Is Equipment Price Deflation a Statistical Artifact?

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Abstract

I argue that methods using relative equipment price deflation to measure productivity growth in machine producing sectors will tend to overestimate the real rate of technological change because they incorrectly rely on marginal cost pricing. In order to illustrate my argument, I introduce an endogenous growth model in which heterogeneous final goods producers can choose what technology to use. The various technologies are supplied by monopolistically competing machine suppliers. This market structure, combined with capital skill complementarities, implies that the best machines are marketed to the best workers and are sold at the highest markup. The endogenously determined markups are such that standard methods will tend to overestimate the degree of technological progress in the machine producing sector.

keywords: imperfect competition, price indices, vintage capital.

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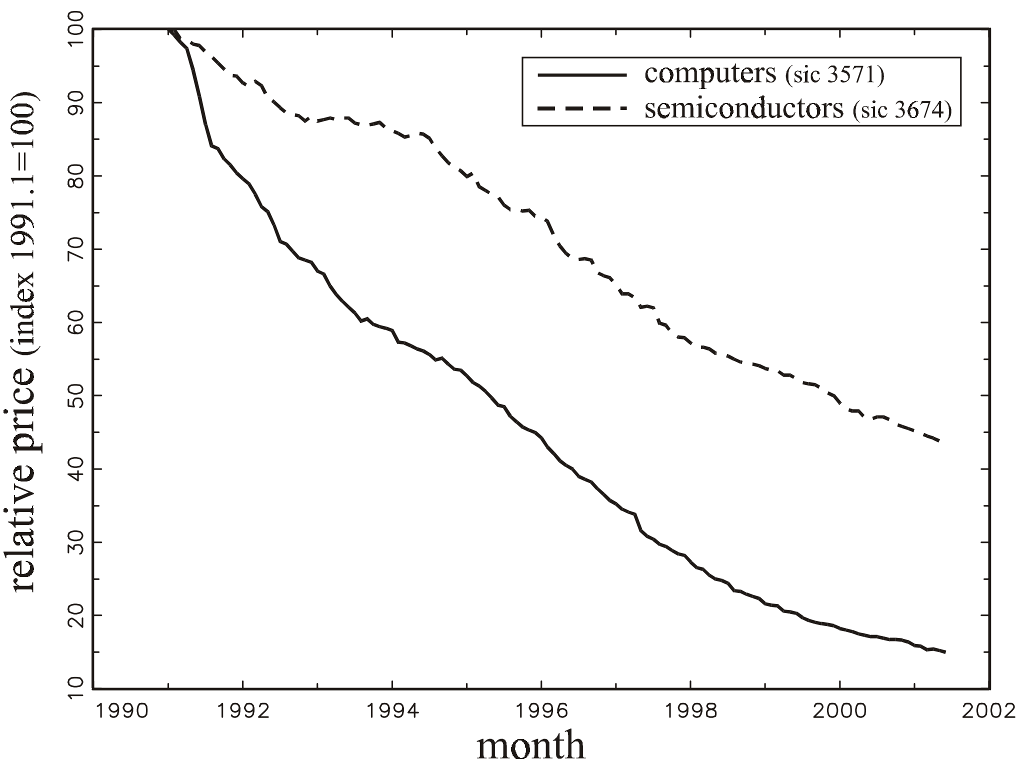
1 Introduction and motivation

Many recent studies argue that one of the main driving forces behind the record economic expansion of the 1990's is the unprecedented productivity growth in the information technology producing sectors. Virtually all of these studies, including Oliner and Sichel (2000), Gordon (2000), Jorgenson and Stiroh (2000), and Violante, Ohanian, Ríos-Rull, and Krusell (2000) argue that the unprecedented productivity growth in the IT producing sectors is reflected in the steady decline of their output prices relative to GDP or consumption goods. Their argument is similar to that of Greenwood, Hercowitz and Krusell (1997), who like to refer to it as investment specific technological change. Namely, if the productivity growth rate of the investment goods producing sector is consistently higher than that of the final goods producing sector, then this will lead to a steady decline in the relative price of investment goods.

Two of the price indices that are most often referred to in the context of the IT sector are those for computers and semiconductors, which I have plotted in Figure 1. As can be seen from this figure the relative prices of computers and semiconductors have dropped an astounding 90% and 60% respectively in the period of January 1991 through July 2001. If all markets would be perfectly competitive, as is assumed by all growth accounting studies referred to above, then this suggests that productivity growth in the IT producing sectors rapidly outpaces that of the final goods sector. This is a big if, however. Because, if these markets would be perfectly competitive then none of the IT producers would be able to make the profits necessary to recoup the expenses they made on research and development of their products. Consequently, it is interesting to explore theoretical models that allow for imperfect competition between the suppliers of different investment goods.

This is exactly what I do in this paper. I introduce an endogenous growth model in which heterogeneous final goods producers can choose what technology to use. The various technologies are supplied by monopolistically competing machine suppliers. This market structure, combined with capital skill complementarities, implies that the best machines are marketed to the best workers and are sold at the highest markup. The endogenously determined markups are positively correlated with the level of productivity embodied in a machine.

I use this model to show how in a world where there is *no* technological progress in the machine producing sector, the methods applied by the



note: relative price is ratio of PPI for indicated SIC's and CPI-U

Figure 1: Relative prices of computers and semiconductors

Bureau of Labor Statistics as well as the Bureau of Economic Analysis for the construction of the Producer Price Indices and Investment Price Indices will tend to find a steady decline in the relative price of investment goods similar to that observed in the data. In my theoretical model this measured productivity growth is completely spurious, however, and is induced by the structure of the varying markups across different vintages of machines.

The structure of this paper is as follows. In Section 2 I introduce the market for machines with heterogeneity on the demand side and imperfect competition on the supply side. I use this setup to derive the equilibrium demand and price schedule. This is the core of the paper. In Section 3 I combine the technology choice setup of Section 2 with the consumer choice decision and patent race structure needed for a general equilibrium framework. I define the competitive equilibrium of this economy and derive its balanced growth path. In Section 4 I then show that, even though on this balanced growth path there is no productivity growth in the machine producing sector, the empirical methods applied by the BLS and BEA would find spurious price declines in the relative price of machines. I illustrate this result with a numerical simulation. In Section 5, I provide some tentative evidence in support of my hypothesis. In Section 6, I conclude with some suggestions for further research.

2 Market for machines

In this section I introduce the market for machines with imperfect competition that forms the backbone of this paper. This section consists of three parts. In the first part I describe the demand side of the market for machines in which workers of different types decide on their optimal technology choice. In the second part I introduce the supply side of the market in which monopolistic competitors decide on the profit maximizing price of their machines. In the third part I combine the demand and supply sides of the market and define the Pure Strategy Nash equilibrium outcome, prove its existence and uniqueness, and derive its main properties.

2.1 Machine users

I will take a certain degree of heterogeneity on the demand side of the market as given. This heterogeneity takes the form of different productivity types for workers. Each worker's type is denoted by h. I will assume there is a continuum of workers of measure one that is uniformly distributed over the unit-interval, such that $h \backsim unif\,(0,1)$.

Final goods are produced by the combination of one worker, of type h, with one machine, which embodies a technology level equal to A_t. The output of such a combination is hA_t. In order to avoid having to consider intractable intertemporal optimization problems and having to make assumptions about possible second hand markets, I will assume that machines fully depreciate in one period. This assumption basically implies that the machines considered here are equivalent to intermediate goods in the sense of Aghion and Howitt (1992) and Romer (1990). The workers can not use these machines for nothing. The price of a machine of type $A_{t-\tau}$ at time t is denoted by $P_{t,\tau}$.

Where this paper distinguishes itself from others in the literature is that I will allow workers to choose the technology that they are using from a menu of available technologies. That is, workers have the choice between all the types of machines that have been introduced so far. Let A_t be associated with the machines introduced at time t, then, at time t, the workers can choose from the 'technology-menu' $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$. The notational convention that I will use in this paper follows Chari and Hopenhayn (1991) in the sense that τ represents 'vintage age'. That is, A_t represents the frontier technology level and $A_{t-\tau}$ is the frontier technology level of τ periods ago. For notational convenience, I will every once in a while switch between the notation of technology in its levels, i.e. A_t, and technology growth rates, i.e. $g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$. Throughout, I will assume that there is no technological regress such that $g_t > 0$ for all t.

In order to maximize his income, a worker of type h will choose a technology from the technology choice set, which is defined as

$$\Upsilon_t(h) = \left\{ \tau \in \mathbb{N} \,\middle|\, \tau \in \arg\max_{s \in \mathbb{N}} \{hA_{t-s} - P_{t,s}\} \right\}$$

The resulting labor income of a worker of type h at time t equals

$$y_t(h) = hA_{t-\tau} - P_{t,\tau}, \text{ for all } \tau \in \Upsilon_t(h)$$

2.2 Machine producers

Machine designs are assumed to be patented for M periods. During the first M periods of a machine design's life, the particular machine is supplied by a monopolist firm. After the patent expires the machine design is public domain and there is perfect competition in the supply of these machines. I will assume that units of the consumption good are the only input needed in machine production, this to avoid having to deal with the selection of workers across sectors. The production of a continuum of mass X of machines of type $A_{t-\tau}$ requires the use of $\frac{c_\tau}{2} A_{t-\tau} X^2$ units of the consumption good, where $c_\tau \geq 0$. That is, when $c_\tau > 0$ a machine producer faces decreasing returns to scale. Note that the cost function is scaled by c_τ which depends on the vintage age, in order to allow for learning by doing.

The set of buyers of machines of type $t-\tau$, which I will denote by $D_t(\tau)$, is given by

$$D_t(\tau) = \left\{ h \in [0,1] \middle| \tau \in \underset{s \in \{0,1,2,\ldots\}}{\arg\max} \left(hA_{t-s} - P_{t,s}\right) \right\}$$

In order to properly define demand, I will have to deal with measures of sets. For this reason, I will use the notation $\mu(\mathcal{H})$ for the Lebesque measure of the set $\mathcal{H}$. Total demand for the machine of type $t-\tau$ at time t is then given by

$$X_{t,\tau} = \mu(D_t(\tau))$$

The question that is left is how these machine producers end up choosing the prices of their machines. Throughout this paper, I will focus on Pure Strategy Nash equilibria. For the particular problem at hand here this implies that, taking the prices of the other machines, i.e.

$$\mathbf{P}'_{t,\tau} = \{P_{t,0}, \ldots, P_{t,\tau-1}, P_{t,\tau+1}, \ldots\},$$

and the levels of the technologies, i.e.

$$\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\},$$

, as given, the machine producer to type $A_{t-\tau}$ chooses the price of his machine to maximize profits. This implies that $P_{t,\tau}$ is an element of the best response set

$$BR_t\left(\tau; \mathbf{P}'_{t,\tau}, \mathbf{A}_t, r_t\right) = \left\{ P_{t,\tau} \in \mathbb{R}_+ \middle| P_{t,\tau} \in \underset{P \in \mathbb{R}_+}{\arg\max} \left\{ PX_{t,\tau} - \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \right\} \right\}$$

Because patents expire after M periods, these best response sets only apply to $\tau = 0, \ldots, M-1$. For machines that were designed M or more periods ago, perfect competition implies that price must equal average cost, and that free entry drives both to zero, such that $P_{t,\tau} = 0$ for $\tau \geq M$. The corresponding profits are

$$\pi_{t,\tau} = PX_{t,\tau} - \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \text{ for all } P_{t,\tau} \in BR_t\left(\tau; \mathbf{P}'_{t,\tau}, \mathbf{A}_t, r_t\right)$$

for $\tau = 0, \ldots, M-1$.

