in $(U,_1\mathbf{a})$. The auxiliary constraint $\Gamma(U)$ is imposed in order to guarantee the constraint correspondence to be compact valued. A possibility is the following. The incentive compatibility constraint (2) requires $U^e \geq U^u + f(a(0))$, so we can always choose appropriately two constants $k_1, k_2 > 0$, and add to (2) the constraints $U^u \geq U - k_1$ and $U^e \leq U + k_2$. The continuity of W implies that we can always find a γ such that (16) is verified. Case 2: Now suppose $a(0) \neq a'(0)$. The idea here is that we do not check for continuity in this case, that is, we set γ such that, whenever $a(0) \neq a'(0)$, then $\|s - s'\| > \gamma$.

This can always be done since $|\alpha_i - \alpha_k| > 0$ for any i, k with $k \neq i$. To see why, notice that the extreme case is when the two sequences differ only in period zero, and $|a(0) - a'(0)| = \Delta_{\min}$ where $\Delta_{\min} = \min_{i,k,i\neq k} |\alpha_i - \alpha_k|$ is the minimal distance between the actions. Since the two sequences are identical from period one on we can only say that, $\|s - s'\| \geq \Delta_{\min} > 0$, but this is enough. In summary, the choice of γ is done according to the continuity properties of W, with the restriction $\gamma < \Delta_{\min}$.

Since u^{-1} is bounded, if we start from a bounded V then TV will remain bounded.

Now, it can be checked directly that the operator satisfies the Blackwell's sufficient conditions, thus T defines a contraction in the complete metric space of the bounded and continuous functions with the sup norm, in the space "reduced" space $S = \mathcal{A} \times \mathcal{U}$. The continuity of the low $m(0, \cdot)$, allow us to complete the proof by applying Lemma 9.5 and Theorem 9.6 of Stokey and Lucas (1989) which guarantee that the contraction mapping result is still true in the original space $\mathcal{A} \times \mathcal{U} \times \mathcal{H}$, with human capital. ■

Proof of Theorem 2 Given the continuity result we obtained in Theorem 1, to show Theorem 2 it suffice to show the compactness of $\mathcal{A}$. That is

Lemma 2 *$\mathcal{A}$ is compact in topology induced by the metric $d_\delta(x, y)$ for any $\delta \in (0, 1)$.*

Our strategy of proof is as follows. We start by noting that if the set of search effort levels A is made of only two elements (i.e. if $N = 2$), then the set of sequences $\mathcal{A}$ corresponds to the Cantor set, which is known to be compact in the topology induced by the metric $d_\delta(x, y)$ for $\delta = \frac{1}{3}$. From this we then show that the Cantor set is topologically equivalent to the set $\mathcal{A}$ with $N > 2$, with the topology induced by a generic $\delta \in (0, 1)$.

Before starting the proof we recall a couple of definitions and a known result.

Definition 2 *A property $\mathcal{P}$ is said topological or topologically invariant, if when the topological space (X, τ) has this property, then also each homeomorfic space (Y, τ') has this property.*

Definition 3 *X and Y are said to be homeomorfic or topologically equivalent if there exists a bijective (one-to-one and onto) function $f : X \to Y$ such that both f and f^{-1} are continuous. The function f is said homeomorfism.*

Theorem 5 *A one-to one continuous function from a compact space onto a Hausdorff space is a homeomorphism.*

Proof. See Theorem 2.34 in Aliprantis and Border (1994), page 41. ■

Now we are ready to start the proof.

Step 1. It is easy to see that, if A is made of two elements, then $\mathcal{A}$ is the Cantor set $\Delta \equiv \{0, 1\}^{\mathbb{N}}$. If we start by endowing the set $\{0, 1\}$ with the discrete topology (which is both compact and Hausdorff), then it is well known that the metric $d_\delta(x, y) = \sum_{n=0}^{\infty} \delta^n |x(n) - y(n)|$ with $\delta = \frac{1}{3}$ induces the product topology on Δ (see, for example, Aliprantis and Border 1994, page 93) so from the properties of the Hausdorff spaces and using the Tychonoff Product Theorem we have that Δ is both Hausdorff and Compact.

Step 2. We are now going to show that Δ with the topology induced by the metric $d_{\delta=\frac{1}{3}}$ is homeomorphic to the topological space (Δ, τ_δ) where τ_δ is the topology induced by the d_δ metric with a generic $\delta \in (0,1)$. In particular we are going to show that the identity function from (Δ, τ_δ) onto $(\Delta, \tau_{\delta'})$ defines an homeomorphism. So consider the function $f : \Delta \to \Delta$ that maps $f(x) = x$ for any $x \in \Delta$. Of course, this function is one-to one and onto. It suffice now to show that f is continuous. Since any metric space is Hausdorff, we can then recall Theorem 5 to complete the proof.

To show that f is continuous we have to show that for any $x \in \Delta$ and any $\varepsilon > 0$ there exists a $\gamma > 0$ such that if $d_\delta(y,x) < \gamma$ then $d_{\delta'}(f(y), f(x)) = d_{\delta'}(y,x) < \varepsilon$. This can easily done as follows. First, notice that for any $\varepsilon > 0$ it is always possible to find a $T < \infty$ such that if $y(t) = x(t)$ for any $t \leq T$ then $d_{\delta'}(y,x) < \varepsilon$. Indeed if x and y are such that $y(t) = x(t)$ for any $t \leq T$, then $d_{\delta'}(y,x) \leq \frac{(\delta')^T}{1-\delta'}$, so we are done. Second, notice that for any T there exists a $\gamma > 0$ such that if $d_\delta(y,x) < \gamma$ then the first T elements are identical. Indeed, writing the distance in extensive form

$$d_\delta(y,x) = \sum_t \delta^t |y(t) - x(t)|$$

with $y(t),\ x(t) \in \{0,1\}$, is easy to see that the minimum distance between two sequences such that they does not meet the requirement is where they differ only in the T position element. In this case, the distance between the two sequences is at least $\delta^T > 0$. So with $\gamma \leq \delta^T$ we are done.

