

Firm Risk and Leverage-Based Business Cycles ^{*}

Sanjay K. Chugh [†]
University of Maryland

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Abstract

I quantify the properties of cyclical fluctuations in the cross-sectional dispersion of firm-level risk, and I quantify the properties of cyclical fluctuations in aggregate leverage ratios, along with the debt and equity components separately, in the U.S. non-financial corporate sector. Using the estimated “risk shock” process as an input to a baseline DSGE financial-accelerator framework, I assess how well the model explains business-cycle fluctuations in the financial conditions of non-financial firms. In the model, risk shocks calibrated to micro data can account for virtually all of the business-cycle volatility of leverage, debt, and equity. In terms of aggregate quantities, however, pure risk shocks account for only a small share of GDP fluctuations in the model, less than two percent. Instead, it is standard TFP shocks that explain virtually all of the model’s real fluctuations. Hence, the results suggest a type of dichotomy present at the core of a standard class of DSGE financial frictions models: risk shocks lead to large financial fluctuations, but these are largely isolated from macro fluctuations.

Keywords: leverage, second-moment shocks, time-varying volatility, credit frictions, financial accelerator, business cycles

JEL Classification: E10, E20, E32, E44

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[†]email address: chughs@econ.umd.edu.

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1 Introduction

In this paper, I modify an existing class of general-equilibrium financial accelerator models in a way that leads to empirically-relevant fluctuations in firms' leverage ratios, along with other measures of their financial conditions. Specifically, I show that “risk shocks” can usefully be employed in a baseline DSGE model of financial frictions to explain financial fluctuations. Such shocks, through their effects on leverage ratios, also have the potential to cause fluctuations in aggregate macroeconomic quantities, completely independently from standard TFP and other “first-moment shocks” common in the macro literature. However, I do not treat risk shocks as a free parameter. The empirical discipline I bring to bear on the model relates to and contributes to a distinct recent literature that has studied how time-variation in the *cross-sectional distribution of firm-level outcomes* — “risk shocks” — may in and of themselves drive business cycles.

There are four main results, two from empirical work and two from the theoretical model that quantifies the link between the main empirical findings. First, I characterize business-cycle fluctuations in firm-level risk using U.S. micro data. Specifically, based on the data constructed by Cooper and Haltiwanger (2006), I characterize the time variation in the cross-sectional dispersion of firm-level productivity; I identify this time variation as risk fluctuations. I find that firm risk is strongly countercyclical with respect to GDP, consistent with the findings of Bloom, Floetotto, and Jaimovich (2009) and Bachmann and Bayer (2009). I also find that firm risk is quite volatile over the business cycle: measured by the ratio of the standard deviation of *innovations* in risk to average risk, the volatility of annual firm risk is 17 percent. By this metric, volatility of firm risk is similar to that measured by Bloom, Floetotto, and Jaimovich (2009), but substantially larger than that measured by Bachmann and Bayer (2009). The estimated risk shock process is used as an input to the theoretical model.

Second, using Compustat data, I construct cyclical measures of the aggregate leverage ratio in the U.S. non-financial business sector, which constitutes a large share of the demand side of credit markets. Because basic statistics on the cyclical properties of aggregate leverage — most notably its cyclical volatility — are largely lacking in the macro literature, constructing these statistics seems to be of interest in its own right.¹ Using non-financial firms selected from Compustat, I find that the aggregate leverage ratio over the period 1987-2009 was about unity and that its cyclical volatility

¹Some empirical studies that speak to the same sorts of issues I examine in this paper are Levin, Natalucci, and Zakrajsek (2004), Covas and den Haan (2006), Korajczyk and Levy (2003), Hennessy and Whited (2007), and Levy and Hennessy (2007). With the exception of Covas and den Haan (2006), none of these papers presents business-cycle statistics on the aggregate leverage ratio, although in principle they each could given the data they study. In the online Appendix of their paper, Covas and den Haan (2006) present the cyclical correlation of firms' leverage with GDP, although not its cyclical volatility. As described further below, the results I find corroborate their finding regarding correlation with GDP.

was about 5 percent, over four times as volatile as GDP. Furthermore, leverage is acyclical to mildly countercyclical with respect to GDP.² The cyclical properties of leverage, along with those of debt and equity separately, provide metrics against which the performance of the theoretical model is assessed. More broadly, these nascent stylized facts may provide guidance to other business-cycle modeling efforts in which financial frictions and leverage fluctuations potentially play a prominent role.

The other two contributions of the paper are theoretical. The first main result from the model is that empirically-relevant risk shocks drive virtually all of the business-cycle volatility of the model’s financial-market aggregates, and the quantitative fit with the data in this regard is remarkably tight. In the model, these financial fluctuations have the potential to drive, or at least be associated with, real fluctuations. Such “leverage-based business cycles” could arise through fluctuations in firms’ balance-sheet conditions that are induced by risk shocks. Hence, the transmission channel that the model emphasizes is explicitly a financial channel: if there were no financial frictions, there is no channel by which risk shocks could affect real fluctuations at all. This latter aspect of the model is similar to the qualitative business-cycle model of Williamson (1987).

However, the second main result from the theoretical model is that pure risk shocks, in which average TFP is held constant, lead to very small fluctuations of standard macro aggregates such as GDP. The volatility of GDP conditional on risk shocks alone is less than two percent of GDP volatility conditional on shocks to average TFP alone. Thus, risk shocks and the “leverage-based business cycles” they have the potential to cause do *not* seem to be an important phenomenon when viewed through the lens of a baseline financial-accelerator model calibrated to firm-level data. This result emerges despite the fact that the underlying risk shocks in the model, which are calibrated based on micro data, are fairly large compared to other micro evidence on risk fluctuations. The results from the theoretical model thus suggest a type of dichotomy present at the core of a standard class of DSGE financial frictions models: risk shocks lead to large financial fluctuations, but these are largely isolated from macro fluctuations.

Bloom, Floetotto, and Jaimovich (2009) and Bachmann and Bayer (2009) — henceforth, BFJ and BB, respectively — are two prominent studies in the recent “risk shocks” literature. Regarding theory, the main question I take up in this paper is broadly similar to theirs: studying the extent to which changes over time in cross-sectional dispersion of productivity can lead to aggregate fluctuations. However, the focus in this paper is on quantifying the role of financial factors per se in transmitting risk shocks to economic activity. In the model I present, the *only* way for risk shocks to possibly transmit into fluctuations of GDP and other macro aggregates is through leverage

²I define the leverage ratio as total end-of-quarter book-value of debt to total end-of-quarter book-value of equity for all non-financial firms in Compustat that report positive revenue and positive debt in a given quarter.

— hence the terminology “leverage-based business cycles.” In contrast, the transmission channels in the models of BFJ and BB are non-financial; their models feature no financial frictions and instead emphasize the role of firm-level factor adjustment costs in transmitting risk fluctuations into aggregate quantities.

In studying the joint business-cycle dynamics of real and financial outcomes, this paper contributes to a large emerging literature. For example, Jermann and Quadrini (2009) also aim to jointly explain some salient facts regarding real and financial fluctuations. In their empirical work, Jermann and Quadrini (2009) document the cyclical properties of flows of firms’ equity and debt issuance. However, they do not report the cyclical behavior of the debt-to-equity ratio, which is one point of focus of this paper.³ The medium-scale monetary policy model of Christiano, Motto, and Rostagno (2009) also employs the risk shock highlighted in this paper, but they estimate the parameters of the process based on aggregate macro and financial data, rather than using direct firm-level evidence. In terms of main results, while I find that a miniscule share of GDP fluctuations can be attributed directly to risk shocks, Christiano, Motto, and Rostagno (2009) find that nearly 20 percent of GDP fluctuations stem from risk shocks. Much of the difference in results seems due to their much larger macro-estimates of risk fluctuations than micro evidence indicates.⁴ As well, some of the difference may also be due to the host of nominal rigidities, real rigidities, and “news shock” events present in their model, from which I abstract in order to isolate the role of risk shocks.

It is clear that in order to consider fluctuations in cross-sectional dispersion, the model must have some notion of heterogeneity and cannot be a strict representative-agent economy. In the Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999) class of models on which I build, the heterogeneity is in borrowers’ idiosyncratic ability to repay their loans, which in turn stems from idiosyncratic productivity. This feature is central in these models because with no cross-sectional heterogeneity of borrowers’ ability to repay, there is no risk at all from the point of view of lenders, and hence no financial friction. In typical quantitative analysis of these models, parameters for the distribution are chosen based on evidence on long-run risk premia or the like, but then the distributional aspect of the model invariably fades into the background.

I instead place this feature of the model in the foreground by emphasizing the time-variation in cross-sectional dispersion of firms’ productivity, using firm-level evidence to discipline the cal-

³Jermann and Quadrini (2009) use financial data from the Flow of Funds Accounts of the Federal Reserve Board, whereas I use Compustat data.

⁴To be clear, the magnitude of risk fluctuations I find in the Cooper and Haltiwanger (2006) micro data is *large* compared to the *micro* evidence of studies such as BFJ and BB, but it is *small* compared to the *macro* evidence of studies such as Christiano, Motto, and Rostagno (2009).

ibration. Fluctuations in firm-level risk presents lenders with time-varying risk of their overall loan portfolios, and hence leads them to extend more or less credit to borrowers — i.e., extend more or less leverage. While risk shocks turn out to account quite well for financial fluctuations in the model, risk-induced financial fluctuations are almost completely isolated from real fluctuations. This result is perhaps unsettling because the agency-cost setup is a common building block of richer DSGE models of financial frictions. The results obtained here suggest that in richer agency-cost models that *do* find spillovers between financial fluctuations and real fluctuations, the linkages are not driven by the basic agency-cost friction itself, but rather by other features of the model that interact with the friction.