2.3 Equilibrium and its properties

Now that the demand and supply side of the machine market are well defined, I can now focus on the resulting Pure Strategy Nash equilibrium. Such an equilibrium consists of a price schedule, i.e. $\mathbf{P}_t^* = \left\{P_{t,0}^*, \ldots, P_{t,M-1}^*\right\}$ and a collection of corresponding demand sets, i.e. $\{D_t^*(0), D_t^*(1), \ldots\}$. I will derive the equilibrium in two steps. In the first step I show that, independent of the price schedule, the demand sets $D_t(\tau)$ have some important properties. In the second step I use these properties to derive the equilibrium price schedule $\mathbf{P}_t^*$. This equilibrium price schedule is then used to derive equilibrium output, profits, and demand sets.

The main result of the first step is Proposition 1 below

Proposition 1 ***Properties of demand sets***
Independent of the technology menu $\mathbf{A}_t$ and the price schedule $\mathbf{P}_t$, the demand sets $D_t(\tau)$ have the following properties:
(i) For $h' > h$, if $h \in D_t(\tau)$ then $h' \notin D_t(\tau')$ for all $\tau' > \tau$.
(ii) $D_t(\tau) = \emptyset$ for all $\tau > M$.
(iii) Define the set of workers for whom the optimal technology choice is not unique as

$$\widetilde{\mathcal{H}}_t = \{h \in [0,1] \,|\, \exists\ \tau \neq \tau' \text{ such that } h \in D_t(\tau) \wedge h \in D_t(\tau')\}$$

then $\mu\left(\widetilde{\mathcal{H}}_t\right) = 0$.
(iv) $D_t(\tau)$ is connected for all τ.

which is proven in the appendix. The economic interpretation of the results in this proposition are: (i) better workers use better technologies, i.e. there is endogenous assortative matching between workers and machines. Models where this matching occurred as a solution to a planner problem are for example Jovanovic (1999) and Sattinger (1975). (ii) Perfect competition implies that machines of a design for which the patent is expired for more than a year are not demanded anymore. They are obsolete. (iii) Demand functions are properly specified in the sense that the set of workers that is indifferent between technologies is negligible. (iv) If two workers of different types buy the same vintage of machine, then so will all the workers of types in between.

The intuition behind this proposition is probably most clear when one considers a graphic example of how these demand sets are determined. Figure

2 depicts the way these demand sets are determined for the case in which $M = 2$. For simplicity, the time subscript, t, is ignored in the figure. The top panel of figure 2 shows the levels of gross output, i.e. hA_τ, that workers of different types get for the three available technologies, i.e. $\tau \in \{0, 1, 2\}$, the dots mark the points at which the price and gross output levels coincide. The short dashed vertical lines, that extend to the bottom panel, determine the levels of the critical types that get zero income for the various technologies. The bottom panel then depicts the net output levels, i.e. the workers income levels for the three technologies. Workers choose that technology that yields them the highest income level, which implies the demand sets plotted at the bottom.

Now that I have shown that the demand sets have some convenient properties, I can use them to derive the equilibrium price schedule. Before doing so, I first formally define the PSN-equilibrium for the prices in the machine producing sector, which is

Definition 1 ***Equilibrium price schedule***
For a given sequence of technology levels, $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$*, a price schedule* $\mathbf{P}_t^* = \left\{P_{t,0}^*, P_{t,1}^*, \ldots\right\}$ *is an Pure Strategy Nash equilibrium price schedule if*
(i) $P_{t,\tau}^* = 0$ *for all* $\tau \geq M$*.*
(ii) Define $\mathbf{P}_{t,\tau}^{*\prime} = \left\{P_{t,0}^*, \ldots, P_{t,\tau-1}^*, P_{t,\tau+1}^*, \ldots\right\}$*, then* $P_{t,\tau}^* \in BR\left(\tau; \mathbf{P}_{t,\tau}^{*\prime}, \mathbf{A}_t, r_t\right)$ *for all* $\tau = 0, \ldots, M-1$*.*

What I will show in this step is that, for all possible technology paths $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$, there exists a unique equilibrium price schedule. This price schedule is such that all technologies of age M or more recent are used. In particular, the core-proposition of this subsection reads

Proposition 2 ***Solution of equilibrium price schedule***
For any sequence of technology levels, $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$*, there exists a unique equilibrium price schedule with the following properties:*
(i) $P_{t,\tau} > \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \geq 0$ *for all* $\tau = 1, \ldots, M-1$*.*
(ii) $D_t(\tau) \neq \emptyset$ *for all* $\tau = 1, \ldots, M$*.*
(iii) The equilibrium price schedule is unique, and defining $\widehat{P}_{t,\tau} = P_{t,\tau}/A_{t-\tau}$*,*

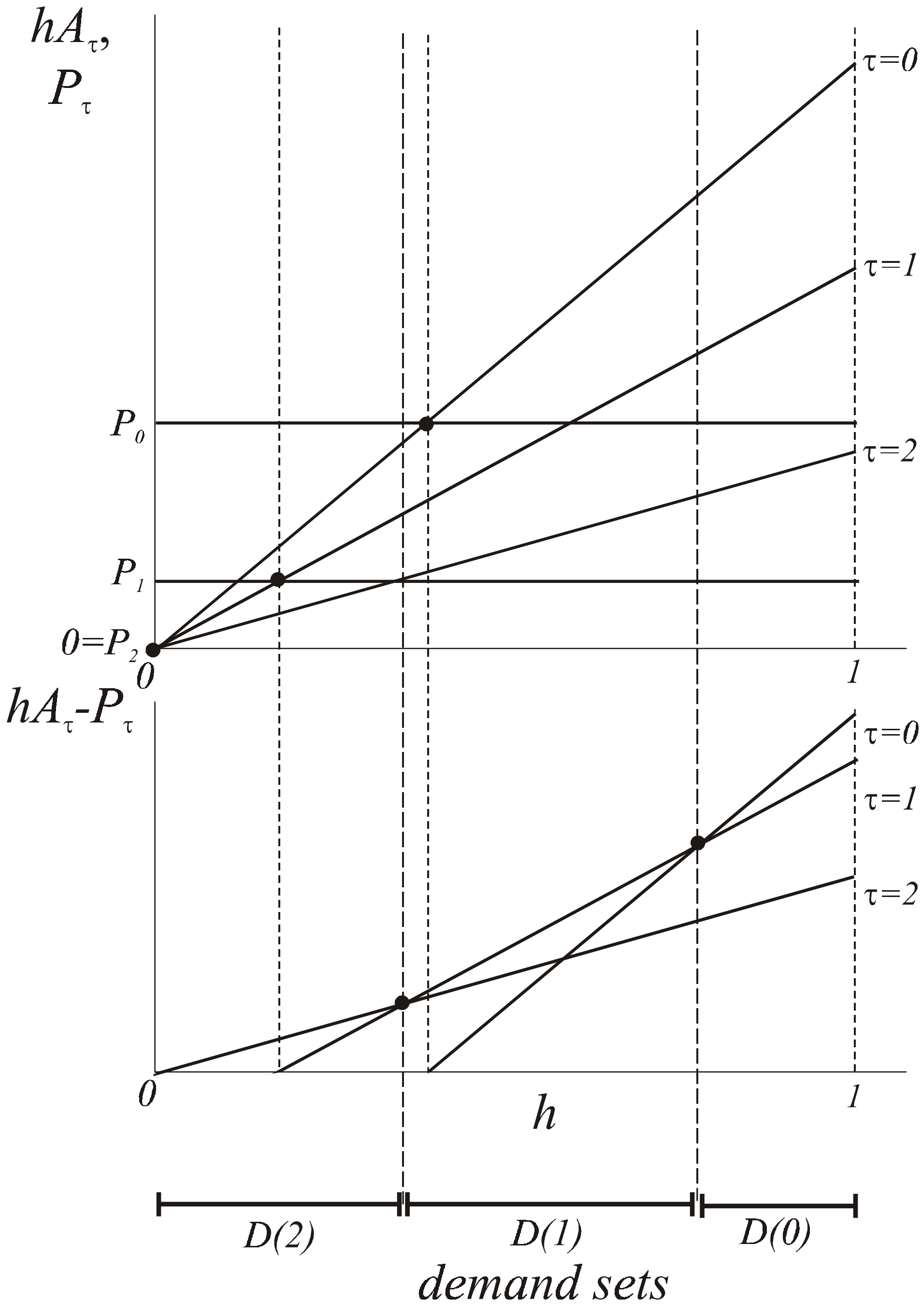


Figure 2: Determination of demand sets

it satisfies

$$\widehat{P}_{t,\tau}=\begin{cases}\left[\frac{1+c_0\left(1+w_{t,0}^1\right)}{2+c_0\left(1+w_{t,0}^1\right)}\right]\left[\frac{1}{1+w_{t,0}^1}+\frac{w_{t,0}^1}{1+w_{t,0}^1}\widehat{P}_{t,1}\right] & \text{for} \quad \tau=0\\ \left[\frac{1+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}{2+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}\right]\left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau-1}+\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau+1}\right] & \text{for} \quad \tau=1,\ldots,M-1\\ 0 & \text{for} \quad \tau=M\end{cases} \tag{1}$$

where

$$w_{t,\tau}^{\tau-1}=\frac{A_{t-\tau+1}}{A_{t-\tau+1}-A_{t-\tau}},\ \textit{and}\ w_{t,\tau}^{\tau+1}=\frac{A_{t-\tau-1}}{A_{t-\tau}-A_{t-\tau-1}}$$

(iv) $\widehat{P}_{t,\tau}$ *is strictly decreasing in* τ*.*
(v) The demand sets satisfy

$$X_{t,\tau}=\begin{cases}\left[\frac{1+w_{t,0}^1}{1+c_0\left(1+w_{t,0}^1\right)}\right]\widehat{P}_{t,0} & \text{for} \quad \tau=0\\ \left[\frac{\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}{1+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}\right]\widehat{P}_{t,\tau} & \text{for} \quad \tau=1,\ldots,M-1\\ w_{t,M}^{M-1}\widehat{P}_{t,M-1} & \text{for} \quad \tau=M\end{cases}$$

The proof of this proposition is again in the appendix. Besides the fact that this proposition proves the existence and uniqueness of the PSN-equilibrium in this market, the most important result of this proposition is (iv). It basically implies that the average cost per efficiency unit is higher for more recent vintages than for older ones. Note that this result is independent of the path of technological progress as well as the cost structure underlying production of the vintages of machines, i.e. $\{c_\tau\}_{\tau=0}^{\infty}$. It is simply due to the imperfect competition between the machine suppliers. This is the result that will underlie the spurious technological change result that I will present in Section 4. First, however, I will implement the market for machines introduced here in an endogenous growth model.