Step 3. Finally, we are going to show that (Δ, τ_δ) is homeomorphic to any $(\mathcal{A}, \tau_\delta)$ where $\mathcal{A}$ is generically defined with $N > 2$, and τ_δ is the topology induced by the d_δ metric. To see how the proof goes we consider the extension to A made of 4 elements. The extension to any other different number of elements can be made similarly.[32] Divide $\mathbb{N}$ in odd and even numbers. Now construct the continuous, one-to one and surjective map between $A = \{\alpha_1, \alpha_2, \alpha_3.\alpha_4\}$ and the two dimensional vectors of $\{0,1\}$ as follows: $f(\alpha_1) = (0,0)$, $f(\alpha_2) = (0,1)$, $f(\alpha_3) = (1,0)$, $f(\alpha_1) = (1,1)$. So we have a map between any $a \in \mathcal{A}$ and a $x \in \Delta^2$. Take a generic element $x \in \Delta^2$ $x = \{x(0), \ldots.., x(t), \ldots.\}$ with $x(t) = (x_1(t), x_2(t)) \in \{0,1\}^2$ so that $x_k(t) \in \{0,1\}$ for $k \in 1,2$. Now set $b(n_t^k) = x_k(t)$ where $k = 1$ corresponds to odd numbers and $k = 2$ corresponds to even numbers. So

$$g(x) = g(x(0), \ldots.., x(t), \ldots.) = b = (b(0), \ldots.., b(t), \ldots.)$$

is our map. This map, by construction, is one-to-one and surjective. What we need to show that its inverse is also continuous. This will again be enough, since if its inverse is continuous, we know that it maps a compact set Δ into the Hausdorff space $\mathcal{A}$, so to complete the proof it will suffice to recall Theorem 5 above.

So, consider the inverse function $h = g^{-1} \circ f^{-1} : \mathcal{A} \to \Delta$. We want to show that for any b is true that for any $\varepsilon > 0$ there exists a $\gamma > 0$ such that $d_\delta(y,b) < \gamma$ implies $d_\delta(h(y), h(b)) = d_\delta(h(y), a) < \varepsilon$.

The idea is exactly as before. First, for each $\varepsilon > 0$ there exists an $T < \infty$ such that if $a'(t) = a(t)$ for any $t \leq T$ then $d_\delta(h(y), h(b)) = d_\delta(a', a) < \varepsilon$. Indeed this is true since, if a'

[32] Notice that, by repeatedly applying the same line of proof, we can increase the number N without limit.

and a are such that $a'(t) = a(t)$ for any $t \leq T$, then $d_\delta(a', a) \leq \delta^T \frac{\alpha_N - \alpha_1}{1-\delta}$, so we are done. Second, we have to show that for any T there exists a $\gamma > 0$ such that if $d_\delta(y, b) < \gamma$ then the first $2T$ elements are identical.[33] Writing the distance in extensive form

$$d_\delta(y, b) = \sum_t \delta^t |y(t) - b(t)|$$

with $y(t),\ b(t) \in \{0, 1\}$, it should now be clear that the minimum distance between two sequences such that they does not meet the requirement is when they differ only in the $2T - 1$ element of the sequence. So the distance between the two sequences is at least $\delta^{2T-1} > 0$, so we are done. ■

Given that existence is proved, it is then easy to combine the continuity and boundedness results of Theorem 2 to show the equivalence between the sequential and the recursive choice of actions. ■

Proof of Proposition 1 It easy to see that the presence of the index H creates only notational complications, so we fix H and eliminate the H index in what follows. Following Grossman and Hart (1983) and changing variable defining $z \equiv u(b)$ the problem becomes

$$\begin{aligned} V(\mathbf{a}, U) &= \sup_{z, U^u, U^e} -u^{-1}(z) + \beta \left[P(a) W(U^e) + (1 - P(a)) V({}_1\mathbf{a}, U^u) \right] \\ s.t. &: z - C(a) + \beta \left[(1 - P(a)) U^u + P(a) U^e \right] \geq z - C(\hat{a}) + \beta \left[(1 - P(\hat{a})) U^u + P(\hat{a}) U^e \right] \\ U &= z - C(a) + \beta \left[(1 - P(a)) U^u + P(a) U^e \right] \end{aligned} \tag{18}$$

where a is the first element in the sequence $\mathbf{a} = \{a(n)\}$. Notice that (since $u^{-1}(.)$ is convex) the objective function is concave and that the constraints set is convex (linear).

Differentiability can be shown as follows. Given the value function is concave we can use Lemma 2 in Benveniste and Scheinkman (1979). For a fixed level of promised utility U_0 we are looking for a differentiable concave function $G(\mathbf{a}, U)$ such that it is well defined in an interval I around U_0 and such that for any $U \in I$ we have $G(\mathbf{a}, U) \leq V(\mathbf{a}, U)$ and $G(\mathbf{a}, U_0) = V(\mathbf{a}, U_0)$. We claim that

$$G(\mathbf{a}, U) = -u^{-1} \left(U + C(a) - \beta \left[(1 - P(a)) U^u + P(a) U^e \right] \right) + \beta \left[p W \left(U_0^e \right) + (1 - p) V \left({}_1\mathbf{a}, U_0^u \right) \right]$$

is the function we are looking for. Indeed, the optimal values U_0^e and U_0^u satisfy the incentive compatibility and the promise keeping constraint can always be satisfied by varying the benefit transfer b, so the function G is well defined and we have $G(\mathbf{a}, U) \leq V(\mathbf{a}, U)\ \forall\ U \in I$ as required. The properties of u imply the concavity and differentiability of $-u^{-1}$. So G is concave and differentiable, and this implies that V is differentiable in U_0 and $V(\mathbf{a}, U_0) = -\frac{1}{u'(b_0)}$. Since V is concave then it is continuously differentiable. ■

Proof of Proposition 2 Set $H = H_0$ and consider period $T(H_0) = \max \{T_S(H_0), T_p(H_0)\}$ as in Assumption **A2.** By assumption, from $T(H_0)$ onwards, the planner face a stationary problem, as the one of Pavoni (2001). Then, either *(i)* we have full-insurance $(a(T(H_0)) =$

[33] Recall we are still considering the case with A made of four elements.