In terms of broader motivation, a widespread recent view is that the cyclical behavior of leverage may be important to both empirical and theoretical understanding of how financial and real outcomes co-move along the business cycle. Geanakoplos (2009), Adrian and Shin (2008), and others have stressed the cyclical behavior of leverage in the financial sector. Mimir (2010) tabulates the cyclical properties of leverage in the financial sector using standard business-cycle filtering tools. Given recent events, a focus on leverage in the financial sector is natural. However, a long tradition in both macro and finance has emphasized leverage in the non-financial corporate sector as being important for aggregate fluctuations, which is the channel studied in this paper.⁵ Lately, there have been hints of evidence that as balance-sheet conditions of financial firms have stabilized, credit demand by and credit supply available to the non-financial sector may soon again be central for aggregate conditions. This paper can be viewed as measuring the extent to which fluctuations in the financial conditions of non-financial firms are related to fluctuations in real activity — the main answer is that it matters little, conditional on risk shocks, in a baseline model of financial frictions.

Finally, a few words regarding terminology are in order. As should be clear from the discussion so far, the idea of “risk shocks” in this paper is variations over time in the cross-sectional standard deviation of firm-level productivity, holding constant average (aggregate) productivity. This is the same notion of “second-moment shocks” that BFJ and BB study. However, it is distinct from another recent conceptualization of “second-moment shocks” emphasized by Justiniano and Primiceri (2008), Fernández-Villaverde and Rubio-Ramirez (2007), and others, in which the standard deviation of the innovations affecting standard macro driving processes such as aggregate TFP, monetary disturbances, etc., vary over time. Crucial in this latter group of studies is that they are all representative-agent economies, so there is no meaningful concept of cross-sectional dispersion and hence of course no possibility of changes in cross-sectional dispersion over time. Focusing on

⁵Bernanke, Campbell, and Whited (1990) is an early empirical study suggesting the importance of non-financial sector leverage in aggregate fluctuations.

the cross section is the main idea in BFJ, BB, and this paper. Gourio (2008) and Christiano, Motto, and Rostagno (2009) also employ the same idea of “firm-level risk” and “risk shocks.” I use the terms “risk shocks,” “firm-level risk,” “second-moment shocks,” and “dispersion shocks” interchangeably.

The rest of the paper is organized as follows. Section 2 presents new empirical evidence on firm-level risk and its business cycle properties. This evidence serves as quantitative input to the model. Section 3 then documents the business cycle behavior of an aggregate measure of the leverage ratio, along with the underlying debt and equity measures, in the U.S. non-financial business sector. This evidence serves as one of the main metrics against which I judge the output of the model. Section 4 presents the baseline model, in which shocks to average TFP and risk shocks are independent from each other. Section 5 intuitively describes why leverage in the model should respond to changes in risk. Section 6 presents quantitative results. Section 7 presents and studies a model extension that features “bundled aggregate shocks,” in which risk fluctuations are correlated with average TFP shocks. Section 8 concludes.

2 Risk Fluctuations

The main goal of this section is to document the properties of business-cycle fluctuations in firm-level risk. The analysis is based on a balanced panel, constructed by Cooper and Haltiwanger (2006), from the Longitudinal Research Database (LRD). The data are annual observations of plant-level measures such as revenue, materials and labor costs, and investment at approximately 7,000 large U.S. manufacturing plants over the period 1974-1988. The starting point for my analysis is Cooper and Haltiwanger’s (2006) measures of plant-level profitability residuals from this panel.⁶

Briefly, Cooper and Haltiwanger (2006) compute for each plant i in year t a residual A_{it} that reconciles exactly the observations of plant i ’s profits and capital stock in year t when described by a profit function that depends only on the capital stock.⁷ The year-specific aggregate residual ω_{mt} is computed as the mean of A_{it} across firms in that year. Plant i ’s profit function in year t thus is viewed as being shifted by both the aggregate shock ω_{mt} and the idiosyncratic shock $\omega_{it} \equiv A_{it}/\omega_{mt}$. In each year, there is thus a cross-sectional distribution of ω_{it} . Denote by σ_t^ω the cross-sectional standard deviation in year t of the idiosyncratic component ω_{it} .

Because plant-level price deflators are unavailable in the dataset,⁸ it is impossible to distinguish true “productivity” shocks from “revenue” shocks, so the residuals mix supply and demand shifts (hence the term “profitability” shocks). As an identifying assumption for the theoretical model

⁶I thank John Haltiwanger for providing their aggregative data on profitability residuals.

⁷The Appendix in Cooper and Haltiwanger (2006) describes in detail the construction of the data and the residuals.

⁸More precisely, they are available only at five-year intervals, too low a frequency for business-cycle analysis.

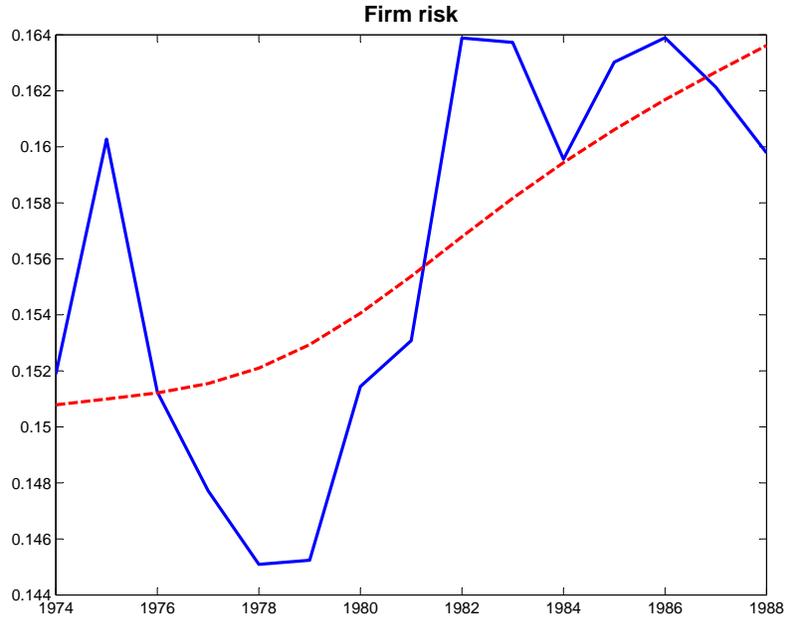


Figure 1: Cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

developed below, I simply interpret these profitability shocks as true productivity shocks.⁹ Furthermore, when deploying the evidence documented here in the model, I identify “plants” as “firms,” abstracting from the fact that a non-negligible share of plant-level output in the LRD represents output of multi-plant firms. With these identifying assumptions, I characterize the business-cycle behavior of both ω_{mt} and of σ_t^ω (the cross-sectional standard deviation of ω_{it}), aspects of the data not studied by Cooper and Haltiwanger (2006).

2.1 Productivity Risk

I first compute the cross-sectional coefficient of variation of productivity (profitability) for each of the 15 years of the sample. Cross-sectional coefficients of variation are used because the residually-computed aggregate mean level of productivity (ω_{mt}) is not unity in the data, but it is normalized to unity in the model below. The time-averaged mean of the cross-sectional coefficient of variation is 0.156, hence I normalize long-run dispersion in the model to $\bar{\sigma}^\omega = 0.156$. Figure 1 plots the time series σ_t^ω , which suggests a modest upward trend in dispersion. (However, the time series is somewhat short.)

⁹A justification for this based on the model presented below is that, in the model, the relative price of all goods is always unity due to perfect competition in goods markets.

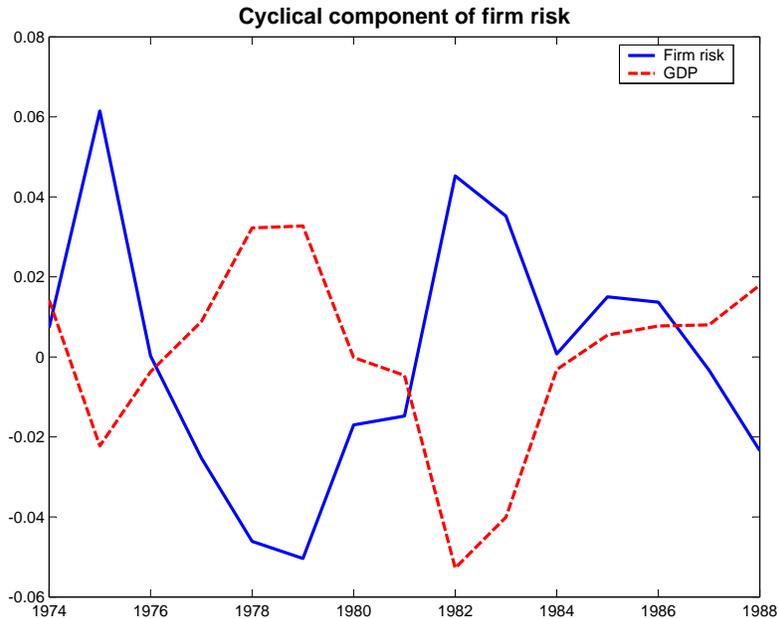


Figure 2: Cyclical component of cross-sectional coefficient of variation of firm-level profitability over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).

