Does poaching distort training?

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Abstract

We analyse the labour market outcome in a competitive search equilibrium model with endogenous human capital formation and endogenous turnover. We derive under which conditions firms have the right incentives to train their employees when trained employees may quit after successful on-the-job search. If firms and their employees are able to co-ordinate efficiently and maximise their joint income, for instance by using long-term wage contracts, search frictions will not distort the training decision. In the absence of long-term wage contracts, there may be too much turnover and too little training. Still, the number of training firms and the amount of human capital investments are constrained optimal, and subsidising human capital investment will therefore reduce welfare.

Finally, the paper also extends the competitive search equilibrium model by including on-the-job search.

1 Introduction

A positive relationship between wages and experience is well documented in the empirical labour literature. This stylised fact indicates that on-the-job training is one essential determinant of worker productivity. Accordingly, the extent to which the market induces firms to invest in general and specific training is crucial for economic welfare. In addition, turnover is important for allocational efficiency, as firms (among other things) may differ in their ability to train workers and to utilise their human capital. It is well known from Becker (1964) that perfect competition leads to an efficient market outcome with respect to investment in training and turnover, provided that there are no credit constraints or minimum wages regulations.

This paper analyses the conditions under which the labour market outcome is efficient in a model with endogenous human capital formation and endogenous turnover in the presence of *search frictions*. To this end, we develop a directed search model in which worker turnover is necessary to obtain an efficient allocation of workers over firms. To be more precise, we study a model in which there exists two kinds of firms; training firms, with a comparative advantage in providing general training to their employees, and poaching firms, with a comparative advantage in utilising the resulting general human capital. Within a competitive search equilibrium framework, where workers with different productivities search in different market segments, we analyse whether training firms have the right incentives to enter the market and to provide the optimal amount of training In contrast with the existing literature, we thus analyse training in a model in which both the on-the-job search intensity undertaken by workers and the number of poaching firms are determined endogenously.

Our first main result is that a sufficient condition for an efficient allocation of resources in this economy is internal efficiency. Internal efficiency means that co-ordination problems within each firm resolved such that employer and his employees are able to maximise their joint expected income. Internal efficiency may come about if workers and firms are able to write long-term binding contracts, or if they are able to bargain efficiently.

This efficiency result contrasts sharply with Acemoglu (1997a). He finds that in the presence of search frictions, turnover creates positive training externalities from human capital investments for future employers. As a result, there is too little general training even though firms and workers can write long-term contracts. He attributes the inefficient outcome to the workers' inability to contract with future employers. Below we argue that Acemoglu's result hinges (among other things) on his assumption that workers with different productivities search in the same market segments and with the same probability rate of finding a job.

Our efficiency result also serves as a convenient starting point when introducing other imperfections than search frictions, and clearifies the reasons why such imperfections may give rise to inefficiencies. We focus on the case where internal efficiency does not hold because training firms set wages for experienced workers so as to maximise their *ex post* profit. In this case, training firms set wages for experienced workers too low, and the equilibrium turnover rate is too high and the human capital investments tend to be too small compared with the socially optimal levels.

Our second main result is that this amount of human capital is still constrained efficient in the following sense: given the entry behaviour by poaching firms, the social and the private return from training coincide. Thus, subsidising training reduces welfare. Although a subsidy increases training, it also increases the number of poaching firms entering the market, thereby exacerbating the inefficiencies created by excessive turnover.

This result also seems to contrast findings in the existing literature. Stevens (1994) argues that poaching creates a wedge between the social and the private returns from training, as long as wages are set below worker productivity. For similar reasons, Acemoglu and Pischke (1999) and Both and Snower (1995) are sympathetic to training subsidies in the absence of government failures. Finally, the view also influences the policy debate. For instance, the OECD (1995, Chapter 7) argues that poaching externalities (among other things) lead to under investment in training, thereby providing a rationale for government subsidies to training firms, for instance in the form of tax breaks for training expenses. Our paper questions this widely held policy recommendation..

The paper is organised as follows. Section 2 presents the model. Section 3 derive the equilibrium assuming internal efficiency. Section 4 analyses the model when firms set wages to experienced workers so as to maximise *ex post* profits. Section 5 discusses some robustness issues while section 6 concludes.

2 Matching technology and equilibrium concept

The matching process and the equilbrium concept that we use are important for our results. Since both are the same in all the different segments of the labour market, we find it

The model's main features, illustrated in figure 1, are as follows:

- *Workers:* There is a continuum risk neutral and *ex ante* identical workers in the economy with measure normalised to one. They enter the labour market as unemployed workers, and leave at an exogenous rate s. New workers enter the market at the same rate, keeping the total number of workers constant.

- *Firms:* There are two types of firms in the economy, training firms and poaching firms. Each firm hires at most one worker, and a novice never starts in a poaching firm. Investments in general training is undertaken by training firms only. The investment is made when the worker is a novice, and the return accrues once the worker is experienced. The productivity of a novice is y^n and that of an experienced worker with human capital level h in a training firm is $y^e(h)$. The productivity in a poaching firm is $y^p(h)$. We assume that $y^n < y^e(h) < y^p(h)$ for all $h \geq 0$.[1] Hence, a poaching firm can utilise experienced workers better than training firms. Other possible interpretations of poaching firms and training firms are given in the discussion section.

- *Time structure:* The model is set in continuous time. A worker that is hired by a training firm will stay inexperienced for a period. The natural way to model a period of time within a continuous-time framework is to let the period length be stochastic: an inexperienced worker employed in a training firm becomes experienced at a rate γ.