$\alpha_0 = 0$), and the utility is not changing, thus we will remain in the full-insurance state forever; or *(ii)* we have $a(T(H_0)) \neq 0$ and in Pavoni (2001) we have shown that the expected discounted utility is decreasing. Thus, the only action path consistent with assumption **A1** in this case is a monotone one, thus it has to converge. Given the available actions set is finite ($N < \infty$), we can without loss of generality assume that after a finite $T^* \geq T(H_0)$ the sequence of actions is constant at $\mathbf{a} = \{\alpha_i, \alpha_i, \alpha_i, \alpha_i, ...\}$. So the set of maximal actions $\mathcal{A}^*(H_0)$ for the period-zero problem, is a subset of the *finite* set of all the sequence of actions which end - after $T^* < \infty$ periods - either with the limiting sequence $\mathbf{a}$ or with the sequence $\mathbf{0} = \{0, 0, 0, 0.....\}$.

From Proposition 1, $V'(\mathbf{a}, U, H) \equiv \frac{\partial V(\mathbf{a},U,H)}{\partial U}$ exists and it is continuous in U. Since $\mathcal{A}^*(U, H)$ is finite, $V'(\mathbf{a}, U, H)$ is also jointly continuous in $(\mathbf{a}, U)$. ■

Proof of Proposition 4 That (8), (9) and (10) are the first order conditions is trivial. Where $V'(U, H) = -\frac{1}{u'(b(U,H))}$ is justified by Proposition 1. So what we have to show is that: (i) actually, we can take the first order conditions, that is the differentiability conditions for taking those FOC are satisfied. Moreover (ii) that $\mu \geq 0$ as claimed.

(i) To simplify the exposition, we rewrite the incentive constraint (2) as follows

$$U^e - U^u \geq \frac{C(a) - C(\hat{a})}{\beta(P(a, H) - P(\hat{a}, H))} \equiv K(a, \hat{a}, H) \quad \text{for any } \hat{a} \in A, \ \hat{a} \neq a. \tag{19}$$

First, notice that, by the strict monotonicity of $C(\cdot)$ and $P(\cdot, H)$, $K(a, \hat{a}, H)$ is well defined and strictly positive. Now, to verify (i) when $a = 0$ is simple and - since is similar to the Case 2 below- is left to the reader. If $a \neq 0$, that is if the implemented action is not the cheapest one, then we can have two cases. Case 1: The maximum is such that the incentive constraint (19) satisfied with equality with respect to some deviation $\hat{a}$. If we rewrite the objective function using (19) with equality for action $\hat{a}$, and (3) we can rewrite the problem as a function of U^u alone

$$\begin{aligned}&\sup_{U^u} \; -u^{-1}\left(U + C(a) - \beta\left[U^u + P(a, H)K(a, \hat{a}, H)\right]\right) + \\ &+\beta\left[P(a, H)W(U^u + K(a, \hat{a}, H)) + (1 - P(a, H))\, V(U^u, H')\right].\end{aligned}$$

The problem is now a free maximization whose objective function is a weighted sum between the differentiable functions u^{-1} and W, and the function $V(U^u, H')$. We can then apply directly Lemma 1 to this problem. Case 2: The optimum is such that the incentive constraint (19) is slack. In this case, we can use (3) to rewrite the problem as a function of both U^u and U^e as follows

$$\begin{aligned}&\sup_{U^u} \; -u^{-1}\left(U + C(a) - \beta\left[U^u + P(a, H)\left(U^e - U^u\right)\right]\right) + \\ &+\beta\left[P(a, H)W(U^e, H) + (1 - P(a, H))\, V(U^u, H')\right].\end{aligned}$$

The choice of U^e is clearly well defined since both u^{-1} and W are differentiable everywhere. For any give choice of U^e, the optimal level U^u is now computed by solving again a *free* maximization over a weighted sum between the differentiable function u^{-1} and the function $V(U^u, H')$, thus Lemma 1 applies also to this case.

(ii) If - for each deviation $\hat{a} \neq a$ - we define $\mu'_{\hat{a}} \geq 0$ the Kuhn Tucker multiplier associated with (19).[34] We have that $\mu \equiv \frac{1}{\beta P(a,H)} \sum_{\hat{a} \in A, \hat{a} \neq a} \mu'_{\hat{a}}$ which is obviously non-negative as claimed. ■

8.2 Proofs of Section 4

Proof of Proposition 4 First, from $F = 0$ it is easy to see that the optimal contract in the employment state problem implies full insurance and the value function W is trivially given by (14) for any range of utilities.

To simplify the verification process, we first rewrite the functional equation associated with the stationary problem. When H is constant V solves the following functional equation

$$V(U) = \max_{a \in A} \{V_a(U)\}$$

where V_a is the today's value for implementing action $a \in A$. And is given by

$$\begin{aligned} V_a(U) &= \sup_{b, U^u, U^e} -b + \beta \left[P(a) W(U^e) + (1 - P(a)) V(U^u) \right] \\ & \quad s.t. \text{ (2) and (3).} \end{aligned}$$

with $W(U)$ given by (14). Now, to verify, the function V in (13) solves the functional equation it suffice to use the usual first order conditions, which are justified by the analysis in Section 3.