Figure 2 displays the HP-filtered components of σ_t^ω and GDP over the period 1974-1988. A clear negative cyclical correlation between the two series is apparent — the contemporaneous correlation between the two series is -0.83 , hence expansions are associated with a decrease in dispersion of firms' idiosyncratic productivity, and recessions are associated with an increase in dispersion of firms' idiosyncratic productivity. Strongly countercyclical firm-level risk is also a robust finding in the micro evidence of BB and BFJ. In terms of volatility, the standard deviation of the cyclical component of σ_t^ω is 3.15 percent over the sample period. With an innocuous abuse of notation, I hereafter use σ_t^ω to denote the cyclical component of cross-sectional dispersion.

In the model presented below, I suppose that σ_t^ω follows the exogenous AR(1)

$$\ln \sigma_{t+1}^\omega = (1 - \rho_{\sigma^\omega}) \ln \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_{t+1}^{\sigma^\omega}, \quad (1)$$

with $\epsilon^{\sigma^\omega} \sim N(0, \sigma_{\sigma^\omega})$. Given $\bar{\sigma}^\omega = 0.156$, the point estimate (using OLS) of the AR(1) parameter is $\rho_{\sigma^\omega} = 0.48$, with a t-statistic of 1.93. With this estimate of ρ_{σ^ω} and the standard deviation of σ_t^ω of 3.15 percent, the standard deviation of the (annual) *innovations* to the cross-firm dispersion process can be computed to be 0.0276. This implies a coefficient of variation (with respect to the mean dispersion $\bar{\sigma}^\omega = 0.156$) of 17.7 percent, which can be directly compared to the empirical evidence reported by BB and BFJ. Computed in a variety of ways, BB find a coefficient of variation of

innovations to firm-level productivity for their entire sample of German firms between two and three percent. However, because the Cooper and Haltiwanger (2006) analysis is of large manufacturing plants, the most comparable result in BB is their finding for the largest (ranked by employment) five percent of firms in their sample. For this sample, BB find a coefficient of variation of firm-level innovations of 5.5 percent (see their Table 8). The 17.7 percent coefficient of variation of plant-level innovations in the Cooper and Haltiwanger (2006) sample is thus substantially larger than the largest firms in BB’s sample.¹⁰ However, this degree of volatility of firm risk lines up much better with the evidence of BFJ, who document using a variety of cross-sectional measures that dispersion of firm outcomes rises very sharply during recessions.

2.2 Average Productivity

For consistency in the way the firm-level data are used as an input to the model, I also characterize the time-series behavior of ω_{mt} , the average productivity (profitability) residual. In the model, this measure will correspond to the standard notion of TFP (i.e., the first moment of the productivity distribution).

Figures 3 and 4 display the actual series, its HP trend, and the cyclical component of average productivity. As noted above, long-run average productivity is normalized to unity in the model, so the vertical scale in Figure 3 is arbitrary.¹¹ The cyclical component of ω_{mt} is highly correlated with the cyclical component of GDP, as Figure 4 shows — the contemporaneous correlation between the two is 0.87. The volatility of the cyclical component of ω_{mt} is 1.26 percent (at an annual horizon). Again with an innocuous abuse of notation, I hereafter use ω_{mt} to denote the cyclical component of average productivity.

In the model presented below, I suppose that ω_{mt} follows the exogenous AR(1)

$$\ln \omega_{mt+1} = \rho_{\omega_m} \ln \omega_{mt} + \epsilon_{t+1}^{\omega_m}, \quad (2)$$

with $\epsilon^{\omega_m} \sim N(0, \sigma_{\omega_m})$. Estimation gives a point estimate $\rho_{\omega_m} = 0.47$, with a t-statistic of 1.84.¹² With this estimate of ρ_{ω_m} and the standard deviation of ω_{mt} of 1.26 percent, the standard deviation of the (annual) *innovations* to the average productivity process can be computed to be 0.0111. Finally, the cyclical correlation between average productivity and the dispersion of productivity (i.e., the concept of firm risk) is -0.97; this extremely strong negative correlation is part of the motivation of the “bundled-shock” model extension considered in Section 7.

¹⁰I thank Rudi Bachmann for pointing out these comparisons.

¹¹And follows directly from the normalizations in the Cooper and Haltiwanger (2006) data.

¹²This differs from Cooper and Haltiwanger’s (2006, p. 623) estimate of the persistence of mean productivity because they do not detrend; the AR(1) coefficient of the unfiltered ω_{mt} series is 0.76.

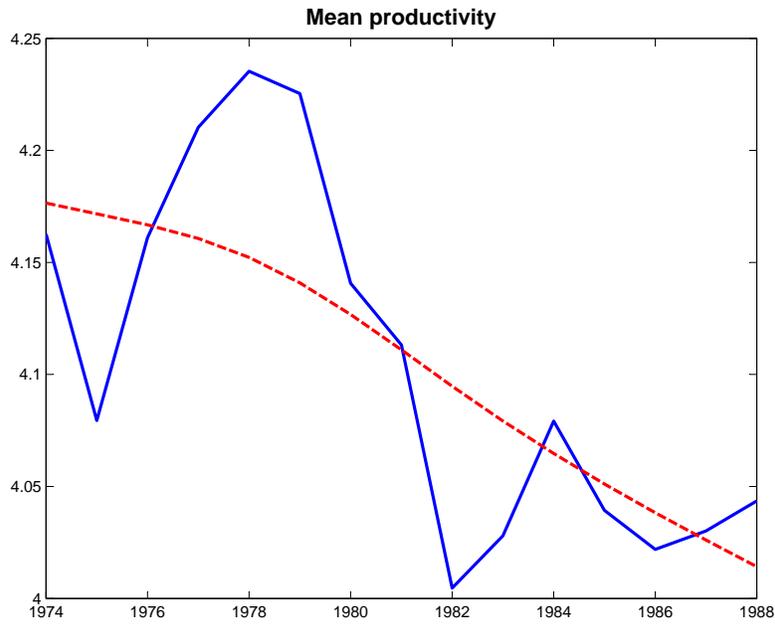


Figure 3: Mean level of firm-level profitability residuals over the period 1974-1988. Data are annual. Trend component constructed with HP filter (smoothing parameter 100). Based on profitability series from Cooper and Haltiwanger (2006).

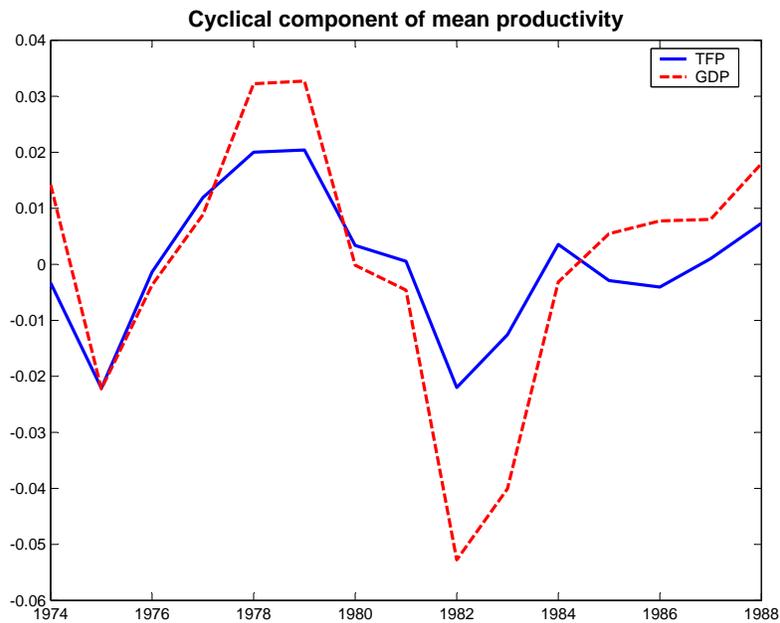


Figure 4: Cyclical component of mean of firm-level profitability residuals over the period 1974-1988. Vertical axis is percentage deviation from HP trend. Computed from profitability residuals constructed by Cooper and Haltiwanger (2006).

In the model developed below, I pursue a quarterly calibration, rather than an annual calibration, because the leverage evidence documented in Section 3 is quarterly. Because the evidence presented in this section is from annual data, I use persistence parameters of $\rho_{\sigma^\omega} = 0.48^{0.25} = 0.83$ and $\rho_{\omega_m} = 0.48^{0.25} = 0.83$, which assumes smoothness in the processes during the year. How this inference of quarterly persistence from annual estimates affects the model calibration of the innovation parameters σ_{σ^ω} and σ_{ω_m} is discussed in Section 6.2.

3 Leverage Fluctuations

In this section, I compute quarterly business-cycle statistics for aggregate measures of the leverage ratio, along with their debt and equity components, of U.S. non-financial businesses over the past 20 years. There are a few other studies that document similar evidence. The closest available evidence is provided by: Levin, Natalucci, and Zakrajsek (2004), who use quarterly Compustat data to construct a time series of non-financial sector leverage over the period 1988-2003; Korajczyk and Levy (2003), who use quarterly Compustat data over the period 1984-1993; and Covas and den Haan (2006), who use Compustat data, although at an annual frequency and with a focus on the behavior of debt and equity separately — that is, on the numerator and denominator of the leverage ratio separately.