- *Matching technology:* The number of searching workers and firms that find each-other in the market is determined by a constant return to scale matching function $x(eu, v)$ that maps a measure of workers u searching with average search intensity e for a measure of v vacancies into a flow x of new matches. Let p and q denote the probability rate for an unemployed worker of finding a job per unit of search intensity and the probability rate that a firm with a vacancy finds a worker, respectively. Since the matching function exhibits constant returns to scale it follows that we can summarise the matching function by a function $q = q(p)$. [2] We assume that unemployed and employed workers search in different submarkets, with different wages, and do not cause congestion for each other. This will be discussed in some details below.

Competitive search equilibrium

We model the labour market as a directed search market. In a directed search market, workers know wages in all firms prior to search, through reputation effects or because firms advertise wages when posting their vacancies. Thus, it is assumed that search frictions are due to other aspects of the search process than collecting information on wages, such as the costs and time delays associated with writing and processing applications, identifying firms with vacancies, the selection of workers by firms etcetera. The competition among workers to obtain a job

[1] This is actually stronger than necessary, we may have that $y^e(h) > y^p(h)$ for some levels of h. However, we would then have too keep track of which workers do on-the-job search and which workers do not, without bringing in new insights.

[2] To see this, note that $p = x(eu, v)/eu = x(1, \theta) = p(\theta)$ where $\theta = v/eu$ is the labour market tightness. At the same time, $q = x(eu, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(p^{-1}(p)) = q(p)$.

in a high-wage firm is harder than to obtain one in a low-wage firm, and the probability rate p that a worker finds a job (with unit search intensity) may therefore depend on the wage w in the firms the worker applies for; $p = p(w), p'(w) < 0$. Correspondingly, a firm may speed up the hiring process by offering a higher wage, that is, $q = q(w), q'(w) > 0$. Finally, our assumption that the matching function exhibits constant returns to scale implies that $q(w) \equiv q(p(w))$.

In the literature, the relationship between wages offered and the rate at which a vacancy is filled is derived in several different settings.[3] Moen (1997) does so in an economy in which a market maker creates submarkets, each characterised by a single wage. Workers and firms are free to choose which submarket to enter. Moen also shows that the labour market may endogenously separate into submarkets through the wages firms advertise. A similar interpretation is given in Mortensen and Pissarides (1999, section 4.1), where a "middle man" (like a job centre) sets the wage. Acemoglu (1997b) and Acemoglu and Shimer (1999 a) and b)) assume that the labour market is divided into regional or industrial submarkets offering potentially different wages. Alternatively, the matching technology may be derived from the urn-ball process (Montgommery 1991, Peters 1991, Burdett, Shi and Wright 1997). In this matching process, each firm posts one vacancy. Workers observe the wages and choose which firm to apply for using mixed strategy. If a firm obtains more than one applicant, it chooses one at random. The higher wage a firm offers, the more applicants it gets, and its vacancy is filled more quickly. Peters (1997) shows that the matching technology based on the urn-ball process are equivalent to the matching process based on submarkets and with an exponential matching function. In what follow we will chose the interpretation given in Moen (1997), and assume that the labour market may endogenously split into different submarkets through the wages the firms advertise.

In our model, the searching workers are heterogeneous along (potentially) several dimensions. Firstly, unemployed workers are inexperienced while on-the-job searching workers are experienced and thus have a higher productivity. Secondly, the amount of training the experienced workers have received may vary. In what follows, we assume that workers with different characteristics search in different market segments. With the interpretation that a market maker (or a middle man) creates the segments, this may be interpreted as if the market maker decides what kind of workers and firms are allowed into which segment. Inderst (2000) shows that if the market maker is able to separate the agents in this way, it is socially optimal to do so. With a wage advertisement story backing the matching technology, the separation assumption may be interpreted as firms advertising truthfully which type of workers they are going to hire, or making wages contingent on worker type.

We will now define the competitive search equilibrium in a general context (without specifying whether the worker search on-the-job or is unemployed). Let

[3] In this paragraph we have borrowed some arguments from Acemoglu and Shimer (1999a).

z denote the income flow to the worker while searching (normalised to zero when unemployed and equal to the wage when employed), $c(e)$ the worker's search cost, and W^i the expected discounted income for a searching worker. We assume that $c(e)$ is increasing and convex and that $c(0) = c'(0) = 0$. Similarly, let W^j denote the expected discounted income after successful search. The asset value equation for the searching worker is then

$$(r+s)W^i = z + ep(W^j - W^i) - c(e) \tag{1}$$

where r is the discount rate. We can write this equations as $W^i = W^i(W^j, p, e)$. Let Y denote the expected discounted joint income for a worker-firm pair that is matched, for the time being regarded as exogenous. The asset value equation for a vacancy is given by

$$rV = q(p)(Y - W^j - V) \tag{2}$$

We can thus write $V = V(p, W^j)$. The equilibrium $(W^{i*}, W^{j*}, p^*, e^*)$ is defined by the three following conditions.

1. Optimal search effort

$$e^* = \arg\max_e W^i(W^{j*}, p^*, e)$$

2. Profit maximisation

$$(p^*, W^{j*}) = \arg\max_{W^j, p} V(p, W^j) \quad \text{subject to} \quad W^{i*} = W^i(p, W^j)$$

3. Zero profit condition:

$$V^*(W^i) = K$$

The profit maximisation condition may need a comment. All submarkets (or firms) that attract workers must give the searching workers their equilibrium expected income W^{i*}. The relationship between wages offered and the arrival rate of workers in those submarkets are thus implicitly defined by the equation $W^{i*} = W^i(p, W^j)$. Since $q = q(p)$, this also defines $q(W^j; W^{i*})$, the trade-off between the wage offered and the arrival rate of workers a firm with a vacancy faces. The profit condition states that the firms maximise expected profit given this trade-off.