3 Endogenous Growth Model

The aim of this section is to implement the market introduced above in a general equilibrium framework with endogenous growth. In order to do so, I have to combine the machine buyers and suppliers with a final goods demanding sector, i.e. consumers, as well as with a sector that creates new machine designs and moves the technological frontier outward, i.e. an R&D sector. These two respective additions form the first two subsections of this section. In the third subsection, I combine all sectors of the economy to define a competitive equilibrium and balanced growth path for it.

3.1 Consumers

Consumers and workers in the final goods sector are basically the same. I will assume that workers of all types each maximize the present discounted value of their lifetime utility and have constant relative risk aversion preferences, such that they choose to maximize

$$\frac{1}{1-\sigma}\sum_{s=t}^{\infty}\beta^{s-t}c_t(h)^{1-\sigma}$$

subject to their lifetime budget constraint

$$k_t(h) = (1+r_t)\,k_{t-1}(h) + y_t(h) - c_t(h) + \Pi_t$$

where $k_t(h)$ is capital holdings at period t of a worker of type h, which is assumed to be the same for all workers of the same type, r_t is the interest rate, $y_t(h)$ is the labor income of a worker of type h in period t, and $c_t(h)$ is the corresponding consumption level, and $\beta \in (0,1)$ the discount factor. The income Π_t represents the income from innovative activities that each household earns, which I will explain in the subsection below.

This problem yields the familiar Euler equation

$$\frac{c_{t+1}(h)}{c_t(h)} = [\beta(1+r_t)]^{\frac{1}{\sigma}}$$

This implies that the consumption growth rates of all workers are the same, independent of their type. As described in Caselli and Ventura (2000), this implies that aggregate consumption is consistent with that of a representative consumer that has CRRA preferences himself.

In the following, capital letters, e.g. Y_t, denote aggregates obtained from aggregation over the various types of workers, i.e.

$$Y_t = \int_0^1 y_t(h)\,dh$$

Then the aggregates K_t, C_t, and Y_t, behave as if they were the solution to a representative consumer solving a utility maximization problem that is identical to that of the workers of each type, but then defined in these aggregates.

3.2 Patent race and innovation

As a simplifying assumption for my general equilibrium framework, I will assume, as do Reinganum (1983), Gilbert and Newbery (1982) and Aghion and Howitt (1992), that the size of the innovation (in each period) is fixed. In particular, the size of the innovation is $g > 0$, in the sense that $A_{t+1} = (1+g)\,A_t$ for all t, as a result of an innovation[1]. Instead of the size of the innovation, what is determined in equilibrium is the R&D intensity with which the innovation is pursued. This intensity is represented by the amount of output spent on the patent race, which I will denote by $X_{t,R}$. The final good is assumed to be the only input into the R&D process.

If one wins the patent race, then one obtains a patent with a value that is equal to the present discounted value of the monopoly profits made on the particular machine design. I will derive this value in more detail later, but for the moment will simply denote it by $V_t \equiv A_t V_t^*$. The patent race that I consider is one in which the probability of winning per unit of output spent is inversely proportional to the total amount of resources devoted to R&D, i.e. $X_{t,R}$. That is, when $X_{t,R}$ is the total amount of resources spent for R&D purposes, the spender of a unit of output on R&D pays a price equal to one and obtains the expected revenue $V_t/X_{t,R}$. I will assume that there is no advantage for the incumbent, such that incumbents and entrants have an equal chance of winning the patent race. Since there are basically M incumbents and a continuum of researchers, the possibility of an incumbent winning the race is zero. Consequently, the freedom of entry and exit in R&D implies the zero profit equilibrium condition $X_{t,R} = A_t V_t^*$.

So what is left to derive is the present discounted value of the monopoly profits made off a machine design. The assumption that g is constant over time implies some important simplifications for the behavior of prices, output, and the value of an innovation. These implications are derived in the proposition below.

Proposition 3 ***value of innovation, output, etc., at constant*** g
If the technological frontier moves out at the same rate, g, in each period then this implies

[1] Throughout, I will assume that $g < \left(\frac{1}{\beta}\right)^{\frac{1}{1-\sigma}} - 1$, such that the consumer's objective function will be bounded.

(i) prices: *the vintage age specific prices per efficiency unit satisfy*

$$P_{t,\tau}/A_{t-\tau} = p_\tau \left(g, \mathbf{c}\right)$$

where $p_\tau\left(g\right) > p_{\tau+1}\left(g\right)$ *for all* $\tau = 0, \ldots, M-1$, $g > 0$, *and* $\mathbf{c} = \{c_0, \ldots, c_M\}$ *is the sequence of production parameters for machines.*
(ii) demand for various vintages: *the demand for different vintages of machines , i.e.* $X_{t,\tau}$*, satisfies*

$$X_{t,\tau} = \widetilde{X}_\tau \left(g, \mathbf{c}\right)$$

only depends on vintage age and not on time.
(iii) output: *aggregate output,* Y_t*, can be written as*

$$Y_t^* = Y_t/A_t = \widetilde{Y}\left(g, \mathbf{c}\right)$$

(iv) profits: *the vintage age specific profits follow*

$$\pi_{t,\tau}^* = \pi_{t,\tau}/A_t = \widetilde{\pi}_\tau\left(g, \mathbf{c}\right)$$

(v) value of innovation: *for the value of the innovation I will assume that the R&D costs incurred today yield a patent for a machine that comes online only in the next period. Consequently, the value of the innovation at time t can be written as*

$$\begin{aligned} V_t^* &= \sum_{s=1}^{M} \left(\frac{1}{1+g}\right)^{s-1} \left(\prod_{j=0}^{s-1}\left(\frac{1}{1+r_{t+j}}\right)\right) \widetilde{\pi}_{s-1}\left(g, \mathbf{c}\right) \\ &= V^*\left(r_t, \ldots, r_{t+M}; g, \mathbf{c}\right) \end{aligned}$$

(vi) rents on innovative activities: *For simplicity, I will assume that each household will take an equal share in each research project, such that there is no uncertainty about the return to their expenditures. Consequently, the income earned from innovative activities equals the sum of the flow profits made on the currently patented machine designs minus the current R&D expenditures, such that*

$$\begin{aligned} \Pi_t &= \sum_{\tau=0}^{M} A_{t-\tau}\widetilde{\pi}_\tau\left(g, \mathbf{c}\right) - X_{t,R} && (2) \\ &= A_t \sum_{\tau=0}^{M}\left(\frac{1}{1+g}\right)^{\tau} \widetilde{\pi}_\tau\left(g, \mathbf{c}\right) - A_t X_{t,R}^* && (3) \end{aligned}$$

where $X^*_{t,R} = X_{t,R}/A_t$.

The functions $p_\tau(g, \mathbf{c})$, $\widetilde{X}_\tau(g, \mathbf{c})$, $\widetilde{Y}(g, \mathbf{c})$ *and* $\widetilde{\pi}_\tau(g, \mathbf{c})$ *are independent of time.*

3.3 Competitive equilibrium

Having derived the solutions to the individual optimization problems of the three sectors in this economy, I am now able to combine these decentralized decisions to define the competitive equilibrium outcome of this economy. Because I have assumed that capital is not used in production, I have abstracted from possible transitional dynamics. Consequently, similar to Romer (1990), the competitive equilibrium defined below constitutes a balanced growth path

Definition 2 ***Competitive equilibrium***
Given $\mathbf{A}_0 = \{A_0, A_{-1}, \ldots\}$, *and* $\{k_0(h)\}_{h=0}^{\overline{h}}$, *a competitive equilibrium in this economy is a path*

$$\left\{\{c_t(h), y_t(h), k_t(h)\}_{h=0}^{1}, \mathbf{P}_t, \{D_t(\tau)\}_{\tau=0}^{\infty}, X_{R,t}, \Pi_t, r_t, A_t\right\}_{t=1}^{\infty}$$

such that
(i) Utility maximization*: Given* $\{y_t(h), r_t, k_0(h)\}_{t=1}^{\infty}$, $\{c_t(h)\}_{t=1}^{\infty}$ *solves the utility maximization problem of the workers of all types* $h \in [0, 1]$.
(ii) Optimal technology choice*: In every period, given* $\mathbf{P}_t$ *and* $\mathbf{A}_t$, *the demand sets* $\{D_t(\tau)\}_{\tau=0}^{\infty}$ *are determined by the workers' optimal technology choice decision introduced in subsection 3.2.*
(iii) Price equilibrium*: In every period, given* $\mathbf{A}_t$, $\mathbf{P}_t$ *is the price equilibrium.*
(iv) Patent race equilibrium*: In every period, the research intensity* $X_{R,t}$ *solves the patent race equilibrium.*
(v) Rents on innovative activity*:* Π_t *is determined by (2).*
(vi) Capital market clearing*: In every period, the interest rate* r_t *clears the capital market such that* $K_t = 0$.
(vii) Technological progress*: In every period,* $A_{t+1} = (1+g) A_t$.

In order to derive the competitive equilibrium of this economy it is easiest to rewrite the competitive equilibrium dynamics in terms of transformations of variables that will be constant on the equilibrium path. These transformations turn out to be output, capital, and consumption per efficiency unit, i.e. $Y^*_t = Y_t/A_t$, $K^*_t = K_t/A_t$, and $C^*_t = C_t/A_t$, prices per efficiency unit, i.e. $\widehat{P}_{t,\tau}$, the interest rate, i.e. r_t, and the research intensity and income per efficiency unit, i.e. $X^*_{t,R} = X_{t,R}/A_t$ and $\Pi^*_t = \Pi_t/A_t$. In terms of these variables, a competitive equilibrium is defined as a combination of variables

$$\left\{Y^*, K^*, C^*, \widehat{P}_\tau, \Pi^*, X^*_R, r\right\}$$

such that

$$\begin{aligned}
Y^* &= \widetilde{Y}(g, \mathbf{c}) &(4)\\
K^* &= 0 &(5)\\
X^*_R &= V^*(r, \ldots, r; g, \mathbf{c}) &(6)\\
\Pi^* &= \sum_{\tau=0}^{M} \left(\frac{1}{1+g}\right)^{\tau} \widetilde{\pi}_\tau(g, \mathbf{c}) - X^*_R &(7)\\
(1+g)K^* &= (1+r)K^* + Y^* - C^* + \Pi^* &(8)\\
C^*/C^* &= \left(\frac{1}{1+g}\right)[\beta(1+r)]^{\frac{1}{\sigma}} = 1 &(9)
\end{aligned}$$

and $\widehat{P}_\tau$ is the PSN equilibrium in the machine market. The following proposition establishes its existence and uniqueness.

Proposition 4 ***Existence and uniqueness of competitive equilibrium*** *For all $g > 0$, $\mathbf{c} \in \mathbb{R}^M_+$, there exists a unique septuple $\left\{Y^*, K^*, C^*, \widehat{P}_\tau, \Pi^*, X^*_R, r\right\}$ that satisfies equations (4) through (9) and where $\widehat{P}_\tau$ is the PSN equilibrium.*

In the next section, I will show how equipment price indices, as measured by the BEA and BLS, will behave on the competitive equilibrium path of this economy.