With the exception of Covas and den Haan (2006), these other studies do not report standard business cycle statistics, such as volatilities and cross-correlations with standard macro aggregates, using filtering procedures common in business-cycle analysis. Constructing metrics using this standard macro approach is the goal here. In the online Appendix to their study, Covas and den Haan (2006) present cyclical correlations of a few measures of leverage with respect to GDP, but not the cyclical volatility of leverage. Relative to Covas and den Haan (2006) and Levin, Natalucci, and Zakrajsek (2004) — henceforth, LNZ — the evidence presented here extends the analysis through 2009 and also documents both business-cycle volatilities and correlations of leverage, providing some metrics against which the predictions of business-cycle models that feature endogenous leverage may be judged, including the model I study below. In more finance-oriented and firm-level applications, Hennesy and Whited (2007) and Levy and Whited (2007) also document some of the type of evidence on which I focus.¹³

Like LNZ and Korajczyk and Levy (2003), I use quarterly Compustat data on publicly-traded non-financial U.S. firms. I examine the period 1987:Q3 — 2009:Q1, corresponding broadly to the Great Moderation period. In each quarter, every non-financial firm that reports positive debt and

¹³An important distinction between Hennesy and Whited (2007) and Levy and Whited (2007) relative to the type of model-based lens through which LNZ and I view the data is that in the former, external financing can be either in terms of debt or equity, whereas in the latter external financing is only in the form of debt.

positive sales is selected.¹⁴ The measure of debt is the book value of firms' total long-term debt, and the measure of equity is the book value of total shareholder equity. In each quarter, I compute aggregate debt and aggregate equity as the simple sum of debt and equity over all firms selected in that quarter. The aggregate leverage ratio is then defined as the ratio of aggregate debt to aggregate equity in each quarter.

Figures 5 and 6 plot the time series of aggregate leverage, its HP trend component, and its cyclical component. The mean leverage ratio over the sample is 1.03, and Figure 5 shows that there has been an upward trend since the late 1980's.^{15,16} Based on the cyclical component of leverage plotted in Figure 6, the standard deviation of aggregate leverage is 4.7 percent in the non-financial business sector. Figure 7 plots the cyclical components of the aggregate debt and aggregate equity components separately. At around five percent each, the cyclical volatility of debt and equity separately are similar to that of leverage.

Table 1 presents correlations over the business cycle of aggregate leverage, aggregate debt, and aggregate equity with standard macro aggregates. Perhaps counter to conventional wisdom, the contemporaneous correlation of leverage in the non-financial business sector with GDP is slightly countercyclical. Non-financial firms do not seem to load up on leverage during expansions; in fact, somewhat the opposite. This finding is consistent with that in Levy and Hennessy (2007), who show that leverage ratios in highly-constrained firms are countercyclical, while leverage ratios in less-constrained firms are acyclical. Moreover, Table 2 shows that leverage is only mildly countercyclical with respect to GDP at leads and lags of up to four quarters.

The magnitudes in Tables 1 and 2 are small enough that one may broadly read the evidence as showing virtual acyclicity of leverage in the non-financial business sector.¹⁷ This finding is also

¹⁴Thus, the data are not a panel. The number of firms selected according to this criteria grows through the sample: in 1987:Q3, the selection method picks out 1,596 firms, while in 2009:Q1 the selection method picks out 3,446 firms.

¹⁵This latter aspect of the leverage ratio I construct differs from LNZ, who show in their Figure 3 that the leverage ratio displays a downward trend during the period 1988-2000, which is not evident here. Some differences may be definitional ones (for example, they use the market value of common equity as their measure of equity, in contrast to my metric of total shareholder equity) and some may be sample selection and construction issues (for example, they use a sales-weighted average of firm-level leverage ratios, whereas I focus directly on an aggregative measure of leverage, ignoring the cross-sectional dimension of leverage).

¹⁶I also note that the mean leverage ratio I compute is substantially larger than that computed by Levy and Whited (2007, Table 1), which may be at least partly, and perhaps almost entirely, attributable to the different sample selection methods employed. Yet another (early) point of comparison for the results presented in Figures 5 and 6 is Bernanke, Campbell, and Whited (1990), who computed aggregate non-financial sector leverage in the late 1980's of about 0.4; as Figure 5 shows, I find that it was about 0.7 in the late 1980's.

¹⁷Note that the recent evidence of Adrian and Shin (2008), who document procyclicality of leverage amongst the five large U.S. investment banks leading up to the most acute phase of the financial crisis in September 2008, is for the supply side of the credit markets — lenders. The evidence I present is for the demand side of credit markets — (corporate) borrowers. Hence there is no inconsistency between these findings and Adrian and Shin (2008). In

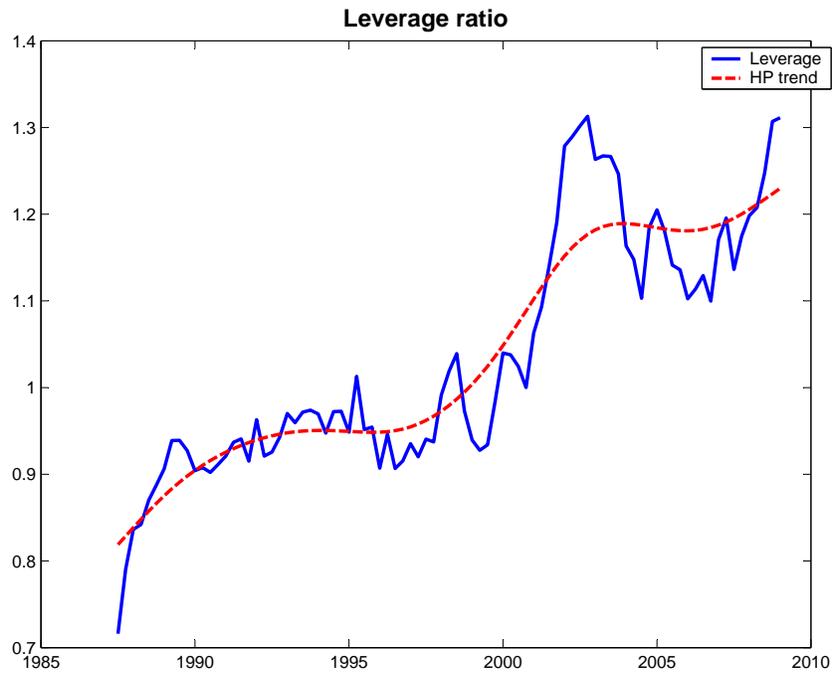


Figure 5: Leverage ratio in U.S. non-financial business sector, 1987Q3-2009Q1. Mean = 1.03. Trend component constructed with HP filter (smoothing parameter 1600).

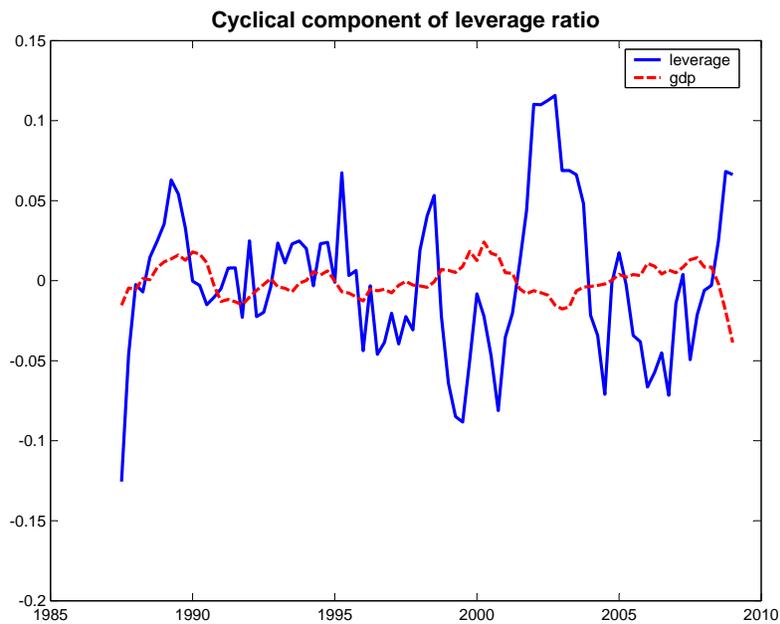


Figure 6: Cyclical component of leverage ratio in U.S. non-financial business sector, 1987Q3-2009Q1. Vertical axis is percentage deviation from HP trend. Standard deviation = 4.7 percent.

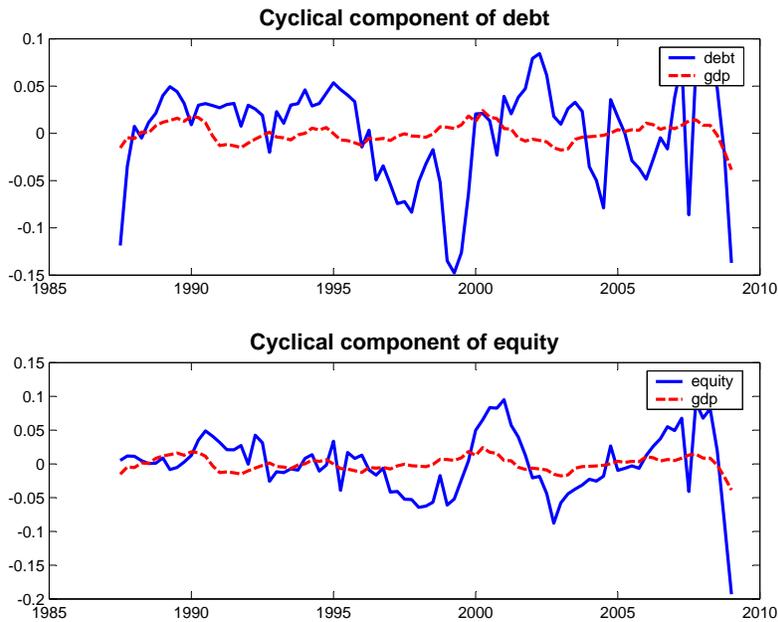


Figure 7: Cyclical components of debt and equity in U.S. non-financial business sector, 1987Q3-2009Q1. Vertical axis is percentage deviation from HP trend. Standard deviation of debt = 5.2 percent, standard deviation of equity = 4.5 percent.