Free entry and equation (2) imply that $W^j = Y - K\frac{r+q}{q}$. Inserted into equation (1) this gives

$$(r+s)W^{i*} = z + e^*p(Y - K\frac{r+q(p^*)}{q(p^*)} - W^{i*}) - c(e^*) \tag{3}$$

Note that the competitive search equilibrium allocation is such that V is maximised given W^i, while free entry ensures that $V = K$. It is then straightforward to show that in equilibrium, W^i is maximised given that $V = K$. Thus, the equilibrium value of p maximises (3) with respect to p.[4]

[4] A similar result is derived in Acemoglu and Shimer (1999).

To be more precise, let the *feasible set* of pairs (W^j, p) be defined as $\Phi = \{(W^j, p) | V(W^j, p) \geq K$. Let $W^i(W^j, p) \equiv \max_e W^i(W^j, p)$. We then have that

Lemma 1 *In the competitive search equilibrium, $W^i(W^j, p)$ is maximised given that $(W^j, p) \in \Phi$.*

We will now derive the welfare properties of the model under the following assumptions:

Assumption 1: The social and private value of a match are equal.

Assumption 2: The worker's income flow while searching is equal to his marginal productivity at this stage.

Assumption 2 implies that the income flow of a searching worker in the unemployed search market equals his value of leisure (normalized to zero) and that the income flow of a worker in the on-the-job search market is equal to his productivity. Let b denote the inflow of workers to the search market (which is equal to s in the unemployed-search market) and N^i the stock of searching workers in that market.

The income flow in this economy is aggregate production less the cost of creating vacancies. The flow of new hires has intensity epN^i. By assumption, the social net present value of a match is equal to Y, and the social value of the flow of new hirings is thus $YepN^i$. In steady state, a corresponding number of vacancies are created at total cost epN^iK. Furthermore, it takes time before the vacancies are matched with workers, and due to discounting the cost of having a stock v of vacancies is vKr. Total hiring costs can thus be written as $epN^i(r+q)K/q$. The planner's objective function is therefore

$$R(N^i) = \int_0^\infty [epYN^i + zN^i - epN^i \frac{r + q(p)}{q(p)} K - c(e)N^i] e^{-rt} dt$$

The planner maximises this expression with respect to p, given the constraint

$$\dot{N}^i = b - (s+p)N^i$$

Define the social value of an additional searching worker as $R'(N^i)$. In the appendix we show that the following then holds:

Proposition 1 *Given assumption 1 and 2 above, the following holds:*

a) The socially optimal allocation maximises W^i given that $(p, W^j) \in \Phi$.

b) The competitive search equilibrium allocation is socially efficient

c) The social value of an additional worker entering the search market is equal to his equilibrium expected income W^{i*}.

The last statement ensures that the private gain from entering the search market for a worker is identical to the social value, and thus ensures that the entry decision of a worker is optimal.

It follows that the competitive search equilibrium can be defined as the solution to the problem of maximising $W^i(W^j, p)$ given that $(W^j, p) \in \Phi$.[5] We will use this extensively below.

Before we start to analyse our model, we introduce the term *internal efficiency*. Until now we have treated the joint expected income Y as exogenous. However, several decisions a worker-firm pair takes may influence their joint income Y. We may therefore write the $Y = Y(x)$, where x is a vector of decision variables made by the worker and the firm (here the choice of human capital investments and of search behaviour). A worker-firm pair (or just a firm) is behaving internally efficient if it chooses x in such a way that $Y(x)$ is maximised, when the market parameters (p and the distribution of wages) are taken as given. Internal efficiency thus implies that within-firm co-ordination problems are sorted out.

3 Equilibrium with internal efficiency

In this section, we derive and evaluate the equilibrium of the model. We first just assume internal efficiency. Towards the end of the section we show how internal efficiency can be arranged through various contractual arrangements.

3.1 Equilibrium with exogenous human capital investments

We first derive the equilibrium outcome when the investments in human capital is exogenous. To this end, we set $h = 0$ (normalisation) and denote the productivity of an experienced worker in a training firm and in a poaching firm by y^e and y^p, respectively. The joint expected discounted income of the novice and the training firm, Y^n, is then defined by

$$(r + s)Y^n = y^n + \gamma(Y^e - Y^n) \tag{4}$$

where Y^e denotes the joint expected income once the worker is experienced. Hence, internal efficiency means that Y^e is maximised. Y^e is given by

$$(r + s)Y^e = y^e + ep^e(W^p - Y^e) - c(e) \tag{5}$$

where p^e is the rate at which an experienced worker in a training firm finds a job in a poaching firm, and W^p is the expected discounted income of a worker

[5] Moen (1997) shows that the model may have more then one equilibrium, which all are equivalent from a welfare perspective as they yield the same value of W^i. To avoid uninteresting technicalities we assume that the equilibrium is unique.

employed in a poaching firm. Note that Y^n is maximised if and only if Y^e is maximised. Internal efficiency is thus obtained if and only if

1. The search effort e is chosen so as to maximise Y^e.

2. If choosing between different submarkets with different combinations of W^p and p^e (which wage to apply for), the worker-firm pair joins the submarket that maximises Y^e.[6]

Thus, if the worker-firm pair behave internally efficient, their search behaviour coincides with the worker's privately optimal search behaviour if his wage is equal to y^e. Thus, the equilibrium value of e is given by $e^* = \arg\max_e[y^e + p^e e^*(W^p - Y^e) - c(e^*)]$.

Furthermore, the equilibrium in the on-the-job search market can be characterised as the solution to the maximisation problem

$$\max_{p^e} Y^e(W^p, p^e), \quad (W^p, p^e) \in \Phi^p$$

where $\Phi^p = \{(W^p, p^e)|V^p(W^p, p^e) \geq K^p\}$, and K^p is the cost of opening poaching vacancy.