4 Spurious productivity growth

So, what would happen if the BEA and BLS would measure equipment price indices in the economy above? Before I analyze this question in detail, I start off by considering what would be a reasonable price index and productivity index in this economy. In order to consider this it is important to realize that on the balanced growth path

Implication 1 ***Average price paid per efficiency unit is constant over time***
The total number of efficiency units sold in the market equals

$$\sum_{\tau=0}^{M} A_{t-\tau} X_{t,\tau} = A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} \widetilde{X}_{\tau} \left(g, \mathbf{c} \right) = A_t \overline{X} \left(g, \mathbf{c} \right) \tag{10}$$

while the total revenue in the market equals

$$\sum_{\tau=0}^{M} P_{t,\tau} X_{t,\tau} = A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} p_{\tau} \left(g, \mathbf{c} \right) = A_t \overline{p} \left(g, \mathbf{c} \right)$$

such that the average price of an efficiency unit equals $\overline{p} \left(g, \mathbf{c} \right) / \overline{X} \left(g, \mathbf{c} \right)$ *and is independent of time.*

Implication 2 ***Average production cost per efficiency unit is constant over time***
The total production costs of all the efficiency units sold in the market equals

$$\sum_{\tau=0}^{M} A_{t-\tau} c_{\tau} X_{t,\tau}^{2} = A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} c_{\tau} \widetilde{X}_{\tau}^{2} \left(g, \mathbf{c} \right) = A_t \overline{C} \left(g, \mathbf{c} \right)$$

such that the average production cost per efficiency unit equals $\overline{C} \left(g, \mathbf{c} \right) / \overline{X} \left(g, \mathbf{c} \right)$ *and is again independent of time.*

These two implications are important because they suggest that (i) because the average production cost per efficiency unit is constant there is no productivity growth in the machine producing sector, (ii) because the average price paid per efficiency unit is constant, any reasonable quality adjusted investment price index should be constant over time.

I will now proceed with the following thought experiment in this section. Suppose that the BEA and BLS would observe the quality of machines, i.e. A_t, perfectly and would apply their methods to the construction of an investment price index in this economy, how would the resulting investment price index behave? As it turns out, for all the methods used by the BEA and BLS the resulting price index would not be constant, but would instead be steadily declining.

The methods applied by the BEA and BLS can basically be categorized in two types. The first is matched-model price indices, while the second is hedonic price indices. The following two subsections deal with each of these separately. For brevity purposes, I will not discuss the construction of these price indices in much detail. A thorough overview of their construction methodology is in Dulberger (1989).

4.1 Matched-model indices

Price changes measured by a matched model index are basically a weighted average of the price changes of the models that were in the market in the current and previous period. Denote P_t^M as the matched-model price index in this economy, then

$$P_t^M = (1 + \pi_t^M) P_{t-1}^M$$

where investment price inflation, i.e. π_t^M, equals

$$1 + \pi_t^M = \frac{\sum_{\tau=1}^M P_{t,\tau} X_{t,\tau}}{\sum_{\tau=1}^M P_{t-1,\tau-1} X_{t,\tau}}$$

By substituting in the equilibrium properties we obtain that

$$1 + \pi_t^M = \sum_{\tau=1}^{M} \omega_\tau \left(\frac{p_\tau (g, \mathbf{c})}{p_{\tau-1} (g, \mathbf{c})} \right) = 1 + \overline{\pi}^M$$

where the weights ω_τ equal

$$\omega_\tau = \frac{\left(\frac{1}{1+g}\right)^\tau p_{\tau-1} (g, \mathbf{c}) \widetilde{X}_\tau (g, \mathbf{c})}{\sum_{\tau=1}^M \left(\frac{1}{1+g}\right)^\tau p_{\tau-1} (g, \mathbf{c}) \widetilde{X}_\tau (g, \mathbf{c})}$$

But since result (iv) of Proposition 2 implies that for all $\tau = 1, \ldots, M$, $p_\tau (g, \mathbf{c}) < p_{\tau-1} (g, \mathbf{c})$, we obtain that, independent of g and $\mathbf{c}$, $1 + \overline{\pi}^M < 1$. Consequently, because of the varying markups, depending on where models are in the relative quality ladder, a matched-model price index will be steadily declining in this economy, i.e. investment price inflation is negative $\overline{\pi}^M < 0$.

4.2 Hedonic price indices

Hedonic price indices are used by the BEA and BLS to quality adjust the price indices for computer equipment, and software. The basic idea of hedonic price indices is to measure certain observable quality indicators of equipment and then use regression analysis to consider what part of price increases can be attributed to the changing quality composition of the equipment sold and what part is due to pure inflation.

There are two types of hedonic price indices. The first type is based on a sequence of separate cross sectional regressions, each for a specific period. This is the methodology that the BLS applies for the construction of some of its Producer Price Indices. Holdway (2001) contains an excellent explanation of the BLS' methodology. The second type consists of hedonic price indices based on pooled cross-sectional regressions. These are applied by the BEA for the construction of its investment price indices used in the National Income and Product Accounts. Wasshausen (2000) contains a detailed description of the evolution of the hedonic regressions used by the BEA over the years.

In order to address the behavior of these two price indices in my theoretical model, I first describe what the model implies for the cross-sectional behavior of prices. Throughout, I will assume that the quality index A_t is observed correctly, which is doubtful in the actual application of hedonic price methods. Prices in this model satisfy

$$\ln P_{t,\tau} = \ln A_{t-\tau} + \ln p_\tau \tag{11}$$

Bearing in mind that (11) is the underlying data generating process, I will consider the application of log-log hedonic regressions. These are regressions where $\ln P_{t,\tau}$ is regressed on $\ln A_{t-\tau}$ and, possibly, some dummy variables.

A cross-sectional hedonic regression, as used by the BLS, in the context of my theoretical model would be for a specific t and of the form

$$\ln P_{t,\tau} = \beta_1 + \beta_2 \ln A_{t-\tau}$$

the resulting regression coefficient for quality will then equal

$$\widehat{\beta}_2 = 1 + \frac{\sum_{\tau=0}^{M-1} \left(\ln A_{t-\tau} - \overline{\ln A_{t-\tau}}\right)\left(\ln p_\tau - \overline{\ln p_\tau}\right)}{\sum_{\tau=0}^{M-1} \left(\ln A_{t-\tau} - \overline{\ln A_{t-\tau}}\right)^2} > 1 \tag{12}$$

where

$$\overline{\ln A_{t-\tau}} = \frac{1}{M-1} \sum_{\tau=0}^{M-1} \ln A_{t-\tau} \text{ and } \overline{\ln p_\tau} = \frac{1}{M-1} \sum_{\tau=0}^{M-1} \ln p_\tau$$

and the summation runs up till $M-1$, because $P_{t,M}=0$ and thus can not be taken a logarithm of. The positive bias in the estimate $\widehat{\beta}_2$ is an example of an omitted variable problem where the omitted variable, i.e. $\ln p_\tau$, is positively correlated with the regressor, i.e. $\ln A_{t-\tau}$. This positive correlation is again implied by result (iv) of Proposition 2.

How does this positive bias affect measured investment price inflation? To answer this question, I will compare the implication for the sequence of frontier machines over time. The theoretical model implies that $\ln A_t = (1+g) + \ln A_{t-1}$ and, as can be seen from (11), $\ln P_{t,0} = (1+g) + \ln P_{t-1,0}$. Consider a hedonic price index for the frontier machine, which I will denote by P_t^H. The percentage change in the hedonic price index, i.e. π_t^H, is equal to the percentage change in the prices minus the part that is attributable to the quality change. That is,

$$\pi_t^H \approx \Delta \ln P_t^H = \Delta \ln P_{t,0} - \widehat{\beta}_2 \Delta \ln A_t = \left(1-\widehat{\beta}_2\right)(1+g) < 0$$

where Δ is the first difference operator. Hence, the positive bias in the estimate of β_2 leads to spurious investment price deflation. The extent of this deflation is increasing in the correlation between $\ln p_\tau$ and $\ln A_{t-\tau}$. Note that if $\beta_2 = 1$ would be estimated correctly, then this method would lead to the proper result that there is no investment price deflation whatsoever. Furthermore, the balanced growth properties of the model imply that $\widehat{\beta}_2$ and thus π_t^H are independent of time, i.e. $\pi_t^H = \overline{\pi}^H$.

Instead of data on a single cross-section for each year, the BEA pools these cross-sections for the quality adjustment of the price indices used for some types of computer equipment in the NIPA. Wasshausen (2000) contains a detailed description of the hedonic regressions used. In the context of the theoretical model here, the BEA's pooled cross-sectional regressions boil down to the regression of $\ln P_{t,\tau}$ on $\ln A_{t-\tau}$ and time dummies. That is, the regression equation is

$$\ln P_{t,\tau} = \sum_{t=1}^{T} \delta_t D_t + \beta_2 \ln A_{t-\tau}$$

where $D_t = 1$ in period t and 0 otherwise. It is fairly straightforward to show that, because of the balaced growth properties of the model, in this equation the estimated coefficient on quality, i.e. $\widehat{\beta}_2$, is the same as in (12). Moreover, the estimated time-varying intercepts, i.e. $\hat{\delta}_t$'s, will satisfy

$$\hat{\delta}_t = \hat{\delta}_{t-1} + \left(1-\widehat{\beta}_2\right)(1+g)$$

If quality adjusted inflation is directly measured by the changes in the estimated time-varying intercepts, then the estimated equipment price inflation using this method is

$$\pi_t^H \approx \hat{\delta}_t - \hat{\delta}_{t-1} = \left(1 - \widehat{\beta}_2\right)(1+g) < 0$$

which is equal to that measured by the simple cross-sectional method. Again, this method leads to the measurement of spurious equipment price deflation due to imperfect competition.

In the simple theoretical model in this paper, this bias can be eliminated by the inclusion of vintage age dummies. That is, the hedonic regression equation

$$\ln P_{t,\tau} = \sum_{t=1}^{T} \delta_t D_t + \sum_{\tau=0}^{M-1} \theta_\tau D_\tau + \beta_2 \ln A_{t-\tau} \tag{13}$$

where $D_\tau = 1$ if machine of vintage age τ and zero otherwise, would lead to the appropriate regression result that $\hat{\delta}_t = \hat{\delta}_{t-1}$ for all $t \geq 1$. Berndt, Griliches and Rapaport (1993) use vintage age dummies in their empirical studies of PC prices and find limited significance. One has to realize, however, that vintage age dummies are a good proxy for the extent of the markup in the theoretical model here, because I have assumed that only one new machine design is invented in each period. Furthermore, the coefficients θ_τ in (13) are constant because I assume that g is constant over time. Though these two assumptions are innocuous for the expositional purpose of the theoretical model introduced in this paper, they would be unrealistic to make for the purpose of an empirical analysis. For example, in the case of PC's, where Berndt, Griliches, and Rapaport (1993) use dummies for vintage age in years, the frequency of introduction of new models is much higher than once a year.

This is not the first paper to point out that markups might affect hedonic price indices. Pakes (2001), for example, contains an illuminating discussion of the same topic. What is different here is that I show, in the specific theoretical context of my model, that these indices have a bias with a known sign and how this bias comes about through imperfect competition.

4.3 A numerical example

In the two subsections above, I have shown that the methods applied by the BEA and BLS lead to an upward bias in measured investment price deflation in the theoretical model introduced in this paper. I have, however, not addressed the possible magnitude of this bias. In this subsection, I use a simple numerical example to get at this magnitude.