Following a similar argument as the one in Pavoni (2001) it is also easy to verify that all the conditions to apply Theorem 4.3 of Stokey et al. (1989) are verified, so V is actually the vale function for our problem.

Computing the closed form for the non-stationary model (when H is changing) is not simple. The main complication, is that, in each period t, the multiplier associated with the incentive compatibility constraint can be either positive or zero. And the way of computing the B_n coefficients vary considerably between the two cases. However, the verification process can be done by using the first order conditions together with the algorithm we give below.

For didactical reasons, we now present the details of the verification process, with the algorithm for computing the optimal program, for the specific case when there are only two actions. $a_t \in \{0, 1\}$. The extension to the finitely many actions case is immediate.

Full-insurance regime One reason for $\mu = 0$ is that, in some period t, it is not optimal to give incentives, thus, $a_t = 0$ and the worker is fully-insured. This can always occur when the utility level U_t is sufficiently high. In this case, $B(H_t)$ equals $B_0(H_t) = L^{\frac{\beta P(0,H_t)}{1-\beta(1-P(0,H_t))}}$ and the corresponding payments are $w_{t+s} = b_{t+s} = b_t$ for any $s \geq t$.

Incentive binding regime For low levels of utility, and when the hazard rate does not decreases too fast with unemployment duration, the incentive compatibility constraint

[34] Notice that (19) is linear for any $\hat{a}$, hence satisfies the constraint qualification requirement needed to apply the (local) Kuhn-Tucker theorem.

(2) is binding ($\mu > 0$). In this case, given $B(H_{t+1})$, the new value $B(H_t)$ is derived according to the non-stationary counterpart of (15), i.e.

$$B(H_t)^{\frac{1}{\beta}} = [\exp\{D(H_t)\}]^{\frac{1-\beta}{\beta}} [P(1,H_t)\exp\{l\}\exp\{(1-\beta)E(H_t)\} + (1-P(1,H_t))B(H_{t+1})], \tag{20}$$

where $H_{t+1} = m(0,H_t)$, $E(H_t) = \frac{c}{\beta(P(1,H_t)-P(0,H_t))}$, and $D(H_t) = \beta P(0,H_t)E(H_t)$.

Incentive-slack regime From the policy functions and the envelope conditions, we can derive the following relationship between unemployment benefit payments in two successive periods

$$\frac{b_{t+1}}{b_t} = \frac{B(H_{t+1})}{B(H_t)} B(H_t)^{-\frac{1-\beta}{\beta}} [\exp\{D(H_t)\}]^{\frac{1-\beta}{\beta}}. \tag{21}$$

The first term $\frac{B(H_{t+1})}{B(H_t)}$ in the right hand side, comes from the non-stationarity of the model, and for the fully-stationary model it is equal to one. The second multiplicative expression in (21), i.e. $B(H_t)^{-\frac{1-\beta}{\beta}} [\exp\{D(H_t)\}]^{\frac{1-\beta}{\beta}}$, corresponds to the decrease in the utility level during unemployment. In Corollary 1 we showed that $\frac{b_{t+1}}{b_t} \leq 1$, and that we can have only two possibilities. Thus, (i) if $\frac{b_{t+1}}{b_t} < 1$, then the incentive compatibility constraint is binding and we are in the previous regime. Moreover, (ii) it cannot be that $\frac{b_{t+1}}{b_t} > 1$, so the only way to give incentives in this case is to make the incentive compatibility constraint verified with strict inequality (slack). In this last case, the optimal contract implies $w_{t+1} = b_t = b_{t+1}$ and the coefficient $B(H_t) \neq B_0(H_t)$ is computed from the promise keeping constraint (3).

The algorithm The algorithm we propose to compute the multiplicative coefficients and the optimal program is a backward one. Hence we need a starting point. In fact, the final sequence has to be computed using a shooting argument.

Given two initial levels U_0 and H_0, the process goes as follows. First, Assumption **A2** implies that there will be a $T = T(H_0) = \max\{T_p(H_0), T_S(H_0)\}$ such that either $B(H_T) = B_0(H_T) = L^{\frac{\beta\underline{\hat{p}}}{1-\beta(1-\underline{\hat{p}})}}$ or solves (15) for $p = \underline{p}$ and $\hat{p} = \underline{\hat{p}}$. Second, start by computing $B(H_{T-1})$ using (20), and compute the ratio $R_T = \frac{b_T}{b_{T-1}}$ using (21). If $R_T < 1$, then we can go to the next step, otherwise we set $b_{T-1} = b_T$ and compute $B(H_{T-1})$ using the promise keeping constraint. Third, a sequence of coefficients $B(H_t)$ implies also both a sequence of expected wages $\mathcal{S}(H_t)$ - which is computed using the hazard rates implied by the effort levels relative to each corresponding regime - and a sequence of utility levels. Thus, we now can check whether, for some t, it is optimal to switch to the full-insurance regime (by computing the threshold utility level). If this is true for more than one period, then define $t^* = \max \mathcal{T}$ - where $\mathcal{T}$ is the (finite) set of all such switching points -, set $B(H_{t^*}) = B_0(H_{t^*})$ and start again from $t^* < T(H_0)$ to compute the new coefficients. When a "consistent" sequence of coefficients is obtained, then we are done. Obviously this can be done for each initial levels of worker's expected discounted utility U_0 and human capital endowment H_0. ■