From proposition 1, it follows both that the on-the-job search market is efficient (given the inflow of experienced workers) and that the social value of an additional experienced-worker and firm pair in a training firm is equal to its private counterpart Y^e. From equation (4) it thus also follows that the social value of a match between a training firm and a novice is also equal to its private counterpart Y^n.

Finally, let us turn to the submarket for unemployed workers. Unemployed workers maximise their expected discounted income W^u, defined as in equation (1). If we denote by W^n and p^u the expected discounted income for a novice in a training firm and the probability rate of finding a job in the unemployed search market, respectively , the expected discounted income of a searching worker can be written as $W^u = W^u(W^n, p^u)$. Let V^t and K^t denote the value of a training vacancy and the cost of creating one, respectively. The equilibrium in the unemployed search market then solves the problem

$$\max_{p^u} W^u(W^n, p^u), \qquad \text{S.T.} \quad (W^n, p^n) \in \Phi^t$$

Where $\Phi^t = \{(W^n, p^u)|V^t(W^n, p^u) \geq K^t\}$, and K^p is the cost of opening poaching

[6]The point may need a comment, as in equilibrium there is only one wage offered in each market segment. Suppose that for some reasons, some worker-firms instead would trade off W^p and p^e so as to maximise a different objective function $\widetilde{Y}^e(W^p, p^e)$. Then, by the very definition of the competitive search equilibrium, a new submarket would open up offering a feasible combination of W^p, p^e that would maximise $\widetilde{Y}^e$ (for a further discussion of the competitive search equilibrium concept we refer to Moen 1997).

vacancy.

Suppose there are no unemployment benefits. This implies that the income flow of an unemployed workers equals his value of leisure, i.e. Assumption 1 holds. Since the social and the private value Y^n of a match in a training firm coincide (see above), Assumption 2 also holds. Hence, it follows from Proposition 1 that the equilibrium is efficient. Given that the value of leisure is normalised to zero, the equilibrium exists provided that the parameters satisfy the subsequent condition.

Proposition 2 *Suppose that $y^n > (r+s+\gamma)K^t$ and that $y^p - y^t > (r+s)K^p$. Suppose also that firms are internally efficient. Then a non-trivial equilibrium exists in which a positive measure of workers are employed in each type of firms. Furthermore, this equilibrium is socially efficient, with a socially efficient number of training firms and a socially optimal amount of turnover.*

3.2 Explicit human capital investments

In order to obtain a finite number of on-the-job search market segments, we assume that the level of general human capital investments h takes a finite number of values $k = 1, ..., m$. The restriction to discrete investment levels avoids technical problems associated with empty segments. The cost of the investments is ah, where each worker's cost parameter a is an independent draw from a known discrete distribution. The draw of a takes place after the worker is hired but before the investment decision h is undertaken.[7]

As stated earlier, we assume that workers with different training levels search in different market segments. Consider a worker with general human capital level h_k. Internal efficiency implies that the on-the-job search behaviour maximises Y^e given by (5), with y^e replaced with $y^e(h_k)$.

It follows from lemma 1 that *equilibrium in the on-the-job search market for workers with human capital level* h_k solves the maximisation problem

$$\max_{p_k^e} Y^e(p_k^e, W_k^e; h_k), \ \ (p_k^e, W_k^e) \in \Phi^p(h_k)$$

where $\Phi^p(h_k) = \{(p_k^e, W_k^e) | V^p(p_k^e, W_k^e; h_k) \geq K^p$. Again, it follows from proposition (1) that all of the on-the-job search market segments are efficient, and that the marginal social value of a worker with human capital level h_k is given by the private equilibrium value $Y^e(h_k)$.

In training firm hiring a worker that turns out to have a cost parameter a, the joint expected discounted income is given by (from equation (4))

$$(r+s)Y^n = y^n - ah_k + \gamma(Y^e(h_k) - Y^n)$$

[7]We assume that it is optimal for the firm to withhold all worker types

Internal efficiency implies that the training firm chooses h_k so as to maximise Y^n. Let the corresponding optimal value of Y^n as a function of a be written as $Y^n(a)$, and let $\overline{Y}^n$ denote its expectation with respect to a. Since the social value of an additional experienced worker is equal to the private value $Y^e(h)$, it follows that $\overline{Y}^n$ is also the expected social value of a match between an unemployed worker and a training firm. The unemployed-search market may now be described exactly as before, but with Y^n replaced with $\overline{Y}^n$. The following result thus applies

Proposition 3 *The equilibrium level of general human capital investments is socially optimal*

Finally, note that our specification of human capital investment may be extended to allow for firm-specific human capital investments: Let $m \in (m_1, ..., m_k)$ denote the amount of firm-specific human capital investment, and write the productivity of an experienced worker as $y^e(h, m)$. By applying exactly the same arguments as above it can be shown that the amount of firm-specific human capital investments is socially optimal. Hence, training firms do not have too strong incentives to invest in training which ensures that the worker is more productive in the incumbent firm than in a poaching firm (given internal efficiency). In some sense the opposite is true: a particular investment level is more attractive the more productive is the worker in a poaching firm (all else equal) as this will increase the worker's future income prospects and thus the joint expected income for the training firm and its employee.

3.3 Arrangements that ensure internal efficiency

In this subsection, we will analyse various arrangements that will lead to internal efficiency, and thereby to an optimal amount of training.