As one can see from the explanation in the previous subsection, the only parameters that are relevant for this bias are the growth rate of the technological frontier, i.e. g, the vintage age dependent cost parameters, collected in the vector $\mathbf{c}$, and the patent length M.

As Jaffe (1999) reports, the standard patent length in the U.S. since 1994 is 20 years, so I will fix $M = 20$. I will illustrate the results for two different values of g. The first is $g = 0.021$ which is the average post-war annual growth rate of U.S. real GDP per capita. The second is $g = 0.587$, which is the annual growth rate of microprocessor speeds implied by Moore's law, i.e. the prediction that the speed of microprocessors will double about once every 18 months.

What is left to choose is the cost structure, $\mathbf{c}$. I will allow for possible learning by doing in the sense that

$$c_\tau = \frac{c_0}{(1+\gamma)^\tau} \text{ where } \gamma > 0$$

Here γ reflects the per period percentage gain in efficiency in the production of machines of a particular vintage due to learning by doing. I will illustrate the outcome of the model for different rates of learning by doing and different values of c_0. The values of γ and c_0 that I use are chosen purely for illustrative purposes to show the effect the cost parameters have on the equilibrium outcome and the deflation bias.

Table 1 lists the parameter combinations and implied measured investment price inflation rates for matched-model and hedonic price indices. Before I discuss these deflation rates in detail it is useful to first consider the equilibrium results plotted in Figures 3 and 4. Each of these figures plots the price as a function of the vintage (upper-left), the price per efficiency units as a function of the number of efficiency units (lower-left), the market share for each vintage (upper-right), and the diffusion curve (lower-right). Diffusion is defined as the percentage of people using vintages that are as good as or better than a particular vintage. Figure 3 plots these variables for the Moore's law case. In the case of no production costs, i.e. $c_0 = 0$, the producer of the frontier vintage in this case is the dominating market force. This producer can basically decide what part of the market he conquers and what part he leaves for his competitors. Consequently, the frontier vintage

Table 1: Numerical results

case	g	c_0	γ	$\overline{\pi}^M$	$\overline{\pi}^H$
			Moore's law		
A1	0.587	0	0	-67.2%	-383.8%
A2	0.587	30	0.125	-4.8%	-40.3%
A3	0.587	30	0.25	-7.0%	-88.2%
A4	0.587	60	0.25	-4.5%	-59.2%
			real GDP growth		
B1	0.021	0	0	-72.9%	-6424.8%
B2	0.021	5	0.25	-14.3%	-1244.5%
B3	0.021	5	0.5	-20.4%	-2751.8%
B4	0.021	10	0.5	-18.9%	-2241.9%

absorbs more than half of the market. If the supplier of the frontier vintage faces significant production costs then decreasing returns to scale might force him to raise his price and to lower his market share in order to cover costs. This effect can be seen because the supplier of the frontier vintage has a lower market share in the case of A4 than in A3. Learning by doing gives the suppliers of older vintages a relative competitive edge over those of new vintages. Consequently, an increase in the learning by doing rate shifts market share from newer to older vintages. This can be seen when one compares the market share curves for cases A2 and A3, where the increased learning by doing increases the market shares of vintages 6 through 18 at the expense of the other ones. The lower-left hand panel of Figure 3 plots $p_\tau(g, \mathbf{c})$. As you can see, $p_\tau(g, \mathbf{c})$ is relatively flat for the cases A2, A3, and A4 and has a lower correlation with $A_{t-\tau}$ plotted on the x-axis of that panel. Consequently, the downward bias in the hedonic price index is much smaller in cases A2-A4, as is listed in Table 1, than in case A1. In fact, in case A1 the bias in the estimated coefficient in the hedonic regression is so severe that it implies an infeasible equipment deflation rate of -383%.

Figure 4 depicts similar results for the case that $g = 0.021$. Again it shows how an increase in c_0 shifts market power away from the frontier vintages, while an increase in the rate of learning by doing benefits the middle vintages relative to the newer ones. The lower-right hand panel of the figure shows how cases B2-B4 exhibit s-shaped diffusion. Most importantly, the lower-left hand panel suggests that in all cases $p_\tau(g, \mathbf{c})$ is highly positively correlated

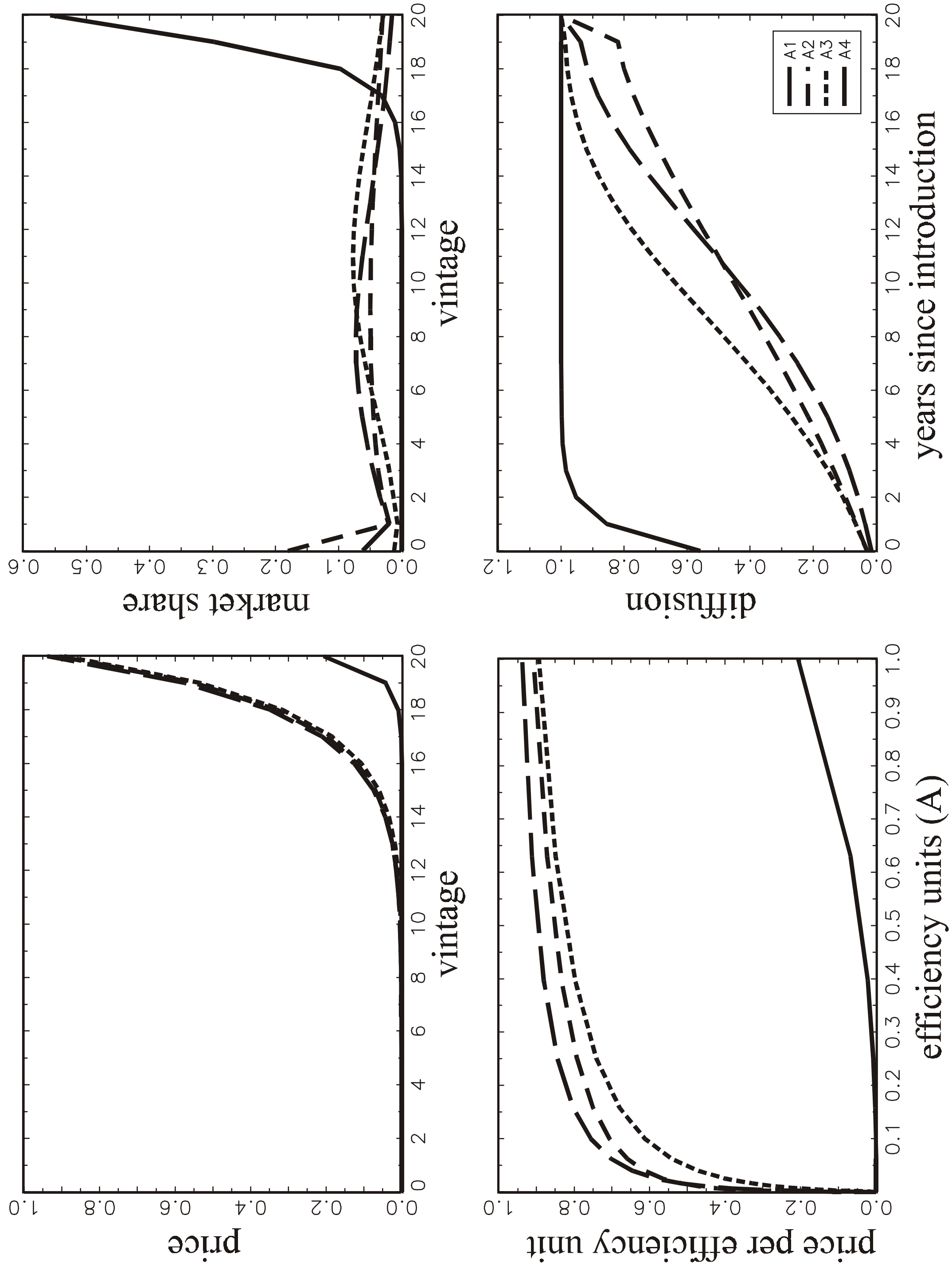


Figure 3: Equilibrium outcome for Moore's law ($g = 0.587$)

with $A_{t-\tau}$. This manufests itself is the sever downward biases in the hedonic price indices listed for the GDP growth case in Table 1. These biases are aggrevated again because they are also decreasing in the sample standard deviation of $\ln A_{t-\tau}$, which is much smaller when $M = 20$ and $g = 0.021$ than in the Moore's law example where $g = 0.587$.

The conclusion of this simple numerical exercise is thus that in my theoretical model the potential biases in measured investment price deflation can be enormous. For hedonic price indices they can even be so enormous that they lead to infeasible deflation rates. Furthermore, these biases turn out to be extremely sensitive to the underlying market structure.

5 Do we observe varying markups in the data?

So, in the context of the theoretical model introduced above the BLS and BEA methodologies tend to overestimate investment price deflation and therefore productivity growth in the capital goods producing sector. The question that remains, however, is how important are the factors that drive this result in the actual data? In this section I will provide some anecdotal evidence on these factors. The evidence that I present here is only meant to be tentative. The importance of varying markups definitely deserves a more thorough empirical examination than there is room for in this paper.

As I argued in the introduction, IT producing firms need to make profits on their sales in order to recover the sunk cost of the R&D that led to the invention of the products they sell. Table 2 lists the reported operating margins for 7 of the leading IT producing firms for 2000 and the first quarter of 2001. Each of them reports operating margins that are bigger than 5%. Microsoft has by far the highest operating margins sometimes even exceeding 40%.

The existence of markups, however, does not necessarily imply the overestimation of investment price deflation. What drives this overestimation in my theoretical framework is that the average price per efficiency unit is declining in the quality of the machine sold.