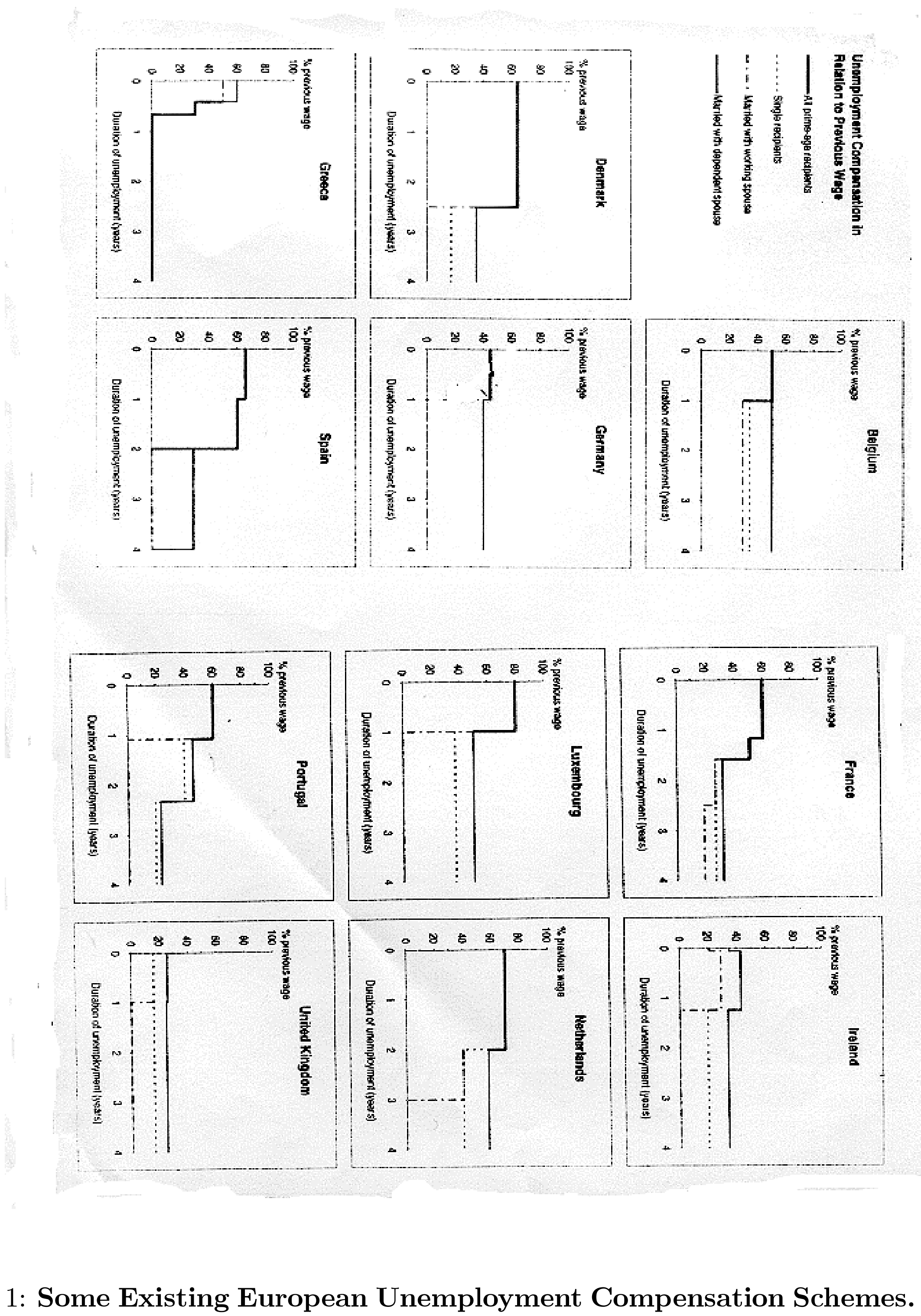

Figure 1: **Some Existing European Unemployment Compensation Schemes.** *Source* Reissert, B. and G. Schmid (1994), "Unemployment Compensations and Active Labor Market Policy: The Impact of Unemployment Benefits on Income Security, Work Incentives and Public Policy," in *Labor Market Institutions in Europe*, G. Schmid Ed., Sharpe, US.

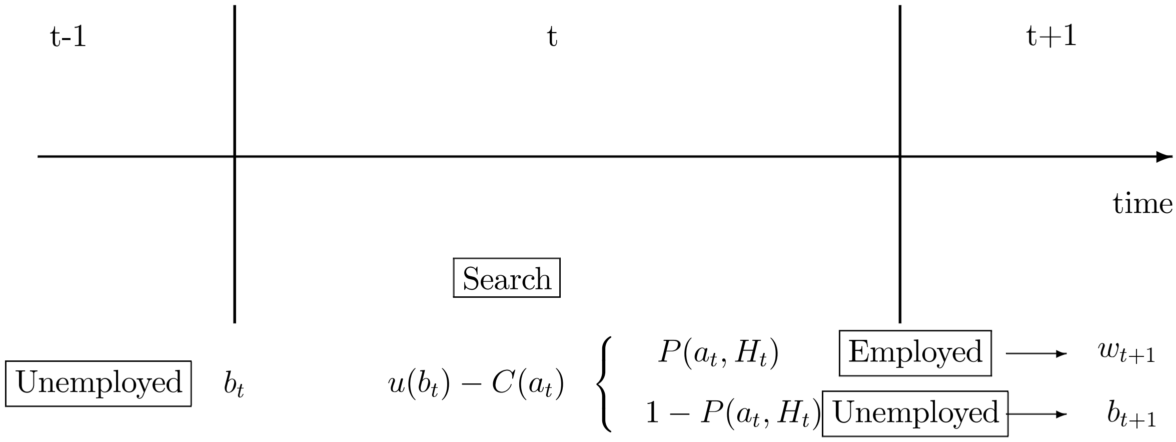

Figure 2: **The timing of the model in the unemployment state.** At the beginning of period t the unemployed worker receives the unemployment benefit b_t and is required to supply the costly job-search effort a_t. This effort affects the probability of being employed at the beginning of the next period $P(a_t, H_t)$. If at the beginning of next period the worker is employed he receives a net wage $w_{t+1} = S(H_{t+1}) - \tau_{t+1}$ otherwise, if he still unemployed, the worker receives the unemployment benefit b_{t+1}, and so on.