Firms advertise long-term wage contracts

We first assume that firms are able to advertise and commit to long-term wage contracts. A long-term wage contract consists of a wage w^n that the worker will obtain as a novice and a vector $w^e = (w^e(h_1), ..., w^e(h_m))$ of wages to experienced workers, depending on his training level. Finally, the contract specifies the share of the training cost that the worker bears, μ.

For any expected income W^n offered to a worker, it is in the firm's interest that $\overline{Y}^n - W^n$ is maximised, i.e., that the contract leads to internal efficiency. In order to obtain optimal on-the-job search behaviour, the firm sets $w^e(h_k) = y^e(h_k)$. In addition, such a wage schedule makes the worker residual claimant of the return on human capital. Hence, in order to obtain optimal human capital investments the worker bears the entire investment costs ah, i.e. $\mu = 1$.

It follows that the expected income of a worker starting to work in the firm is given by

$$(r+s)EW^n = w^n + \max_h E[-ah + \gamma(Y^e(h_k) - W^n)] \quad (6)$$

Note that as $w^e(h_k)$ is determined so as to obtain internal efficiency, w^n has to be used to scale total compensation to the worker. As before, the equilibrium in the unemployed-search market maximises W^n given that $V^t = K^t$.

Firms sell off jobs

The simplest way to obtain efficiency is that the firms advertise a "price" P the worker has to pay in order to become residual claimant of the income from the firm. To simplify the argument, assume that the trade happens before a is realised. The expected income to the worker will then be $W^n = EY^n(a) - P$. As a worker will have the right incentives, internal efficiency is obtained. Analogously with what we found before, the firm will now trade off a high price P with a low arrival rate of workers (buyers).

Quitting fees

The arrangements described above may in some circumstances be difficult to implement, for two reasons: Firstly, it requires that the compensation to a large extent is postponed until he becomes experienced. If workers are restricted in the credit market, this may be costly. Secondly, it may be difficult for a firm to commit to paying the entire match surplus to the worker when the worker becomes experienced.

These difficulties may be avoided if the firm instead use *quitting fees* to ensure optimal on-the-job search behaviour. Suppose the wage to an experienced worker, w^e, is less than his productivity y^e. Without quitting fees, the worker will do too much on-the-job search. By contrast, if the worker has to pay a fee $F = (y^e - w^e)/(r+s)$ to the firm if quitting, it is straightforward to show that the incentives to do on-the-job search is correct.

On the other hand, with quitting fees and wages below marginal product, a worker is no longer the residual claimant on the return from the human capital investments. Efficient investments may then be implemented by bargaining (see below) or if the firms advertise training levels contingent on worker type, $h(a)$, together with the quitting fees $(F_1, ...F_m)$ and a single wage rate w for both experienced and unexperienced workers. The firm will then advertise the optimal training levels and quitting fee in order to achieve efficiency, and adjust w so as to attract workers. Again, the equilibrium outcome is optimal.

Efficient bargaining

In our model, there is symmetric information between the worker and the employee. Standard Nash bargaining will therefore typically lead to internal efficiency. Advertising a single wage rate $\overline{w}$ together with bargaining will therefore typically be sufficient to ensure an optimal allocation of resources.

To see this, suppose that firms advertise an unconditional wage rate $\overline{w}$ and an (unconditional) human capital investment $\overline{h}$, and that the employer and the employee renegotiate the wage contract after the worker is hired and after a is revealed but before the investment in human capital is made. Assume first that the firm has all the bargaining power and makes a take-it or leave it offer to the worker in this renegotiation game.

The worker accepts any arrangement where his pay-off at least matches the pay-off that he obtains with the intial contract $(\overline{w}, \overline{h})$. Denote this latter pay-off by $W^n(\overline{w}, \overline{h})$. The pay-off to the firm is then $Y^n(a, h) - W^n(\overline{w}, \overline{h})$. Being the residual claimant, the firm chooses an arrangement that maximises Y^n. This may be a wage contract of the form (w^n, w^e), where w^e is set equal to the productivity of the worker when he is experienced, while w^n is adjusted so that the worker accepts the contract. The firm may alternatively propose a contract with quitting fees that ensure optimal on-the-job search behaviour.

Internal efficiency also emerges in the case where both the worker and the firm have some bargaining power, provided that there is symmetric information, and that the firm can manipulate, the worker's pay off in the bargaining game through its advertised contract $(\overline{w}, \overline{h})$.

Finally, consider the case when wages will be renegotiated at the point when the worker becomes experienced. Efficient on-the-job search behaviour may still be obtained as long as we allow for quitting fees. Since $Y^e(h_k)$ is maximised, it follows that the level of h will be optimally chosen.

4 Ex post determination of wages

We have shown that internal efficiency may be obtained through various arrangements. In this section, we analyse the effects on human capital investments when internal efficiency is not obtained. More specifically, we study the effects on the level of training when wages *ex post* are set too low and hence there is too much turnover. We thus follow up a common concern in the literature that there may be excessive turnover or too little investment in general training because wages for various reasons are below the workers' productivity (e.g. Stevens (1994), OECD (1995), and Booth and Chatterji (1998)).

We assume that the only variable the firm can commit to *ex post* is wages, and that the firm is free to set a new wage when the worker becomes experienced. Thus, training firms set wages for experienced workers so as to maximise *ex post* profit. We do not allow for quitting fees. When setting the wage for experienced workers, a firm takes into account that lowering wages implies higher search intensity and thereby a higher quit probability rate. The firm thus faces a trade-off between wage costs and the quit rate of the worker. Consider a worker with human capital investment level h_k. The quit rate for this worker is $e_k p_k^e$, where p_k^e is the arrival rate of job offers in the search market for a worker with training level h_k and e_k the search effort for such a worker. p_k^e is determined by the entry decision of training firms, and is regarded as exogenous for an individual firm.