	GDP	C	I	leverage	debt	equity	
Std. dev. (%)	1.04	0.89	5.17	4.71	5.21	4.54	
Auto. corr.	0.84	0.87	0.79	0.75	0.66	0.70	
Corr. matrix	GDP	1	0.87	0.90	-0.35	0.10	0.54
	C		1	0.71	-0.21	0.24	0.56
	I			1	-0.45	-0.07	0.42
	leverage				1	0.57	-0.37
	debt					1	0.51
	equity						1

Table 1: Business cycle statistics for standard macro aggregates (GDP, consumption, and gross investment) and aggregate debt, equity, and leverage ratio in U.S. non-financial business sector. Based on HP-filtered cyclical components over the period 1987:Q3-2009:Q1.

	$t - 4$	$t - 3$	$t - 2$	$t - 1$	t	$t + 1$	$t + 2$	$t + 3$	$t + 4$
leverage	-0.05	-0.15	-0.27	-0.34	-0.35	-0.35	-0.30	-0.28	-0.24
debt	0.25	0.17	0.09	0.10	0.10	-0.13	-0.31	-0.45	-0.48
equity	0.42	0.44	0.48	0.54	0.55	0.32	0.03	-0.18	-0.28

Table 2: Correlations of leverage, debt, and equity in U.S. non-financial business sector with GDP at various horizons. Based on HP-filtered cyclical components over the period 1987:Q3-2009:Q1.

consistent with the conclusion of Covas and den Haan (2006) that leverage is acyclical. A further interesting comparison is to Hennesy and Whited (2007, Table 1), who document essentially zero covariance between non-financial firms’ leverage and physical investment over the period 1988-2001. The covariance between aggregate leverage and aggregate investment in the sample used here is -0.0012, in line with the value of -0.0018 they find.

This evidence amounts to a first step in constructing measures of aggregate leverage in a way familiar to standard business cycle analysis. Future work may refine these aggregative measures and examine alternative measures.¹⁸ For the purposes of the rest of this paper, I take the following as stylized facts that emerge from this evidence: the cyclical volatility of leverage, debt, and equity in the non-financial business sector are all four to five times larger than the volatility of GDP; and there is little correlation, or at most a mild negative correlation, over the business cycle of leverage with standard aggregate quantities.

4 Model

As described in the introduction, the model is based on the agency-cost formulation of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997, 1998), and Bernanke, Gertler, and Gilchrist (1999). The model I construct is most directly based on the “output model” of Carlstrom and Fuerst (1998), in which all prices are flexible, firms require short-term working capital (formally, intraperiod) to finance their production costs, and there are no other rigidities or frictions whatsoever. This fact, my finding of acyclicity, or mild countercyclicity, of non-financial sector leverage is consistent with the one piece of evidence Adrian and Shin (2008) document for non-financial firms: their Figure 2.3 also displays mild countercyclicity of non-financial sector leverage (although note that their notions of cyclicity are with respect to market asset values, rather than with respect to GDP). See Mimir (2010) for a standard business-cycle accounting of financial-sector balance-sheet conditions.

¹⁸In addition to, for example, parsing these results into long-term vs. short-term leverage, etc, yet another dimension of analysis would be examining leverage behavior amongst publicly-traded firms (which are what Compustat covers) vs. privately-traded firms. Davis, Haltiwanger, Jarmin, and Miranda (2007) show that firm-level outcomes may be very different for public vs. private firms.

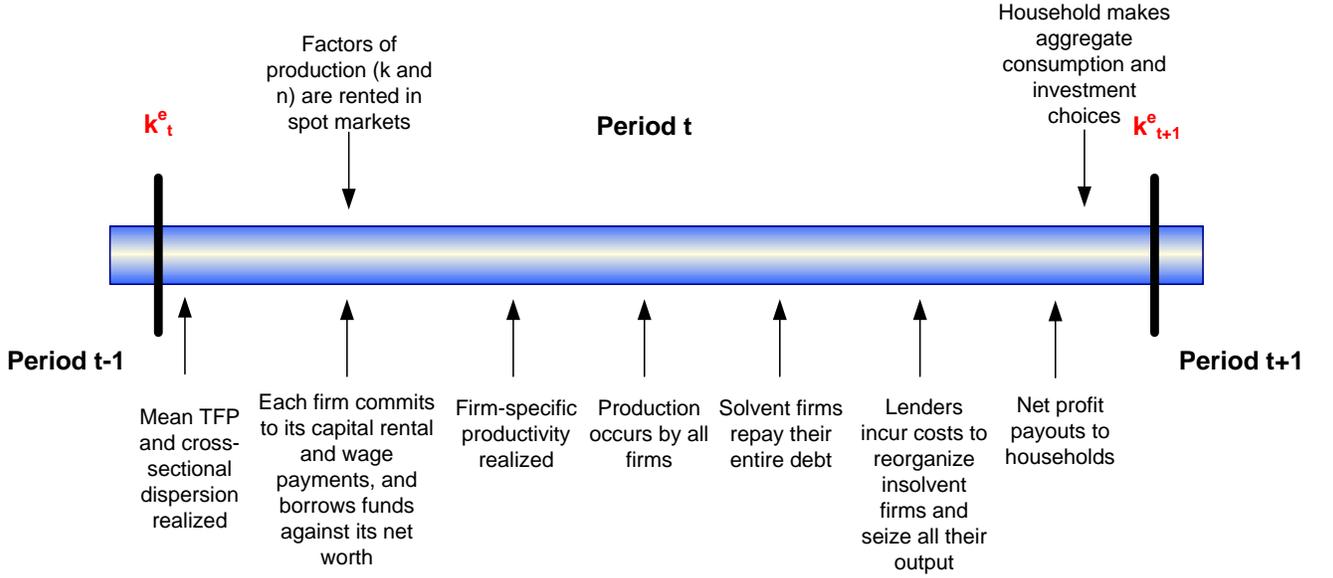


Figure 8: Timing of events in model.

provides the cleanest starting point to highlight the role of shocks to firm risk, so from here on I speak as if I am building “just” on the Carlstrom and Fuerst (1998) — henceforth, CF — analysis, recognizing that it is meant to capture an entire literature of work.

I turn now to a detailed description of the economic environment and the equilibrium. As an aid to the ensuing description, Figure 8 illustrates the timing of events in the model. Because the model is virtually identical to the CF (1998) model, with only a couple of modifications made to map the model to the data in a cleaner way, readers familiar with the CF output model may choose to skip to the analysis beginning in Section 5.

4.1 Households

There is a representative household in the economy that maximizes expected lifetime discounted utility over streams of consumption and leisure,

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(1 - n_t)], \quad (3)$$

subject to the sequence of flow budget constraints

$$c_t + k_{ht+1} = w_t n_t + k_{ht} [1 + r_t - \delta] + \Pi_t. \quad (4)$$

The functions $u(\cdot)$ and $v(\cdot)$ are standard strictly-increasing and strictly-concave subutility functions over consumption and leisure, respectively. The rest of the notation is as follows. The household’s

subjective discount factor is $\beta \in (0, 1)$, c_t denotes the household's consumption, k_{ht} denotes the household's capital holdings at the start of period t , w_t is the real wage, r_t is the market rental rate on capital, and δ is the depreciation rate of capital. The household also receives aggregate dividend payments Π_t from firms as lump-sum income, the determination of which is described below.¹⁹

Emerging from household optimization is a completely standard labor supply condition

$$\frac{v'(1 - n_t)}{u'(c_t)} = w_t, \quad (5)$$

and a completely standard capital supply condition

$$u'(c_t) = \beta E_t \{u'(c_{t+1}) [1 + r_{t+1} - \delta]\}, \quad (6)$$

which follows as usual from the household's period- t first-order conditions with respect to c_t and k_{ht+1} . The one-period-ahead stochastic discount factor is defined as $\Xi_{t+1|t} = \beta u'(c_{t+1})/u'(c_t)$, with which firms, in equilibrium, discount profit flows.

4.2 Firms

There is a continuum of unit mass of firms, each of which produces output by operating a constant-returns technology. Firms are heterogenous in their productivity. Firm i produces output using the technology $\omega_{it}F(k_{it}, n_{it})$: k_{it} is the firm's purchase of physical capital on spot markets, n_{it} is the firm's hiring of labor on spot markets, and ω_{it} is a firm-specific productivity realization.

Each period, firm i 's idiosyncratic productivity is a draw from a distribution with cumulative distribution function $\Phi(\omega)$, which has a time-varying mean ω_{mt} , a time-varying standard deviation σ_t^ω , and associated density function $\phi(\omega)$, all of which are identical across firms. Time-variation in ω_{mt} corresponds to the usual notion of TFP shocks, in the sense of exogenous variation in the mean of firms' technology. The time-varying volatility σ_t^ω is the key innovation in the model compared to CF. Given the first and second moments ω_{mt} and σ_t^ω common across firms, idiosyncratic productivity for a given firm is i.i.d. over time, an assumption made for tractability.²⁰

¹⁹I could also introduce shares in order to directly price streams of dividends paid by firms to households; but this extra detail is unnecessary for the main points, so it is omitted.