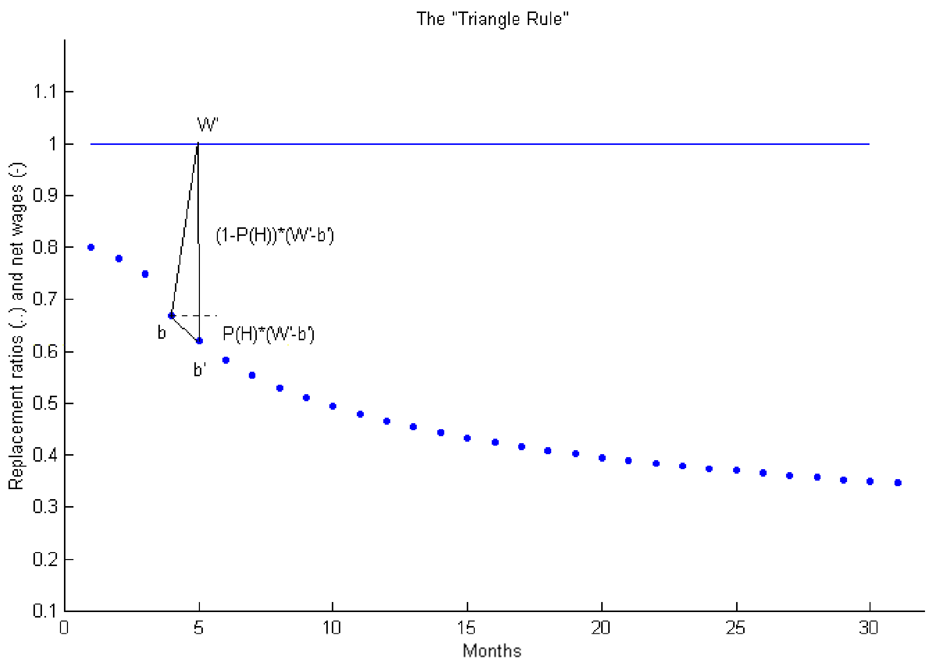

Figure 3: An example of the triangle rule.