For a given wage w^e, the worker chooses search effort so as to maximise his expected income, given by

$$(r+s)W^e(w^e;h_k) = w^e(h_k) + e_k p_k^e[W^p(h_k) - W^e(h_k)] - c(e_k)$$

We can write $e_k = e_k(w^e;h_k)$, and obviously $\frac{\partial e_k}{\partial w^e} < 0$. First order conditions for maximum is given in the appendix. A firm's profit is given by

$$(r+s)J^e(w^e,e_k;h_k) = y^e(h^k) - w^e(h_k) - e_k p_k^e J^e(h_k) \tag{7}$$

The wage is thus set so as to maximise $J^e(w^e;h_k.e_k)$ given that $e_k = e_k(w^e;h_k)$. It is easy to see that the firm will always set $w^e(h_k) < y^e(h_k)$. If not, the profit is at most zero. In contrast, the profit is strictly greater than zero for $w^e(h_k) < y^e(h^k)$. Furthermore, we assume that $w^e(h_k)$ is unique (in the appendix we show that this always holds whenever $c'(e)$ is convex).

The equilibrium in the on-the-job search market for workers with training level h_k then solves the problem

$$\max_{W^p,p^e} W^e(W^p,p^e;w^e h_k), \quad (W^p,p^e) \in \Phi^p(h_k)$$

where Φ^p is defined as above. In the appendix we show that p^e and e are both higher than in the full commitment case, we thus have excessive turnover.

We assume that the worker and the firms are able to obtain internal efficiency at the point in time when the human capital investments are undertaken. The investments thus maximise joint expected discounted income for the worker as novice and the firm, i.e., maximises $Y^n(h;a)$, taking into account that the wages will be set as described above when the worker becomes experienced. By definition, Y^n is strictly less than when the firm is internally efficient, and therefore too few training firms will be created. The next proposition summarises our findings:

Proposition 4 *Suppose wages for experienced workers are set ex post as described above. Then, relative to the socially efficient equilibrium, the following holds:*

1. *There is too much on-the-job search (e_k is too high)*

2. *Given the number of training firms in the market, there are too many poaching firms entering the market (p_k^e is too high)*

3. *Too few training firms enter the market*

What about the amount of training in each firm? It turns out that it is difficult to show that the amount of training always falls. To understand why, note that the distortions created by excessive turnover may vary non-monotonically with the amount of training in the firm: If the worker is very well trained, the optimal turnover-rate may well be very high, and the distortions created by an even higher turnover may not be that great. At lower training levels, the turnover rate may be fairly low, and the distortions created by excessive turnover may be greater. One cannot therefore rule out that the training level, for some levels of a actually will increase with *ex post* wage setting.

To get unambiguous results, we have to impose more restrictions on the model. First, there are only two levels of human capital: zero or one. Second, only workers with human capital $h = 1$ do on-the-job search. In this case, excessive turnover due to *ex post* wage setting reduces the private return from investing in human capital. Fewer workers will therefore be trained:

Lemma 2 *Given $h \in \{0, 1\}$, $e_{h=0} = 0$ and $e_{h=1} > 0$, the amount of training (the cut-off level of a) with ex post wage setting is lower than the first best optimal level.*

Similar results in the literature has often been used to rationalise training subsidies. We will show that with endogenous entry of poaching firm, this is no longer the case. To be more specific, we consider the effects on aggregate production of a training subsidy. In the appendix we show the following result:

Proposition 5 *A training subsidy reduces aggregate production in the economy, i.e., reduces welfare*

The point is that as long as the inefficiencies created by a too high turnover rate and too many poaching firms entering the market exists, this reduces the social as well as the private return from human capital investments. Furthermore, given these distortions, the training level provided by the market is optimal. A training subsidy will increase the amount of training, but it will also increase the number of poaching firms entering the market. The total effect is negative.

To be more precise, we first characterise the private value of having one more experienced worker with human capital level h_k. The social value, if his productivity was equal to his wage $w^e(h_k)$, would have been $W^e(w^e(h_k))$. His actual productivity is $y^e(h_k) > w^e(h_k)$, and the overshooting $y^e(h_k) - w^e(h_k)$ is allocated to the firm. It thus follows that the social value of one more experienced worker is equal to $Y^e(w^e(h_k))$, that is, the social and the private value coincide. If the planner could reduse the amount of turnover he could also increase Y^e, but by definition that is not an option. The constrained social and the private value of a match in the unemployed search market, and the socially and privately optimal human capital investments, therefore also coincide. The proposition follows.

As mentioned before, this result contrasts with earlier contributions (e.g. Stevens (1994)). Our result differs because we endogenise the number of poaching firms, while earlier papers treat this number as an exogenous parameter.

5 Discussion

In this section we discuss some fundamental mechanisms and assumptions in the model, regarding the matching technology, the wage determination process and the production technology.

Matching technology

A crucial assumption in our model is that workers with different characteristics search in different market segments. This implies that there are no externalities between workers of different types. To illustrate this, suppose a small measure of workers that previously where choosing training level h_i are now, for exogenous reasons, instead choosing a training level $h_j \neq h_i$. The equilibrium response will be that more vacancies will flow into the market segment for experienced workers with training level h_j and less so in the corresponding market segment for workers with training level h_i. The labour market tightness and the advertised wages will stay constant in both segments. Thus, the changes would only affect the expected incomes of the agents that changed their behaviour.

If workers with different characteristics were searching in the same market segment, this would not be the case any longer. Suppose agents with different productivities were searching in the same market. Suppose also that the deviating agents where choosing a higher training level, i.e., $h_j > h_i$. The likely equilibrium response is that more vacancies enter this markets, as workers in average are more productive. This benefits all workers in the market, not only the workers that changed their investments in training. Thus, training gives rise to a positive externality towards other workers and we may end up with underinvestment in training.