There aren't that many capital goods for which quality can be measured in terms of a single indicator, being the empirical equivalent of A_t. One good for which there is such a single quality-indicator is the microprocessor. Though, arguably, the market for microprocessors does not precisely exhibit the market structure considered in the theoretical model in this paper, the

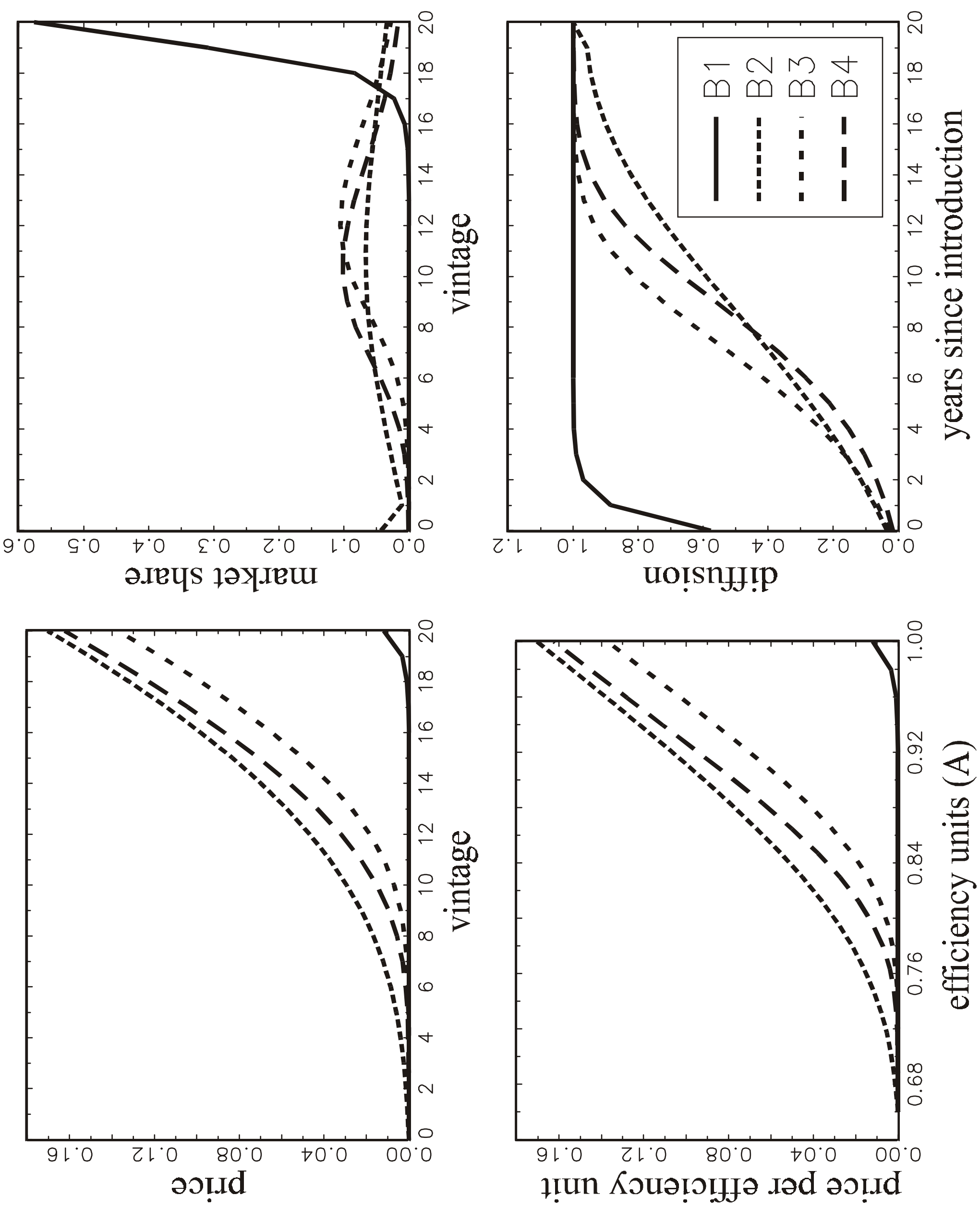


Figure 4: Equilibrium output for GDP growth ($g = 0.021$)

Table 2: Operating margins for IT producers

	2000				2001
Company	*Q1*	*Q2*	*Q3*	*Q4*	*Q1*
Intel	32.7	29.3	32.8	29.8	10.7
IBM	11.0	13.0	12.9	15.1	12.3
AMD	16.5	21.4	21.8	16.6	14.0
Hewlett-Packard	-	7.4	9.2	7.2	5.3
Microsoft	42.2	41.5	44.5	39.8	38.0
Dell	8.6	9.6	9.9	6.8	-
Compaq	3.2	5.8	7.0	5.6	4.5

Note: Operating margin is revenue minus operating costs.
Source: CNNfn, July 2001

observed price schedule does satisfy one of the important equilibrium properties of the theoretical model. Namely, the price per MHz is increasing in the speed of the processor that is bought. Figure 5 plots the prices and prices per MHz of 13 microprocessors, of five different architectures, sold by Intel and AMD in April 1999[2]. The top panel depicts the prices as a function of processor speed, while the bottom panel shows the price per MHz. As can be seen from the bottom panel, the price per MHz is increasing for all types of processors.

Is this increase due to imperfect competition? A thorough answer to this question is beyond the scope of this paper. It is fair to mention, however, that an alternative explanation for this observed price difference is that producers charge price equal to average cost, but that average costs of newer vintages are so much higher because of learning by doing effects. Irwin and Klenow (1994), for example, provide evidence of that a doubling of cumulative past output of memory chips leads to a 20 percent drop in production costs. Since cumulative output of previous vintages is much higher than that of the newest vintage, this might explain part of the increase. One has to realize, though, that the increased marginal cost of the newest features is not only observed for capital goods, but is also observed for many consumer goods, like electronics, where it is hard to argue that learning by doing applies to the same extent. An example of such an electronics product for which markups

[2]Source: Electronic Engineering Times, April 19, 1999, “Intel, AMD Slash Processor Prices”.

are increasing as a fraction of price in the level of advancement of the model is the Palm Pilot. In the spring of 2001 Palm charged a price of \$399 for its high-end model, the Palm V, on which it made a profit of \$150, a 38% markup. For its low-end model, the m-100, it charged \$149 for a profit of \$26, a markup of 17%[3]. There are even more striking examples of how markups affect prices. The most extreme is Cockburn and Anis (1998), who show that generic Arthritis drugs are cheaper than patented ones, even though clinical trials suggest that the generic ones are of superior quality.

[3] Source: BusinessWeek, June 4, 2001, "Palm's Market Starts to Melt Down in it's Hands"

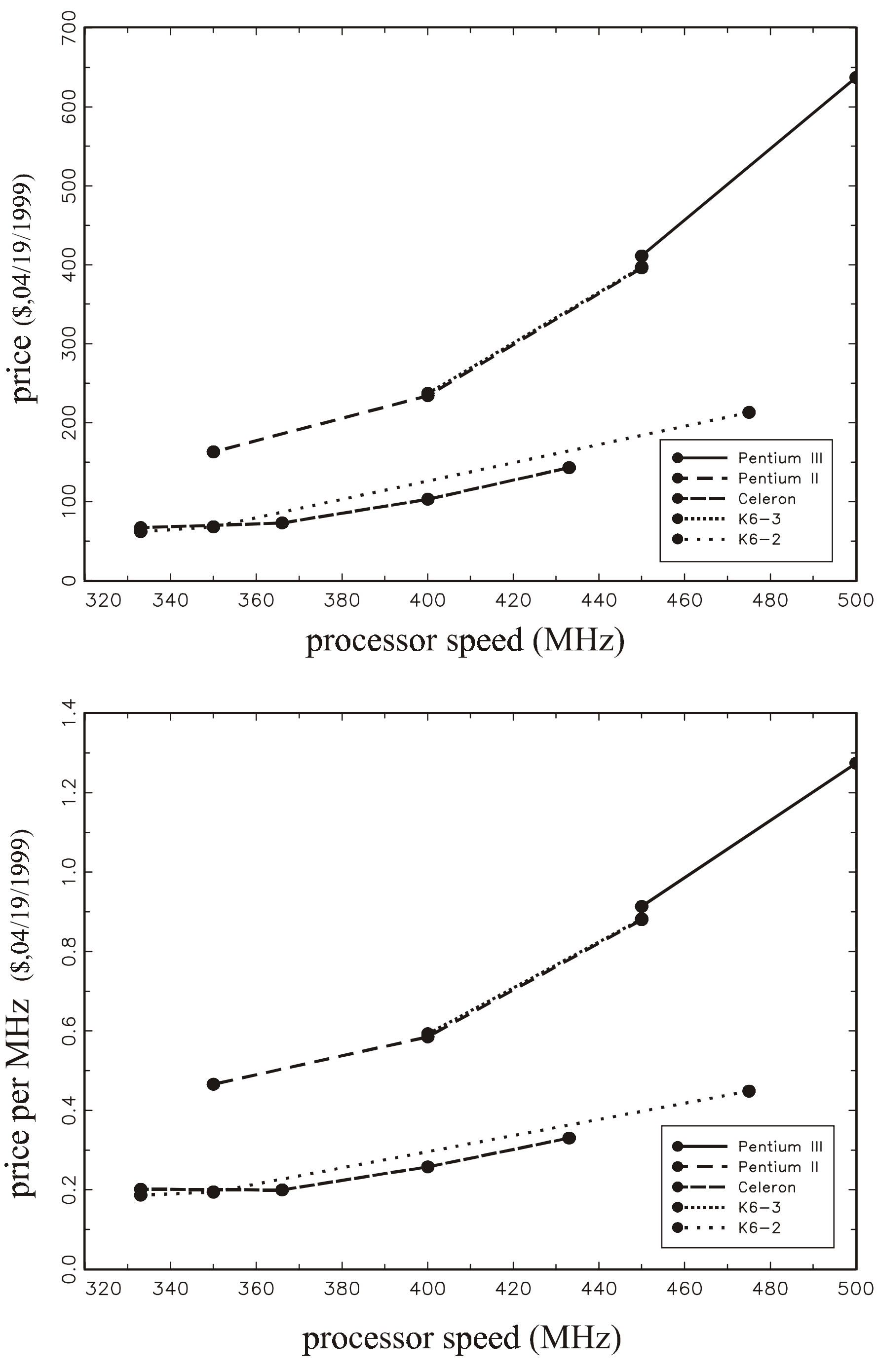


Figure 5: Price per MHz is increasing with speed of processor

6 Conclusion

In this paper I argued that, by not taking into account the fact that equipment markets are not perfectly competitive, the BEA and BLS are likely to overestimate investment price deflation. To illustrate my argument, I introduced an endogenous growth model in which suppliers of different vintages of machines imperfectly compete for the demand of a heterogeneous set of workers. This market structure results in a price schedule which would lead the BLS and BEA to find investment price deflation, even though the model economy does not exhibit any investment price deflation at all. The measured deflation in the model economy is a complete statistical artifact.

The theoretical example given in this paper is an extreme case. In practice there is good reason to believe that the quality of investment goods has been steadily improving. Quality adjustments of capital goods, however, are currently treated with double standards. On the one hand, there are computer equipment and software to which the BEA and BLS extensively apply the quality adjustment methods discussed in this paper. While on the other hand, there are the other capital goods for which there is no serious effort to quality adjust.

This paper suggests that real investment in computer equipment and software is likely to be overstated because of the bias discussed here. Thus, everything else equal, the results in this paper suggest an overestimation of real output growth and productivity growth in the IT producing sector. However, real output growth and productivity for other capital goods producing sectors is likely to be underestimated because it is virtually not quality adjusted at all.

The extent to which price deflation for computer equipment and software are overestimated depends on the structure of the markups, which deserves a much more closer look in further research than the tentative evidence provided here.

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1 Proofs

Proof of proposition 1: Properties of demand sets

(i) Consider $h' > h$ and $\tau' > \tau$, then $h \in D_t(\tau)$ implies that

$$\forall\ s \in \mathbb{N}:\ A_{t-\tau}h - P_{t,\tau} \geq A_{t-s}h - P_{t,s}$$

or, equivalently, in terms of marginal benefits and costs

$$\forall\ s \in \mathbb{N}:\ (A_{t-\tau} - A_{t-s})\, h \geq P_{t,\tau} - P_{t,s}$$

Consequently, because for all $\tau' > \tau$ strictly positive technological progress implies $A_{t-\tau'} > A_{t-\tau}$, the marginal benefits from updating for the worker of type h' exceed those of the worker of type h. That is,

$$\forall \tau' > \tau:\ \ (A_{t-\tau} - A_{t-\tau'})\, h \geq P_{t,\tau} - P_{t,\tau'}$$

This implies that it must thus be true that $h' \notin D_t(\tau')$ for all $\tau' > \tau$.