²⁰The assumption of zero persistence of the idiosyncratic component of a firm's productivity is at odds with the evidence of Cooper and Haltiwanger (2006) and others, but it greatly simplifies the computation of the model because the firm sector essentially can be analyzed as a representative agent. This point is discussed further below when I come to the aggregation of the model. This device still allows me to illustrate the main point of the model, which is that variations in cross-sectional productivity dispersion can lead to large fluctuations in aggregate leverage and possibly, in turn, to fluctuations in economic activity. In addition to greatly reducing the computational burden, the assumption of zero persistence in idiosyncratic shocks also retains the simplicity of the CF and Bernanke and Gertler (1989) contracting specifications. If persistent shocks were allowed, it is not clear that the intraperiod loan contracts of these models could not be improved upon by the contracting parties by, say, multi-period contracts. Sidestepping

Firms are owned by households, and the objective of firms is to maximize the expected present discounted value of dividends paid out to households. Denote by Π_{it} the dividend payment made by firm i to households. For descriptive convenience, I decompose Π_{it} into a “non-retained earnings” component Π_{it}^e and an “expected operating profit” component $E_\omega \Pi_{it}^f$; the notation E_ω indicates an expectation conditional on the period- t aggregate state but before idiosyncratic realizations are revealed to any firm.²¹ Thus, $\Pi_{it} \equiv \Pi_{it}^e + E_\omega \Pi_{it}^f$. As described below, the component $E_\omega \Pi_{it}^f$ essentially corresponds to static profits as in a simple RBC model.

Because firms are owned by households, they apply the representative household’s stochastic discount factor (the one-period-ahead discount factor is $\Xi_{t+1|t}$, as defined above) to their intertemporal optimization problem. However, firms are also assumed to be “more impatient” than households by the factor $\gamma < 1$, which can be thought of as a reduced-form way of capturing some sort of principal-agent problem that prevents perfect alignment of the firms’ objectives with households’ preferences. At a technical level, $\gamma < 1$ ensures that firms cannot accumulate enough assets to become self-financing, which would render irrelevant the financial frictions described below. This device for avoiding self-financing outcomes is common in models of financial frictions.

The intertemporal objective function of firm i is thus

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left[\Pi_{it}^e + E_\omega \Pi_{it}^f \right]. \quad (7)$$

The firm problem is now further developed and analyzed.

4.2.1 Firm Financing and Contractual Arrangement

In period t , total operating costs of firm i , which are the sum of capital rental costs and wage payments, are

$$M_{it} = w_t n_{it} + r_t k_{it}. \quad (8)$$

As in CF and as shown in Figure 8, the firm is assumed to commit to all of its input costs after observing the aggregate exogenous state $(\omega_{mt}, \sigma_t^\omega)$, but before observing its idiosyncratic realization ω_{it} and thus before any output or revenue are created.

Part of the financing of the firm’s costs comes from its own accumulated net worth, which is held primarily in the form of capital. The capital that each firm accumulates is rented on spot markets to (other) firms, just like households rent their capital on spot markets. Firm i ’s capital holdings

this issue is yet another reason to assume no persistence in realized idiosyncratic productivity. Note, however, that assuming persistence in shocks to σ_t^ω , as I do, does not pose any of these problems; indeed, shocks to σ_t^ω really are aggregate shocks.

²¹As Figure 8 indicates, firm decisions are made in the first “subperiod” of period t , before idiosyncratic shocks have been realized but after aggregate shocks have been realized, hence the need for E_ω .

at the start of period t are k_{it}^e . Thus, note that k_{it}^e , which reflects the firm’s *savings* decisions, is distinct from k_{it} , which reflects the firm’s *capital demand* decisions for production purposes.

However, the firm’s internal funds (which I refer to interchangeably as its net worth or its equity) are insufficient to cover all input costs. To finance the remainder, a firm borrows short-term — formally, intraperiod — working capital. A firm requires external financing because of the assumption that it is more impatient than households, as described above.²² By acquiring external funds, the firm is able to leverage its net worth in period t ,

$$nw_{it} = k_{it}^e [1 + r_t - \delta] + e_t, \quad (9)$$

into coverage of its operating costs M_{it} . Total borrowing by the firm is thus $M_{it} - nw_{it}$. The component e_t of net worth is a small amount of “endowment income” that each firm receives to ensure its continued operations in the event that it becomes insolvent in the previous period. In closing the model, this endowment is absorbed into the payout Π_{it} the firm pays to its owners, which is the representative household. The payout Π_{it} is thus interpreted as net of the endowment e_t .²³

I describe only briefly the outcome of the contracting arrangement between borrowers (firms) and lenders (households) because it is well-known in this class of models.²⁴ The financial contract is a debt contract, which is fully characterized by a liquidation threshold $\bar{\omega}_t$ and a loan size $M_{it} - nw_{it}$. A firm must be liquidated or “reorganized” if its realized productivity ω_{it} falls below the contractually-specified threshold $\bar{\omega}_t$. Below this threshold, the firm does not have enough resources to fully repay its loan. In that case, the firm is declared insolvent and receives nothing, while the lender must pay reorganization costs that are proportional to the total output of the firm and receives, net of these reorganization costs, all of the output of the firm. Note that all firms, regardless of whether or not they end up requiring reorganization, do produce output up to their full (idiosyncratic) capacity.

Define by $f(\bar{\omega}_t)$ the expected share of idiosyncratic output $\omega_{it}F(k_{it}, n_{it})$ the borrower (the firm)

²²As noted above, this is a standard assumption in this class of models and avoids the self-financing outcome. See, for example, Carlstrom and Fuerst (1997, 1998) and Bernanke, Gertler, and Gilchrist (1999).

²³Thus, equivalently, e_t can be interpreted as a lump-sum transfer of “startup funds” provided by households to firms, as in Gertler and Karadi (2009). By allowing a “firm’s” operations to continue in the event of bankruptcy, the assumption of a startup fund brings great analytical tractability to the model. Thus, the “costs of bankruptcy” in the model are more properly interpreted as “costs of reorganization” without any disruption of its output-producing activities (i.e., bringing in new management to oversee ongoing operations).

²⁴In the context of general-equilibrium settings, familiar expositions appear in Carlstrom and Fuerst (1997, 1998), Bernanke, Gertler, and Gilchrist (1999), and Faia and Monacelli (2007). In partial-equilibrium settings, analysis of this type of contractual arrangement traces back to Townsend (1979), Gale and Hellwig (1985), and Williamson (1987).

keeps after repaying the loan, and by $g(\bar{\omega}_t)$ the expected share received by the lender.²⁵ These expectations are conditional on the realization of the time- t aggregate state, but before revelation of a firm's idiosyncratic productivity ω_{it} . The contractually-specified loan size is characterized by a zero-profit condition on the part of lenders,

$$M_{it} = \frac{nw_{it}}{1 - p_t g(\bar{\omega}_t)}, \quad (10)$$

and the contractually-specified liquidation threshold is characterized by

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}, \quad (11)$$

in which $p_t > 1$ is a “markup” on input costs that arises solely from the external financing needs of the firm.²⁶ Thus, for each unit of capital the firm rents, the cost, inclusive of financing costs, is $p_t r_t$, rather than just r_t . The same is true for each unit of labor that must be paid. Note that neither p_t nor $\bar{\omega}_t$ are firm-specific; this is simply asserted for now, but I return to this below when considering the aggregation of the model. Finally, all contractual outcomes are contingent on the aggregate state $(\omega_{mt}, \sigma_t^\omega)$ of the economy.

CF interpret p_t as a “markup” that drives a wedge between factor prices and marginal products. The analysis below shows that this interpretation also carries over here. However, another informative interpretation of p_t is as an external finance premium. For every unit of cost firms incur for their inputs, they must pay $p > 1$ units inclusive of borrowing costs. Thus, p naturally has an interpretation as an external finance premium.

4.2.2 Operating Profits and Asset Evolution

Firms take as given contractual outcomes when maximizing profits. The *expected* operating profit of firm i in period t is

$$E_\omega \Pi_{it}^f = \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}]. \quad (12)$$

As discussed above, this is an *expected* profit because it is measured before the realization of firm-specific idiosyncratic productivity but after the realization of the aggregate period- t state of the economy, $(\omega_{mt}, \sigma_t^\omega)$. Because the mean of ω_{it} is ω_{mt} , ex-ante revenue of the firm is $\omega_{mt} F(k_{it}, n_{it})$. The idiosyncratic risk ω_{it} and associated financing costs implied by it are captured by the inclusion

²⁵Formally, $f(\bar{\omega}_t) \equiv \int_{\bar{\omega}_t}^{\infty} (\omega_i - \bar{\omega}_t) \phi(\omega_i) d\omega_i = \int_{\bar{\omega}_t}^{\infty} \omega_i \phi(\omega_i) d\omega_i - [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t$ is the share received by the firm, and $g(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} (\omega_i - \mu) \phi(\omega_i) d\omega_i + \int_{\bar{\omega}_t}^{\infty} \bar{\omega}_t \phi(\omega_i) d\omega_i = \int_0^{\bar{\omega}_t} \omega_i \phi(\omega_i) d\omega_i + [1 - \Phi(\bar{\omega}_t)] \bar{\omega}_t - \mu \Phi(\bar{\omega}_t)$.

²⁶The background assumptions of the zero profit condition are that lending is a perfectly competitive activity and entry into the lending market is costless. Formally, the two conditions characterizing the optimal contract result from maximizing (the firm's share of) the return on the financial contract (because the firm, if it remains solvent, is the residual claimant on output), $p_t f(\bar{\omega}_t) M_{it}$, subject to the zero profit condition of the lender, $p_t g(\bar{\omega}_t) M_{it} = M_{it} - nw_{it}$.

of the external finance premium p_t in the above expression.²⁷ Firms take as given the competitively-determined factor prices w_t and r_t .