The critical issue is therefore to what extent our assumption about segmentation in the search market is realistic. When presenting the model, we gave arguments as to why this may be reasonable. The main argument is that as separation into market segments are efficient, and not doing so will imply that all gains from trade are not realised. Since the market maker (or alternatively the individual firms through their job advertisements) have the opportunity to separate the market into different market segments, we would therefore expect this to happen. Furthermore, in the absence of segmentation, workers with different productivities have the same probability of finding a job in a poaching firm, an implication we find unreasonable.

Still, the discussion points to a common weakness in the matching literature, that the matching process is exogenous. It would therefore be interesting to study the training decision in a model in which the matching process was explicitly modelled, for instance by the urn-ball process. Moen (1999) analyses the incentives to invest in education given this matching process, when wages are determined by bargaining. He finds that workers then actually may over-invest in education in order to speed up the job-finding process. We are currently working with a similar model in which firms advertise wages contingent on worker type. Our conjecture is that in equilibrium, the labour market will be segmented in the sense that low-productivity worker will not create search externalities for high-productivity workers, thus reinforcing the results in this paper.

Wage bargaining under the Hosios condition

Suppose now that wages, instead of being determined in a competitive search market, is determined by traditional wage bargaining at the firm level. However, we assume that the Hosios condition holds (Hosios 1990).[8]

The standard assumption in the matching literature seems to be continuous renegotiation of wages, and no quitting fees, see for instance Pissarides (2000). In this case, wages for experienced workers will be set too low, and thus as when wages are set *ex post* this means an inefficiently high turnover rate and an inefficient allocation of resources. However, as we have already seen, with efficient bargaining and enforceable long-term wage contracts in training firms, internal efficiency can be obtained through bargaining. An interesting question is whether optimal resource allocation obtains in a setting with segmented markets, internal efficiency in training firms, and *wage bargaining* under the Hosios condition. Our conjecture is that it does. Under the Hosios condition, wages and labour market tightness will be the same as in a competitive search market in all on-the job search markets (for all training levels). Hence the expected income for a trained worker, and thus also the incentives to invest in training, will be the same as in a competitive search equilibrium model. The efficient outcome of the on-the-job search markets implies that the unemployment search market is also efficient (given that the Hosios condition holds). Acemoglu (1997a) considers bargaining in a setting without segmented markets. That is, workers with different productivities are assumed to have the same probability of finding a new job. This is crucial for his under investment result.

What then if we do not have internal efficiency in firms, for instance because workers and firms continuously renegotiate wages? Again we can draw on the equivalence between bargaining under the Hosios condition and competitive wage setting. Our conjecture is therefore that the results will parallel the results with

[8]The Hosios condition for efficiency with wage bargaining states that efficiency is obtained whenever the absolute value of the elasticity of q (the arrival rate of workers to firms with a vacancy) with respect to the labour market tightness θ is equal to the worker's bargaining power (share of bargaining surplus).

ex post wage setting. Thus, we may now get underinvestment in training, but subsidising training will not improve welfare.

Production technology

The assumptions we have made about the production technology in training firms and poaching firms may seem strict. We will therefore discuss the assumptions in some detail, and give alternative interpretations of training and poaching firms.

Why should a worker be more productive in a poaching firm than in a training firm for any level of training? To some extent this is an empirical question. Still, in a complex world where firms operate at different scales in different parts of the value chain or in different product markets it seems likely that firms have different comparative advantages: some firms are in a better position to train workers than others, and the coexistence of different types of firms then imply that the other firms must have advantages as well, in our two-dimensional case this means that they must be more efficient at utilising trained labour.

Throughout the paper we assume two types of firms, posting and training firms. Alternatively, one could consider a framework in which all firms have the same production technology, and each firm faces the choice of hiring a trained or an untrained worker. In addition, suppose that an experienced worker's productivity also depends on a worker-firm specific component, unknown at the time of the training decision (or, equivalently, if the experienced worker's disutility of work differs across firms). On-the-job searching workers would then be workers who made a bad "draw" and thus have a low worker-firm specific productivity component. In expected terms, these workers would be more productive in a poaching firms. With this interpretation, our analysis also answers the question as to whether firms have sufficient incentives to train their workers themselves rather than to poach already trained workers from other firms. Our answer to this question is confirmative given that training firms operate internally efficient.

6 Conclusion

We have analysed the incentives to invest in general training in a matching model which includes worker turnover and where wages are set in a competitive fashion. We find that as long as employers and their emplyees are able to sort out their within-firm co-ordination problems, the search frictions will not induce inefficiencies and the resulting allocation of resources is optimal. In the absence of internal efficiency there may be underinvestment in training as a result of excessive turnover. However, as the excessive turnover reduces the both the private and the social return from training, the level of investments in training is still constrained efficient, and subsidising training will therefore reduce welfare.

Appendix

Proof of lemma 1

Suppose not. Then there exists a pair $(W^{j'}, p')$ such that $W^i(W^{j'}, p') > W^{i*}$ and at the same time $V(W^{j'}, p') \geq K$. From the continuity of the problem it follows that there exists a pair $(W^{j''}, p'')$ such that $W^i(W^{j''}, p'') > W^{i*}$ and $V(W^{j''}, p'') > K$. But this implies that the profit maximisation condition is not satisfied, and we are not in equilibrium.