(ii) Because patents expire after M periods, all intermediate goods producers of vintages of age M or older face perfect competition. As a result, the price of these vintages is competed down to zero. Consequently all workers will use at least the best technology that is available for free, such that no one will use a technology that is older than M.

(iii) Since $h \backsim unif\,(0,1)$, it suffices to prove that $\widetilde{\mathcal{H}}$ contains a finite number of elements. Since workers will use only technologies $\{0,\ldots,M\}$ there are only a finite number of combinations between which workers can be indifferent. I will show that, if a worker of type h is indifferent between two intermediate goods, then no other worker will be. That is, define the set

$$\widehat{\mathcal{H}}_t(\tau,\tau') = \{h \in [0,1] \mid h \in D_t(\tau) \wedge h \in D_t(\tau')\}$$

such that

$$\widetilde{\mathcal{H}}_t = \bigcup_{\tau=0}^{M-1} \bigcup_{\tau'=\tau+1}^{M} \widehat{\mathcal{H}}_t(\tau,\tau')$$

and

$$\mu\left(\widetilde{\mathcal{H}}_t\right) \leq \frac{1}{\overline{h}} \sum_{\tau=0}^{M-1} \sum_{\tau'=\tau+1}^{M} \mu\left(\widehat{\mathcal{H}}_t(\tau,\tau')\right)$$

I will simply show that $\forall \tau' > \tau : \mu\left(\widehat{\mathcal{H}}_t(\tau,\tau')\right) = 0$. Let $h \in [0,1]$ be such that $h \in D_t(\tau)$ as well as $h \in D_t(\tau')$ for $\tau' > \tau$. In that case

$$A_{t-\tau}h - P_{t,\tau} = A_{t-\tau'}h - P_{t,\tau'}$$

or equivalently

$$(A_{t-\tau} - A_{t-\tau'})\, h = P_{t,\tau} - P_{t,\tau'}$$

This, however implies that for all $h' > h > h''$

$$(A_{t-\tau} - A_{t-\tau'})\, h' > P_{t,\tau} - P_{t,\tau'} > (A_{t-\tau} - A_{t-\tau'})\, h''$$

such that the workers of type $h' > h$ will prefer τ over τ', while workers of type $h'' < h$ will do the opposite. Hence, $\widehat{\mathcal{H}}_t(\tau, \tau') = \{h\}$ and is of measure zero.
(*iv*) Consider $h'' > h' > h$ such that $h'' \in D_t(\tau)$ as well as $h \in D_t(\tau)$. This implies that

$$\forall\ s \in \mathbb{N}: \ (A_{t-\tau} - A_{t-s})\, h'' > (A_{t-\tau} - A_{t-s})\, h' > (A_{t-\tau} - A_{t-s})\, h \geq P_{t,\tau} - P_{t,s}$$

such that

$$\forall\ s \in \mathbb{N} : A_{t-\tau} h' - P_{t,\tau} > A_{t-s} h' - P_{t,s}$$

and thus $h' \in D_t(\tau)$. Hence, $D_t(\tau)$ is connected. ■

Proof of proposition 2: Solution to equilibrium price schedule

I will prove this proposition in two parts. The first consists of my proof of (*i*) and (*ii*). In the second part, I use (*i*) and (*ii*), together with results (*i*) and (*iv*) of proposition 1 to prove (*iii*).
Proof of (i) and (ii): I will prove these parts by induction. The proof applies lemmas 5 and 6. Lemma 5 implies that, no matter what the other machine producers do, the frontier machine producer will always set a price $P_{t,0} > \frac{c_0}{2} A_t X_{t,0}$ and make strictly positive profits. This lemma initializes the induction. Lemma 6 then shows that, independently of what the suppliers of older vintages do, if all suppliers of newer vintages charge a markup and make strictly positive profits, then so will the supplier of vintage $\tau \in \{1, \ldots, M-1\}$. Combining these two lemmas implies that, if there is a Pure Strategy Nash equilibrium, then it must be one in which (*i*) all monopoly suppliers of machines charge a strictly positive markup and make strictly positive profits. That is,

$$(i)\ P_{t,\tau} > \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau} \text{ for all } \tau \in \{0, \ldots, M-1\} \text{ and } (ii)\ \mu(D(\tau)) > 0 \text{ for all } \tau \in \{0, \ldots, M\}$$

Combining (*ii*) with parts (*i*) and (*iv*) of proposition 1, this implies that if there is a Pure Strategy Nash equilibrium, then there exist $\{h_t(0), \ldots, h_t(M-1)\}$ such that

$$D(\tau) = \begin{cases} \left[h_t(0), \overline{h}\right] & \text{for} \quad \tau = 0 \\ \left[h_t(\tau), h_t(\tau-1)\right] & \text{for} \quad \tau \in \{1, \ldots, M-1\} \\ \left[0, h_t(M-1)\right] & \text{for} \quad \tau = M \end{cases}$$

and

$$h_t(\tau) = \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \hat{P}_{t,\tau} - \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}} \hat{P}_{t,\tau+1}$$

which is the basis for the derivation of the form of the Pure Strategy Nash equilibrium in part (*iii*).
Proof of (iii), (iv), and (v): A machine producer of machines of vintage age τ chooses $\hat{P}_{t,\tau}$ to maximize profits

$$\pi_{t,\tau} = A_{t-\tau} \left(\hat{P}_{t,\tau} - \frac{c_\tau}{2} X_{t,\tau} \right) X_{t,\tau}$$

since parts (i) and (ii) have proven that the solution to the PSN-equilibrium is interior, I will simply use standard calculus to find a profit maximizing solution. The associated first order necessary condition for the profit maximization problem is

$$0 = A_{t-\tau}\left[\hat{P}_{t,\tau}\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}} + X_{t,\tau} - c_\tau X_{t,\tau}\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}}\right]$$

such that if there is an interior solution to this problem, it must satisfy

$$\hat{P}_{t,\tau} = \left[c_\tau - \frac{1}{\partial X_{t,\tau}/\partial \hat{P}_{t,\tau}}\right]X_{t,\tau}$$

For the supplier of a non-frontier vintage, i.e. $\tau \in \{1, \ldots, M-1\}$, the demand set satisfies

$$\begin{aligned} X_{t,\tau} &= \left[\frac{A_{t-\tau+1}}{A_{t-\tau+1} - A_{t-\tau}}\hat{P}_{t,\tau-1} - \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}}\hat{P}_{t,\tau} - \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}}\hat{P}_{t,\tau} + \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}}\hat{P}_{t,\tau+1}\right] \\ &= w_{t,\tau}^{\tau-1}\hat{P}_{t,\tau-1} - \left(w_{t,\tau}^{\tau-1} + w_{t,\tau}^{\tau+1}\right)\hat{P}_{t,\tau} + w_{t,\tau}^{\tau+1}\hat{P}_{t,\tau+1} \end{aligned}$$

such that

$$\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}} = -\left(w_{t,\tau}^{\tau-1} + w_{t,\tau}^{\tau+1}\right)$$

Using the above two equations to solve the first order necessary condition yields

$$\hat{P}_{t,\tau} = \left[\frac{1 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}{2 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}\right]\left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau-1} + \frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau+1}\right]$$

which is the first part of the second order difference equation in the proposition. Substituting this expression in that for the demand set yields that

$$X_{t,\tau} = \frac{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}{1 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}\hat{P}_{t,\tau}$$

For the supplier of the frontier vintage, the demand set satisfies

$$X_{t,0} = \left[1 - \left(1 + w_{t,0}^1\right)\hat{P}_{t,0} + w_{t,0}^1\hat{P}_{t,1}\right]$$

such that

$$\frac{\partial X_{t,0}}{\partial \hat{P}_{t,0}} = -\left(1 + w_{t,0}^1\right)$$

Using the above two equations to solve the necessary condition for an interior solution yields

$$\widehat{P}_{t,0} = \left[\frac{1 + c_0\left(1 + w_{t,0}^1\right)}{2 + c_0\left(1 + w_{t,0}^1\right)}\right]\left[\frac{1}{1 + w_{t,0}^1} + \frac{w_{t,0}^1}{1 + w_{t,0}^1}\widehat{P}_{t,1}\right]$$

Substituting this into the expression for the demand set gives

$$X_{t,0} = \frac{1 + w_{t,0}^{1}}{1 + c_0 \left(1 + w_{t,0}^{1}\right)} \hat{P}_{t,0}$$

What is left to show is that $\hat{P}_{t,\tau} > \hat{P}_{t,\tau+1}$. This follows from the fact that for $\tau \in \{0, \ldots, M-1\}$, the second order difference equation that has to be satisfied in equilibrium implies

$$\hat{P}_{t,\tau} = \left[1 + c_\tau \left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)\right] \left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}} \left(\widehat{P}_{t,\tau-1} - \widehat{P}_{t,\tau}\right) - \frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}} \left(\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1}\right)\right] > 0$$

Since $\left(\widehat{P}_{t,M-1} - \widehat{P}_{t,M}\right) > 0$, a simple induction argument can be used to show that $\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1} > 0$ for all $\tau \in \{0, \ldots, M-1\}$.■

Proof of proposition 3: Value of innovation, output, etc., at constant g

(i) Note that, if g is constant, the recursion (1) can be written as a set of linear equations. In matrix form, with the appropriately defined matrices

$$\underset{M+1 \times M+1}{\mathbf{F}(g, \mathbf{c})} \ \widehat{\mathbf{P}}_t = \underset{M+1 \times 1}{\mathbf{G}(g, \mathbf{c})}$$

such that the equilibrium vector with prices equals

$$\widehat{\mathbf{P}}_t = \left[[\mathbf{F}(g, \mathbf{c})]^{-1} \mathbf{G}(g, \mathbf{c})\right]$$

which implies that we can write $\widehat{P}_{t,\tau} = p_\tau (g, \mathbf{c})$ where $p_\tau (g, \mathbf{c}) > p_{\tau+1} (g, \mathbf{c})$ simply because $\hat{P}_{t,\tau}$ is decreasing in τ .

(ii) For the demand sets we obtain that we can write

$$X_{t,\tau} = \begin{cases} \frac{1 + w_{t,0}^{1}}{1 + c_0 \left(1 + w_{t,0}^{1}\right)} p_0 (g, \mathbf{c}) & \text{for} \quad \tau = 0 \\ \frac{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}{1 + c_\tau \left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)} p_\tau (g, \mathbf{c}) & \text{for} \quad \tau = 1, \ldots, M-1 \\ 1 - \sum_{\tau=0}^{M-1} X_{t,\tau} & \text{for} \quad \tau = M \end{cases}$$

However, since when g is constant $w_{t,\tau}^{\tau+1}$ and $w_{t,\tau}^{\tau-1}$ only depend on g and not on t for all $\tau = 1, \ldots, M-1$, we can write $X_{t,\tau} = \tilde{X}_\tau(g, \mathbf{c})$ for $\tau = 1, \ldots, M-1$. In that case however, $X_{t,M} = 1 - \sum_{\tau=0}^{M-1} \tilde{X}_\tau(g, \mathbf{c}) = \tilde{X}_M(g, \mathbf{c})$ and is also constant over time.