Regarding the dynamic aspect of firms, firm i begins period t with assets k_{it}^e , whose beginning-of-period- t market value determines the firm's net worth nw_{it} , as shown in (9). The firm borrows $M_{it} - nw_{it}$ against the value of these assets, and it expects to keep $p_t f(\omega_t) M_{it}$ after repaying its loan.²⁸ Of these "excess" resources, the firm can either accumulate assets or make payments to households. That is,

$$\Pi_{it}^e + k_{it+1}^e = p_t f(\bar{\omega}_t) M_{it}, \quad (13)$$

which highlights that k_{it+1}^e can be thought of as retained earnings. Substituting the contractually-specified quantity of borrowing, $M = \frac{nw}{1-pg(\bar{\omega})}$, this can be re-written as

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} nw_{it}. \quad (14)$$

Further substituting the definition of net worth from (9), the firm's asset evolution is described by

$$\Pi_{it}^e + k_{it+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_{it}^e [1 + r_t - \delta] + e_t]. \quad (15)$$

Finally substituting (12) and (15) into (7), the dynamic profit function of the firm is

$$E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{t|0} \left\{ \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_{it}^e [1 + r_t - \delta] + e_t] - k_{it+1}^e + \omega_{mt} F(k_{it}, n_{it}) - p_t [w_t n_{it} + r_t k_{it}] \right\}. \quad (16)$$

4.2.3 Profit Maximization

Maximization of (16) with respect to capital rental k_{it} and labor hiring n_{it} gives rise to the capital demand condition

$$r_t = \frac{\omega_{mt} F_k(k_{it}, n_{it})}{p_t} \quad (17)$$

and the labor demand condition

$$w_t = \frac{\omega_{mt} F_n(k_{it}, n_{it})}{p_t}. \quad (18)$$

In (17) and (18), the effective payments per unit of each factor are $p_t r_t$ for capital rental and $p_t w_t$ for labor, reflecting firms' need for external financing. Financing costs drive an endogenous time-varying wedge between prices and marginal returns in factor markets, which leads CF to refer to

²⁷As is common in macro models, writing, for example, p_t , is shorthand for the state-contingent equilibrium function $p(\omega_{mt}, \sigma_t^\omega)$. If the distribution of ω were degenerate — that is, if there were no idiosyncratic component of technology — then we would have $p_t = 1 \forall t$, which simply has the interpretation that financing issues are irrelevant as in, say, a baseline RBC model.

²⁸This is because, as noted in footnote 26, the firm keeps the entire (expected) surplus from the contractual arrangement. Hence, in expectation, the firm is left with $p_t f(\omega_t) M_{it}$ after the sequence of borrowing, renting factors of production, producing output, and repaying its loan.

p_t as a “markup.” As discussed above, one can also usefully interpret p_t as the model’s external finance premium. That the external finance premium drives an endogenous time-varying wedge between prices and marginal returns in neoclassical factor markets is a key feature of the model. Note that, although firms may differ in their levels of factor usage, each firm chooses an identical capital-labor ratio because the market prices r_t and w_t and the external premium p_t are identical for all firms and the production technology $F(\cdot)$ is constant-returns.

Maximization of (16) with respect to asset accumulation k_{it+1}^e yields the capital Euler equation for firms,

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} [1 + r_{t+1} - \delta] \right\}. \quad (19)$$

4.2.4 Aggregation

Although firms are heterogenous, tracking the aggregates of this economy is simple. Because the production function $F(\cdot)$ is constant-returns and the monitoring technology is linear (in the quantity monitored), the firm side of the economy can be analyzed as if there were a representative firm that held the average quantity of net worth and hired the average quantity of labor and capital for production.²⁹ This representative firm has a profit function identical to (16) (with firm indices dropped), which clearly gives rise to the same optimality conditions (17), (18), and (19). The (aggregate) profits that get transferred to households are thus

$$\begin{aligned} \Pi_t &= \Pi_t^e + \Pi_t^f = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_t^e [1 + r_t - \delta] + e_t] - k_{t+1}^e + \omega_{mt} F(k_t, n_t) - p_t [w_t n_t + r_t k_t] \\ &= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_t^e [1 + r_t - \delta] + e_t] - k_{t+1}^e + \omega_{mt} F(k_t, n_t) - \omega_{mt} F_n(k_t, n_t) n_t - \omega_{mt} F_k(k_t, n_t) k_t \\ &= \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_t^e [1 + r_t - \delta] + e_t] - k_{t+1}^e. \end{aligned} \quad (20)$$

The second line makes use of the factor price conditions (17) and (18), and the third line follows because $F(\cdot)$ is constant-returns. Thus, note that in this representative-firm foundation of aggregates, firms earn zero aggregate operating profits, so $\Pi_t = \Pi_t^e$.

The result that sufficient linearity in the model makes the distribution of outcomes across firms irrelevant for aggregates is thus the foundation for the claim earlier that the contractual terms p_t and $\bar{\omega}_t$ are not firm-specific. Given a $(p_t, \bar{\omega}_t)$, firms can (and do) differ only in size — a firm with a larger net worth receives a proportionately larger loan and so produces more output. But the size distribution of firms is irrelevant for market prices and hence aggregate outcomes in the model, which makes the agency-cost framework tractable in a DSGE setting.

Finally, the aggregate resource constraint of the economy is

$$c_t + k_{t+1} - (1 - \delta)k_t = \omega_{mt} F(k_t, n_t) [1 - \mu \Phi(\omega_t)], \quad (21)$$

²⁹See CF (1997, 1998) for more details.

in which $k_t = k_{ht} + k_t^e$ is the equilibrium quantity of physical capital at the beginning of period t . Note that aggregate monitoring costs are a final use of output.

4.3 Private Sector Equilibrium

A symmetric private-sector equilibrium is made up of state-contingent endogenous processes $\{c_t, n_t, k_{ht+1}, k_{t+1}^e, k_{t+1}, \Pi_t^e, w_t, r_t, p_t, \bar{\omega}_t\}$ that satisfy the following conditions: the labor-supply condition

$$\frac{v'(1 - n_t)}{u'(c_t)} = w_t; \quad (22)$$

the labor-demand condition

$$w_t = \frac{\omega_{mt} F_n(k_t, n_t)}{p_t}; \quad (23)$$

the capital-demand condition

$$r_t = \frac{\omega_{mt} F_k(k_t, n_t)}{p_t}; \quad (24)$$

the representative household's Euler equation for capital holdings

$$1 = E_t \left\{ \Xi_{t+1|t} [1 + r_{t+1} - \delta] \right\}; \quad (25)$$

the (representative) firm's Euler equation for capital holdings

$$1 = \gamma E_t \left\{ \Xi_{t+1|t} \frac{p_{t+1} f(\bar{\omega}_{t+1})}{1 - p_{t+1} g(\bar{\omega}_{t+1})} [1 + r_{t+1} - \delta] \right\}; \quad (26)$$

aggregate capital market clearing

$$k_t = k_{ht} + k_t^e; \quad (27)$$

the aggregate resource constraint

$$c_t + k_{t+1} - (1 - \delta)k_t = \omega_{mt} F(k_t, n_t) [1 - \mu \Phi(\bar{\omega}_t)]; \quad (28)$$

the contractually-specified loan size

$$M_t = \frac{nw_t}{1 - p_t g(\bar{\omega}_t)}, \quad (29)$$

in which expression (9) for nw_t is substituted in; the contractually-specified liquidation threshold

$$\frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} = -\frac{f'(\bar{\omega}_t)}{g'(\bar{\omega}_t)}; \quad (30)$$

and the evolution of the aggregate assets of firms (equivalently, the assets of the representative firm)

$$\Pi_t^e + k_{t+1}^e = \frac{p_t f(\bar{\omega}_t)}{1 - p_t g(\bar{\omega}_t)} [k_t^e [1 + r_t - \delta] + e_t]. \quad (31)$$

The private sector takes as given the stochastic process $\{\omega_{mt}, \sigma_t^\omega\}_{t=0}^\infty$.

5 Basic Analytics: Firm Risk and Leverage

Before proceeding to the quantitative analysis of the model, it is useful to consider analytically the intuition behind the model's main mechanism. These analytics do not formally prove the main results, which are quantitative in nature. But they shed light on the transmission mechanism, which is quantified in Section 6.

To begin this intuitive consideration, note that conditions (29) and (30), which characterize the terms of the financial contract, can be combined to

$$M - nw = - \left(\frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})g(\bar{\omega}; \sigma^{\omega})}{f(\bar{\omega}; \sigma^{\omega})g_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})} \right) nw. \quad (32)$$

I drop time indices here for ease of notation. The term in parentheses is the leverage ratio because it expresses a firm's total debt obligation, $M - nw$, as a multiple of its net worth (its equity). Thus, define the leverage ratio as

$$\ell(\bar{\omega}; \sigma^{\omega}) \equiv - \frac{f_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})g(\bar{\omega}; \sigma^{\omega})}{f(\bar{\omega}; \sigma^{\omega})g_{\bar{\omega}}(\bar{\omega}; \sigma^{\omega})}. \quad (33)$$

The expected share functions $f(\cdot)$ and $g(\cdot)$ and their derivatives depend on the cross-sectional dispersion σ^{ω} of firm productivity, hence the leverage ratio also depends on σ^{ω} . For this intuitive argument, I emphasize this dependence by explicitly noting it as an argument of these functions.