Proof of proposition 1

The associated Bellman equation is given by

$$rR(N^i) = \max_{p,e}[epYN^i - N^i z - epN^i \frac{r+q}{q}K - c(e)N^i + R'(N^i)(b - (s+p)N^i)] \quad (8)$$

By taking derivative with respect to N^i, and utilising that the constant-return to scale matching function implies that $R''(N) = 0$, we find that

$$(r+s)R'(N^i) = z + ep(Y - \frac{r+q}{q}K - R'(N^i)) - c(e) \quad (9)$$

By comparing (3) and (9) it follows that the expressions for R' and for W^i are equivalent. Now the maximisation problem in (8) can be written as

$$\max_{pe} ep(Y - \frac{r+q}{q}K - R') - c(e)$$

which is equivalent to maximising R'. But it then follows that the planner maximises W^i given by (3), that is, maximises W^i given that $V = K$, just as in the competitive search equilibrium.

Proof of proposition 2

Consider first the on-the-job search market. We have to show that the problem of maximising $Y^e(p^e, W^p)$ given that $(p^e, W^p) \in \Phi^p$ implies that $p^e > 0$. Note that the maximisation problem is independent of the measure of workers working in training firms (as long as it is strictly positive). Suppose first that the optimal solution requires that $p^e = 0$. It follows that $Y^e = y^t/(r+s)$. Then consider the pair $(\varepsilon, \varepsilon + y^t/(r+s))$ where $\varepsilon \geq 0$ is arbitrarily close to zero. Obviously, this yields a higher value of Y^e. We have to show that $(\varepsilon, \varepsilon + y^t/(r+s)) \in \Phi^p$. We know that $\lim_{p \to 0} q(p) = \infty$. Since $c(0) = c'(0) = 0$ it follows that we can make q arbitrarily large by choosing ε arbitrarily small. Thus

$$\lim_{\varepsilon \to 0} V(\varepsilon, \varepsilon + y^t/(r+s)) = \frac{y^p - y^t}{r+s}$$

which, by assumption, is greater than K^p. It thus follows that if there is a positive measure of workers working in training firms, there will be a positive measure of poaching firms entering the market.

Consider then the unemployed search market. We want to show that $p^u = 0$ cannot be a solution to the problem of maximising $Y^n(p^u, W^n)$ given that $(p^u, W^n) \in \Phi^t$. To see this, consider the pair $(\varepsilon, \varepsilon)$ where $\varepsilon \geq 0$ is arbitrarily close to zero. Obviously, $W^u(\varepsilon, \varepsilon) > W^u(0, W^n)$ for any W^n. We know that $Y^n > y^n/(r+s+\gamma)$ (since $Y^e > 0$), thus we must have that (since $\lim_{p \to 0} q(p) = \infty$)

$$\lim_{\varepsilon \to 0} V(\varepsilon, \varepsilon) = \frac{y^n + \gamma(Y^e - Y^n)}{r+s} > \frac{y^n}{r+s+\gamma} > K$$

It thus follows that $(\varepsilon, \varepsilon)$ is feasible for sufficiently small values of ε. Thus, $p^u = 0$ is inconsistent with equilibrium.

First order conditions when wages are set ex post

Taking the derivative of $J^e(h_k)$ (equation (7)) with respect to w^e and setting it equal to zero gives

$$-1 - \frac{de_k}{dw^e(h_k)} p^e J^e(h_k) = 0. \tag{10}$$

The first order condition for the optimal e is

$$p_k^e[W^p(h_k) - W^e(h_k)] - c'(e_k) = 0$$

As $dW^e/de = 0$, equation (1) implies that $dW^e/dw^e = 1/(r+s+e_k p_k^e)$. Taking the derivative of the first order condition for e with respect to $w^e(h_k)$ yield

$$c''(e_k)\frac{de_k}{dw^e(h_k)} = -\frac{p_k^e}{r+s+e_k p_k^e}$$

Rearranging the expression and inserting into (10) gives

$$\frac{(p_k^e)^2}{r+s+e_k p_k^e} J^e(h_k) = c''(e) \tag{11}$$

In what follows, we assume that this equation has a unique solution. (This always holds provided that $c'''(e) > 0$).

Proof of lemma 4

From (3) it follows that, for a given w^e, p^e maximises

$$(r+s)W^e = w^e - c(e^*) + e^* p^e (Y^p - K\frac{r+q(p^*)}{q(p^*)} - W^e(w^e)) \tag{12}$$

It follows that the equilibrium value p^{e*} maximises $p^e(Y^p - W^e(w^e) - \frac{r+q(p^e)}{q(p^e)}K) \equiv f(p^e, W^e(w^e))$. It follows that $f_{p^e,W^e} < 0$. As the second-order conditions for maximum is always satisfied locally, we thus know that $\frac{dp^{e*}}{dW^e} < 0$. From the envelope theorem it follows that $W^e(w^e) = 1/(r+s+p^e e) > 0$. It thus follows that p^{e*} decreases in w^e as well.

Then we show that the equilibrium search intensity is decreasing in w^e. We know that p maximises W^e, and from (12) it follows that p therefore maximises $p^e(Y^p - K\frac{r+q(p^*)}{q(p^*)} - W^e(w^e))$. We can therefore write $W^e(w^e)$ as

$$(r+s)W^e(w^e) = \max_e\{w^e - c(e) + e^e \max_{p^e}[p^e(Y^p - \frac{r+q(p^e)}{q(p^e)}K - W^e(w^e)]$$

From the envelope theorem it follows that the derivative of the second maximand with respect to w^e is equal to $-W^{e\prime}(w^e) = - \; -p^e/(r+s+pe) < 0$. Since the first order condition for e is given by $c'(e) = \max_{p^e} p^e(Y^r - W^e - \frac{r+q(p^e)}{q(p^e)}K)$, it thus follows that $e'(w^e) < 0$.

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Figure 1:

Figure 1. Worker flows in the economy.