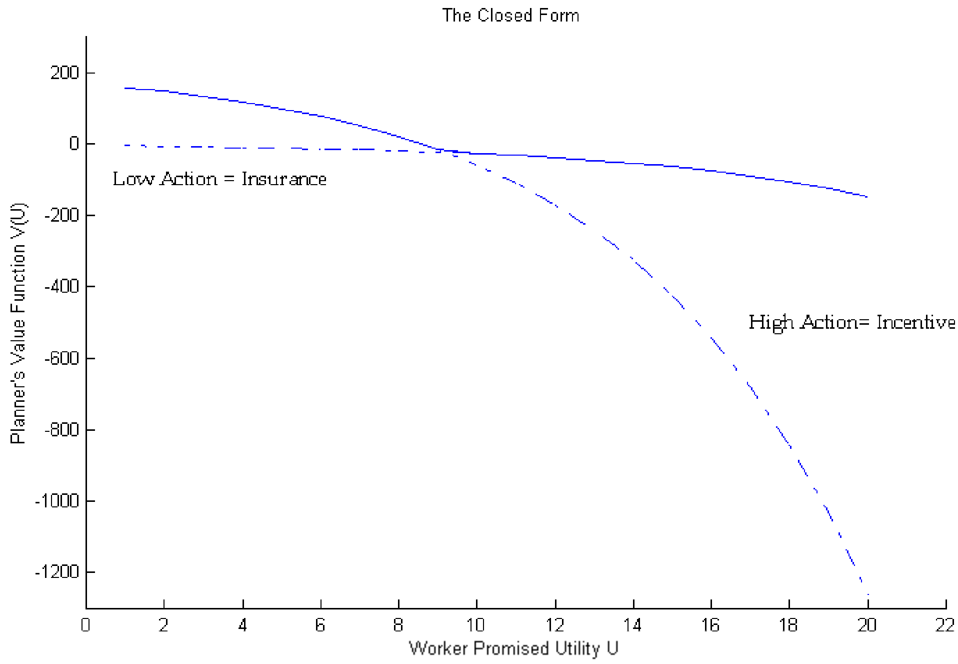

Figure 4: A parametrized example of the Closed form

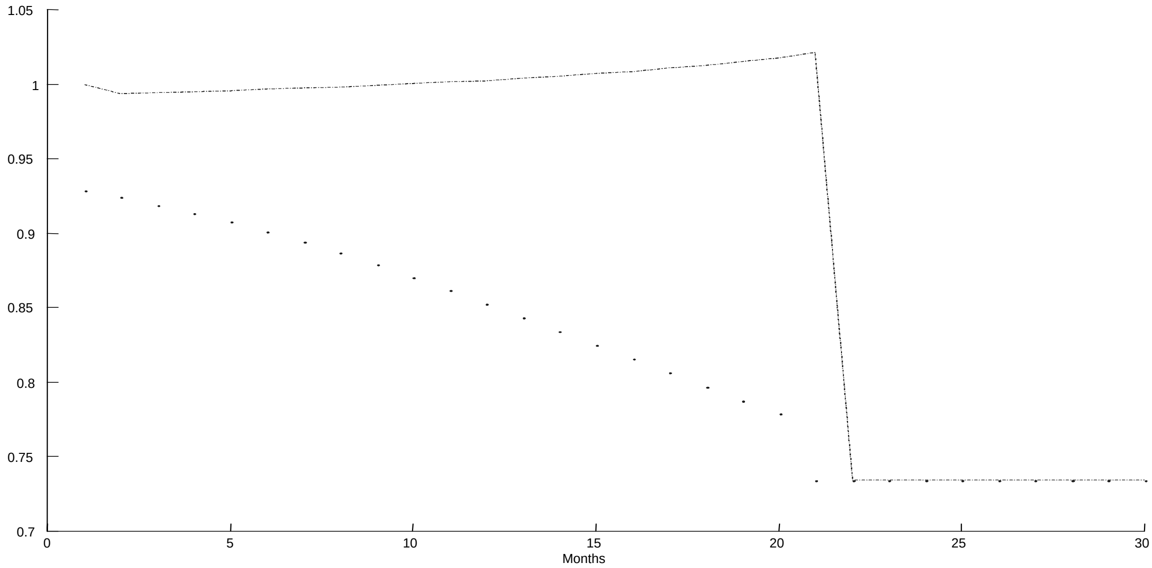

Figure 5: The Spanish Example

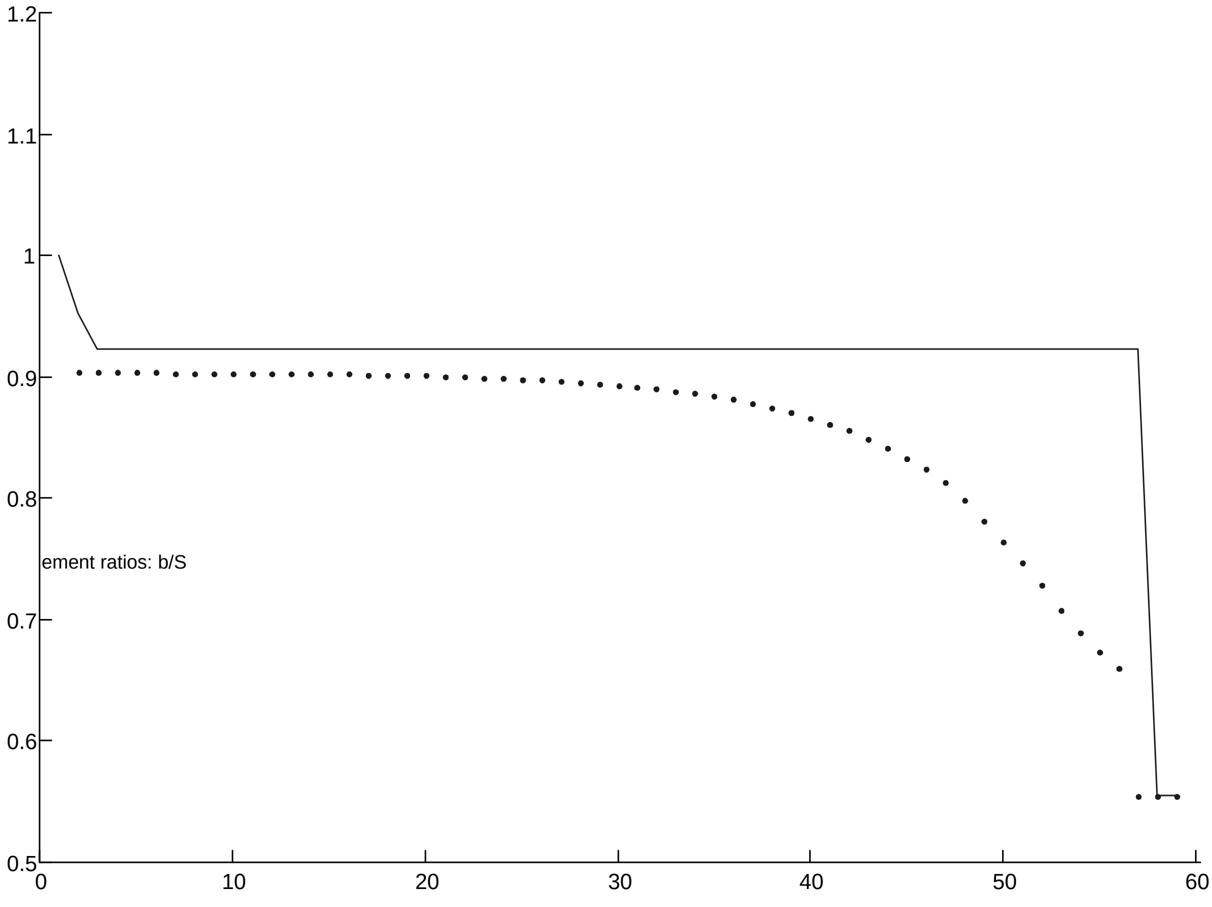

Figure 6: The US Example

Table 1 Selected features of unemployment benefit programmes

Country	Benefit type	Prior employment qualification	Fixed or earning related payment	Payment taxable[1]	Maximum duration of payment
Australia	UA	Nil	Fixed	Yes	Indef
Austria	UI	26 wks/12 mths; 52 wks/2 yrs if first claim, max benefit duration if 156 wks/3 yrs	ER	No, UI and UA set as % of after-tax income	30wk[2]
	UA	Nil	ER		Indef
Belgium	UI	312 days/18 mths[3]	ER[4]	Yes	Indef
Canada	UI	420-700 hrs/yr	ER	Yes	45 wks
	SW	Nil	Fixed		Indef
Czech Republic	UI	12 mths/3 yrs	ER	No	26 wks
Denmark	UI	26 wks/3 yrs	ER	Yes	5 yrs
Finland	UI	26 wks/2 yrs	ER	Yes	500 dys in 4 yrs
	UA	Nil	Fixed	Yes	Indef
France	UI	91 days/12 mths	ER[5]	Yes	60 mths
	UA	5 yrs/10 yrs	Fixed		Indef
Germany	UI	360 days/3 yrs	ER	No	78-832 weekdays[7]
	UA	150 days/yr or exhausted UI benefits	Fixed	No[6]	Indef
Greece	UI	125 days/14 mths	ER[8]	Yes	12 mths
	UA	60/days/2yrs	ER	Yes	3 mths
Hungary	UI	48 mths emp	ER[9]	Yes	2 yrs
Iceland	UI	400hrs/12 mths	Fixed[10]	Yes	5 yrs
Ireland	UI	39 wks/12 mths	Fixed	Yes	15 mths
	UA	Nil	Fixed	No	Indef
Italy	UI[11]	1 yr/2 yr	ER	Yes	180 days
Japan	UI	6 mths/ 12 mths	ER[12]	No	90-300 days[13]
Korea	UI	6 mths	ER	No	30-210 days
Luxembourg	UI	6 mths/12 mths	ER[14]	Yes	1 y/2yrs[15]
Netherlands	UI	26 wks/39 wks for basic benefit and 4 yrs/5 yrs for extended	ER	Yes	6 mths-4.5 yrs[16]
	UA	3 yrs/5 yrs	Fixed	Yes	12 mths
New Zealand	UA	Nil[17]	Fixed	Yes	Indef
Norway	UI	Prior earning requirement	ER	Yes	3 yrs
Poland	UI	180 days/yr[18]	ER	Yes	9-24 mths[19]
Portugal	UI	540 days/2 yrs	ER	No	10-30 mths[20]
	UA	180 days/1 yr	Fixed	No	10-30mths[21]
Slovak Republic	UI	1 yr/3 yrs	ER		6 mths

Figure 7: **Table 1.** *Source* Kalisch, D. W., T. Aman and L. A. Buchele (1998), "Social and Health Policies in OECD Countries: A Survey of Current Programmes and Recent Developments," *Labour Market and Social Policy - Occasional Papers no. 33*, 67299 and 67364. Paris: OECD.

Table 1 Selected features of unemployment benefit programmes (continued)

Country	Benefit type	Prior employment qualification	Fixed or earning related payment	Payment taxable	Maximum duration of payment
Spain	UI	12 mths/6 yrs	ER	Yes	up to 24 mths[20]
	UA	Exhausted UI or worked 6 mths	Fixed	Yes	6 - 18 mths
Sweden	UI	5 mths/12 mths	ER	Yes	300 days[21]
	SW	Nil	Fixed	Yes	150-450 days[21]
Switzerland	UI	6 mths during a base period of 2yrs	ER	Yes	150-400 days[22]
United Kingdom	UI	1 yr/2 yrs[23]	Fixed	Yes	26 wks
	GI	Nil	Fixed	Yes	Indef
United States[24]	UI	Yes[25]	ER	Yes	26 wks

urce: US Social Security Administration (1997), *Social Security Programs Throughout the World*, OECD(1998), *Benefits and Incentives in OECD countries 1995*, OECD *Caring World Synthesis Questionnaire responses*. *OECD Economic Surveys* (various issues).

1. While payment may be treated as taxable income, tax scales may mean no tax is payable on this income.
2. In special cases, up to 52 weeks. If minimum employment contribution duration only 20 weeks.