(iii) Aggregate output in terms of efficiency units can be written as

$$\begin{aligned} Y_t^* &= \frac{Y_t}{A_t} = \frac{1}{A_t} \sum_{\tau=0}^{M} \int_{h \in D_t(\tau)} \left[A_{t-\tau} h - P_{t,\tau}\right] dh \\ &= \frac{1}{A_t} \sum_{\tau=0}^{M} A_{t-\tau} \underbrace{\int_{h \in D_t(\tau)} h dh}_{Z_{t,\tau}} - P_{t,\tau} X_{t,\tau} \end{aligned}$$

Note that

$$\frac{1}{\overline{h}}\int_a^b h dh = \frac{1}{2\overline{h}}\left(a^2 - b^2\right) = \frac{1}{2}\left[\frac{1}{\overline{h}}(a-b)\right](a+b)$$

Applying this to the equation for aggregate yields

$$\begin{aligned} Z_{t,\tau} &= \begin{cases} \frac{1}{2}X_{t,\tau}\left[1+h_t(0)\right] & \text{for} \quad \tau = 0 \\ \frac{1}{2}X_{t,\tau}\left[h_t(\tau-1)+h_t(\tau)\right] & \text{for} \quad \tau = 1,\ldots,M-1 \\ \frac{1}{2}X_{t,\tau}\left[h_t(M-1)\right] & \text{for} \quad \tau = M \end{cases} \\ &= X_{t,\tau}Z^*_{t,\tau} \end{aligned}$$

Using this notation, aggregate output has the representation

$$Y_t^* = \frac{1}{A_t}\sum_{\tau=0}^{M} X_{t,\tau}\left[A_{t-\tau}Z^*_{t,\tau} - P_{t,\tau}\right]$$

where

$$Z^*_{t,\tau} = \begin{cases} \frac{1}{2}\left[1+\left(1+w^1_{t,0}\right)\hat{P}_{t,0} - w^1_{t,0}\hat{P}_{t,1}\right] & \text{for} \quad \tau = 0 \\ \frac{1}{2}\left[w^{\tau-1}_{t,\tau}\hat{P}_{t,\tau-1} - \left(2+w^{\tau-1}_{t,\tau}+w^{\tau+1}_{t,\tau}\right)\hat{P}_{t,\tau} - w^{\tau+1}_{t,\tau}\hat{P}_{t,\tau+1}\right] & \text{for} \quad \tau = 1,\ldots,M-1 \\ \frac{1}{2}w^{M-1}_{t,M}\hat{P}_{t,\tau-1} & \text{for} \quad \tau = M \end{cases}$$

Since again $w^{\tau+1}_{t,\tau}$ and $w^{\tau-1}_{t,\tau}$ only depend on g and not on t and $\hat{P}_{t,\tau} = p_\tau(g,\mathbf{c})$, we can write $Z^*_{t,\tau} = \tilde{z}_\tau(g,\mathbf{c})$. This means that output can be represented as

$$\begin{aligned} Y_t^* &= \frac{1}{A_t}\sum_{\tau=0}^{M}\tilde{X}_\tau(g,\mathbf{c})\left[A_{t-\tau}\tilde{z}_\tau(g,\mathbf{c}) - p_\tau(g,\mathbf{c})\right] \\ &= \sum_{\tau=0}^{M}\tilde{X}_\tau(g,\mathbf{c})\left[\left(\frac{1}{1+g}\right)^\tau\tilde{z}_\tau(g,\mathbf{c}) - p_\tau(g,\mathbf{c})\right] \\ &= \tilde{Y}(g,\mathbf{c}) \end{aligned}$$

(iv) For the profits we obtain that

$$\begin{aligned} \pi^*_{t,\tau} &= \frac{\pi_{t,\tau}}{A_{t-\tau}} = \left(\hat{P}_{t,\tau} - \frac{c_\tau}{2}X_{t,\tau}\right)X_{t,\tau} \\ &= \left(p_\tau(g,\mathbf{c}) - \frac{c_\tau}{2}\tilde{X}_\tau(g,\mathbf{c})\right)\tilde{X}_\tau(g,\mathbf{c}) \\ &= \tilde{\pi}_\tau(g,\mathbf{c}) \end{aligned}$$

(v) and (vi) Follow directly from the explanation in the main text.■

Proof of proposition 4: Existence and uniqueness of competitive equilibrium

The competitive equilibrium equations (4) through (9) can be solved sequentially. That is, (9) pins down the equilibrium interest rate as

$$r = \frac{1}{\beta}(1+g)^{\sigma} - 1 > 0$$

At this constant interest rate the value of a new innovation equals

$$V^*(r,\ldots,r;g,\mathbf{c}) = \left(\frac{1}{1+r}\right)\sum_{s=0}^{M-1}\left(\frac{1}{1+g}\right)^s\left(\frac{1}{1+r}\right)^s\tilde{\pi}_s(g,\mathbf{c}) = X_R^*$$

and the profits from the innovative activities equal

$$\Pi^* = \sum_{s=0}^{M-1}\left(\frac{1}{1+g}\right)^s\left[1-\left(\frac{1}{1+r}\right)^{s+1}\right]\tilde{\pi}_s(g,\mathbf{c}) > 0$$

From proposition 3 we know that for any $g > 0$, $Y^* = \tilde{Y}(g,\mathbf{c}) > 0$ is unique, which yields that steady state consumption equals

$$C^* = Y^* + \Pi^* > 0$$

Hence a competitive equilibrium path exists and is unique.■

Lemma 5 *Independent of $\mathbf{P}'_{t,0}$, the supplier of the frontier vintage will choose $P_{t,0} > \frac{c_0}{2}A_t\mu(D_t(0))$.*

Proof: In order to prove this and the following lemma, it is easiest to consider

$$z_\tau(h) = \max_{s\in\{0,\ldots,M\}\backslash T}(A_{t-s}h - P_{t,s})$$

and

$$\overline{z}_\tau(h) = \max_{s<T}(A_{t-s}h - P_{t,s}),\ \underline{z}_\tau(h) = \max_{s>T}(A_{t-s}h - P_{t,s}),\text{ and } W_\tau(h) = A_{t-\tau}h - P_{t,\tau}$$

then

$$D_t(\tau) = \{h\in[0,1]\,|W_\tau(h)\geq z_\tau(h)\}$$

The properties of $\overline{z}_\tau(h)$ and $\underline{z}_\tau(h)$, which I will not prove here in detail, are (*i*) $\overline{z}_\tau(h)$ and $\underline{z}_\tau(h)$ are continuous on $[0,1]$, (*ii*) $\underline{z}_\tau(0) = 0$, (*iii*) if $P_{t,s} > 0$ for all $s > \tau$, then $\overline{z}_\tau(0) < 0$, and (*iii*) let $h' > h$, then

$$\overline{z}_\tau(h') - \overline{z}_\tau(h) \geq A_{t-(\tau-1)}(h'-h) \text{ and } \underline{z}_\tau(h') - \underline{z}_\tau(h) \leq A_{t-(\tau+1)}(h'-h)$$

For the frontier vintage, let the producer choose $h' = 1-\varepsilon$ such that all workers of type h' and higher will choose the frontier vintage. Independent of $\mathbf{P}'_{t,0}$, this can be done by choosing

$$P_{t,0} \geq [A_t - A_{t-1}]\,h' = [A_t - A_{t-1}](1-\varepsilon) > 0$$

In that case demand for the frontier vintage equals $\mu(D_t(\tau)) = \varepsilon$, while profits equal

$$\begin{aligned}
\pi_{t,0} &= P_{t,0}\varepsilon - \frac{c_0}{2} A_t \varepsilon^2 \\
&\geq \left[[A_t - A_{t-1}](1-\varepsilon) - \frac{c_0}{2} A_t \varepsilon \right] \varepsilon \\
&= \left[[A_t - A_{t-1}] - \left[\left(1 + \frac{c_0}{2}\right) A_t - A_{t-1} \right] \varepsilon \right] \varepsilon
\end{aligned}$$

such that the producer of the frontier vintage makes strictly positive profits, i.e. $\pi_{t,0} > 0$, whenever it chooses

$$0 < \varepsilon < [A_t - A_{t-1}] / \left[\left(1 + \frac{c_0}{2}\right) A_t - A_{t-1} \right]$$

which is always feasible.■

Lemma 6 *If $P_{t,s} > 0$ for all $s < \tau$, then, independent of $P_{t,\tau+1}, \ldots, P_{t,M}$, the supplier of the vintage of age τ will choose $P_{t,\tau} > \frac{c_\tau}{2} A_{t-\tau} \mu(D_t(\tau))$.*

Proof: If $P_{t,s} > 0$, then we know that $\overline{z}_\tau(0) < 0$, and we can distinguish two cases:
(i) $\overline{z}_\tau(1) \leq \underline{z}_\tau(1)$: in that case the suppliers of the more recent vintages than that of age τ have chosen their prices so high that they are being competed out of the market by suppliers of vintages older than τ. In this case the vintage of age τ is essentially in the same situation as the supplier of the frontier vintage in Lemma 5 and the proof of can be applied Lemma 5 again.
(ii) $\overline{z}_\tau(1) > \underline{z}_\tau(1)$: Because $\overline{z}_\tau(0) < 0 = \underline{z}_\tau(0)$ and both $\overline{z}_\tau(h)$ and $\underline{z}_\tau(h)$ are continuous, we know that in this case there must exist an $h' \in (0,1)$ such that $\overline{z}_\tau(h') = \underline{z}_\tau(h')$. Hence, by choosing $P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h')$, the worker would be indifferent between at least three vintages of machine and would obtain an income level of $\overline{z}_\tau(h') = \underline{z}_\tau(h')$. If a worker of type $h < h'$ would use the machine of age τ, then he would obtain

$$A_{t-\tau}h - P_{t,\tau} = \underline{z}_\tau(h') - A_{t-\tau}(h - h') < \underline{z}_\tau(h')$$

and if a worker of type $h > h'$ would the machine of age τ, then he would obtain

$$A_{t-\tau}h - P_{t,\tau} = \overline{z}_\tau(h') + A_{t-\tau}(h - h') < \overline{z}_\tau(h')$$

Hence, the choice of $P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h')$ is the knife-edge case in which the demand set for vintage τ is a singleton. Now, if the supplier of vintage τ chooses

$$P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h') - \delta$$

then it can be easily shown that

$$0 < \mu(D_t(\tau)) \leq \delta \left(\frac{1}{A_{t-\tau} - A_{t-\tau-1}} - \frac{1}{A_{t-\tau+1} - A_{t-\tau}} \right)$$

and that the resulting profits satisfy

$$\pi_{t,\tau} = \left[\{A_{t-\tau}h' - \overline{z}_\tau(h')\} - \delta \left[1 + \frac{1}{2} \frac{c_\tau A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} + \frac{1}{2} \frac{c_\tau A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} \right] \right] \mu(D_t(\tau))$$

Hence, $\pi_{t,\tau}$ is strictly positive whenever

$$0 < \delta < \{A_{t-\tau}h' - \overline{z}_{\tau}(h')\} / \left[1 + \frac{1}{2}\frac{c_{\tau}A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} + \frac{1}{2}\frac{c_{\tau}A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}}\right]$$

which again is a feasible choice independent of the prices chosen by the suppliers of vintages older than age τ.■