3. Qualifying conditions rise with age of claimant up to 600 days in last 36 months.
4. Earnings related ratio declines after initial 12 months, with supplement for those with dependants, term fixed rate thereafter 30% replaceable when annual income exceeds 1.5 times maximum insurable earnings. Remainder taxable.
5. Earnings related amount declines incrementally as unemployment duration increases
6. UI and UA set as proportion of after-tax earnings.
7. Varies according to insured employment period and age of recipient.
8. Supplement for dependants, and total amount subject to a 66.6% wage unskilled worker.
9. Earnings related declines after first yr., subject to notional minimum/maximum amounts.
10. Amount increases according to period of prior employment, supplement for dependent children.
11. Other, more generous payments are also available for some without work, such as wage supplementation payments, CIGO and CIGS, but they are not included here within the scope of unemployment benefits as the employment contract is generally not severed. Those displaced as a result of industry restructuring may also be eligible for a more generous mobility allowance.
12. Higher earnings replacement for prior lower income earners, subject to minimal/maximum limits.
13. Time increases with age, length of insurance, poor employment prospects.
14. Amount reduced if living with person whose wage exceeds 2 times social minimum wage.
15. Extension possible for further 6-12 months for hard-to-place and/or older unemployed.
16. Payment for up to 4.5yrs requires contributions for 3 yrs in last 5 yrs.
17. No employment history requirement for those having completed studies, relieved from military service, completed maternity leave or released from prison
18. Shorter period for young persons without employment history, period increases as prior employment history increases and/or firm bankrupt. 12 months is the norm for most workers (except young, women with 25 years prior employment and men with 30 years employment.
19. Maximum duration of payment depends on age of recipient.
20. Payment duration increases with prior contribution history.
21. Payable up to 450 days if age 55-64 for UI; for SW benefit payable up to 150 days if below age 55, 300 days if age 55-59 and 450 days if age 60-64.
22. Maximum payment period for passive benefits of 150 days if under age 50, 250 days if age 50-60 and 400 days if over age 60. Can be extended to 520 days in total for all through participation in active labour market programmes.
23. Special earnings related contribution history required, rather than just employment qualification.
24. States have own laws, some differences between schemes.
25. About 3/4 of States have minimum earnings requirement over last year, remainder require employment of approx. 15-20 wks in last yr.

Figure 8: **Table 1.** *Source* Kalisch, D. W., T. Aman and L. A. Buchele (1998), "Social and Health Policies in OECD Countries: A Survey of Current Programmes and Recent Developments," *Labour Market and Social Policy - Occasional Papers no. 33*, 67299 and 67364. Paris: OECD.