Figure 9 illustrates why changes in the cross-sectional dispersion of firms' TFP would be expected to cause changes in leverage. Suppose the solid black curve in Figure 9 is the pdf $\phi(\omega)$ before a risk shock occurs. The liquidation threshold $\bar{\omega}$ shown is for this initial distribution. Suppose there is an exogenous reduction in dispersion. If the liquidation threshold $\bar{\omega}$ were to remain unchanged, fewer firms would draw an idiosyncratic $\omega < \bar{\omega}$, which lenders understand because the density $\phi(\omega)$ is common knowledge. This in turn means that fewer firms are expected to be unable to repay their loans, which reduces lenders' risk. Ex-ante, then, lenders would be willing to extend more credit, which implies higher leverage ratios for firms (borrowers). In general equilibrium, $\bar{\omega}$ will of course also change. It is thus a quantitative question how much a given-size change in dispersion σ^{ω} will affect the threshold $\bar{\omega}$ and hence leverage and hence real activity. These questions can only be answered in the full general equilibrium model.

6 Quantitative Analysis

6.1 Computational Strategy

To study the dynamics of the model, I compute a second-order approximation of the equilibrium using my own implementation of the perturbation algorithm described by Schmitt-Grohe and Uribe (2004). Because the main interest is in business cycle fluctuations, such methods are likely to

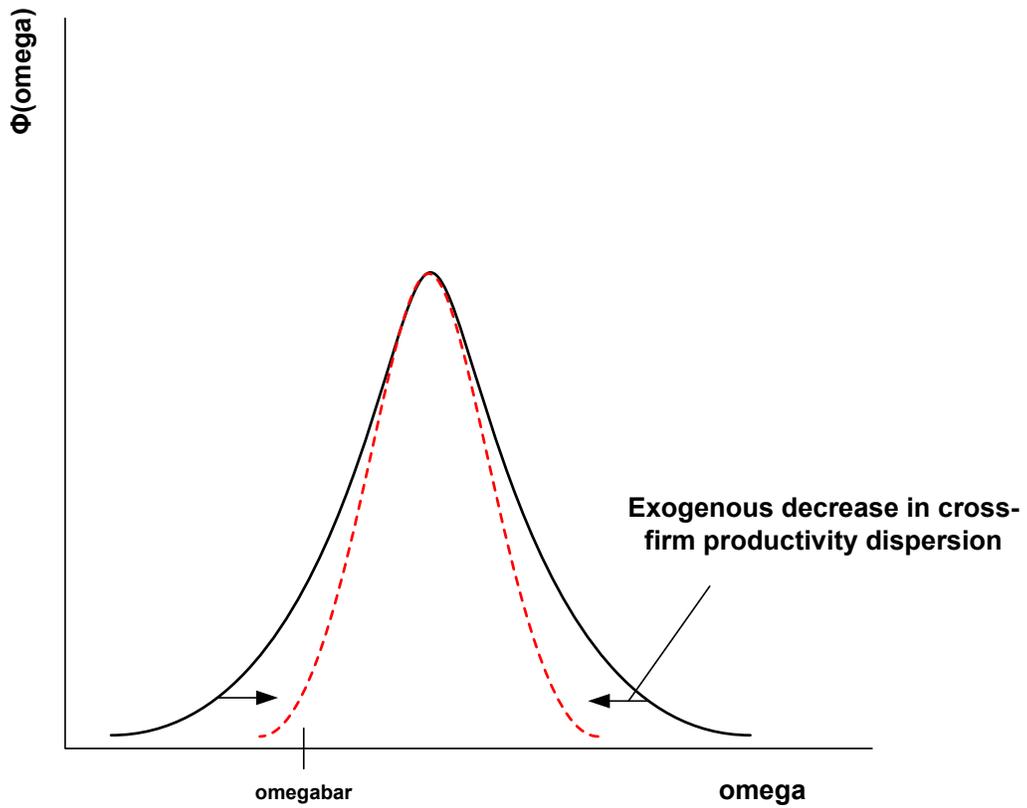


Figure 9: An exogenous decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.

accurately portray the model’s dynamic behavior, as the studies by Aruoba, Fernandez-Villaverde, and Rubio-Ramirez (2006) and Caldera, Fernandez-Villaverde, Rubio-Ramirez, and Yao (2009) suggest.

Changes in cross-sectional risk are indeed aggregate, rather than idiosyncratic, shocks in the model economy. Because I track only aggregate outcomes and do not track any firm-specific outcomes, there is no reason to think that local approximation methods will misrepresent the model’s aggregate dynamics.³⁰ Given a local approximation strategy, I nonetheless compute a second-order approximation given the novelty of the analysis. However, it is useful to note that the results reported below are virtually identical to those I obtain from a linear approximation. The quantitative results reported below are thus fundamentally driven by the model’s mechanism — changes in cross-sectional risk leading to changes in firms’ leverage, which then potentially are transmitted to the real economy — rather than issues about the appropriate approximation method.

Before presenting the dynamic results, I complete the description of the calibration of the model and briefly describe some of its long-run predictions.

6.2 Calibration

The novel aspect of the model calibration is the risk shock process using micro data, as described in Section 2. As described there, long-run dispersion of firm productivity is $\bar{\sigma}^\omega = 0.156$. This is about half the value used by CF (1998, p. 590) and Bernanke, Gertler, and Gilchrist (1999, p. 1368), which are calibrated to aggregate financial data, not firm-level data: the former set $\bar{\sigma}^\omega = 0.37$, and the latter set $\bar{\sigma}^\omega = 0.28$. Thus, direct micro evidence indicates less cross-sectional dispersion than standard macro calibrations of agency-cost models.

As also discussed in Section 2, I assume sufficient smoothness in the average TFP and risk processes so that I can set quarterly persistence parameters $\rho_{\omega_m} = 0.83$ and $\rho_{\sigma^\omega} = 0.83$, even though the data on which the estimation is based are annual. This mismatch between (desired) model frequency and empirical frequency raises the question of the appropriate calibration of the standard errors of the quarterly innovations in the TFP and risk processes.³¹ Given the quarterly frequency of the model and the annual frequency of the productivity data, I simply time aggregate the simulated data from the model, and set parameters σ_{ω_m} and σ_{σ^ω} so that the annualized volatilities of average TFP and dispersion of TFP in the model match their annual empirical counterparts. As documented in Section 2, these are, respectively, 1.26 percent and 3.15 percent. This simulated-

³⁰Recall the discussion above that, given the maintained assumptions of the model, aggregates in the model do not depend on distributions of outcomes at the firm level.

³¹As noted in Section 2, the standard deviation of the annual innovations in the average TFP and risk processes are, respectively, 0.0111 and 0.0276.

Functional Form	Description
$\ln \sigma_{t+1}^\omega = (1 - \rho_{\sigma^\omega}) \ln \bar{\sigma}^\omega + \rho_{\sigma^\omega} \ln \sigma_t^\omega + \epsilon_{t+1}^{\sigma^\omega}$	Exogenous process for firm productivity dispersion
$\ln \omega_{mt+1} = \rho_{\omega_m} \ln \omega_{mt} + \epsilon_{t+1}^{\omega_m}$	Exogenous process for mean of TFP
$u(c) = \ln c$	Consumption subutility
$v(\ell) = \psi \ln \ell$	Leisure subutility
$F(k, n) = k^\alpha n^{1-\alpha}$	Production technology

Table 3: Functional forms for quantitative analysis.

method-of-moments procedure leads to $\sigma_{\omega_m} = 0.008$ and $\sigma_{\sigma^\omega} = 0.0033$.³²

Besides the calibration of the exogenous processes, Table 3 lists all functional forms used in the quantitative experiments, and Table 4 lists all baseline parameter settings. The preference and production parameters are standard in business cycle models. The agency cost parameter is set to $\mu = 0.15$, which is the same as the calibrated value in Covas and den Haan (2006) and in line with the estimate $\mu = 0.12$ by Levin, Natalucci, and Zakrajsek (2004). The value for firm’s “additional” discount factor is set to $\gamma = 0.99$, which allows the model to match a long-run annualized external finance premium of about two percent. This value of γ is larger than the calibrated values of CF and BGG and seems due to the much lower calibrated value of $\bar{\sigma}^\omega$ here.

6.3 Long-Run Dispersion and Long-Run Equilibrium

I compute the long-run deterministic (steady-state) equilibrium numerically using a standard non-linear equation solver. The main comparative static exercise I conduct is presented in Figure 10, which plots the long-run (steady-state) equilibria as a function of long-run cross-sectional dispersion $\bar{\sigma}^\omega$. All other parameters are held fixed at those presented in Table 4.

Figure 10 shows that the long-run response of the economy to changes in $\bar{\sigma}^\omega$ is non-monotonic. For low dispersion of idiosyncratic productivity, GDP falls as dispersion rises, but for high dispersion, the comparative static result reverses. The nonmonotonicity is also evident in the long-run behavior of the finance premium (lower right panel) as well as other standard aggregate quantities such as gross investment and consumption (for brevity, the latter are not shown in Figure 10). This effect is not due to any nonmonotonicity of the contract terms, as debt (upper middle panel) is strictly decreasing in $\bar{\sigma}^\omega$, and the bankruptcy threshold $\bar{\omega}$ (not shown) and hence bankruptcies

³²It is interesting to note that $\sigma_{\omega_m} = 0.008$ is quite similar to the calibration of the size of quarterly innovations in the aggregate TFP process in a baseline RBC model, in which a benchmark value is 0.007. Here, of course, $\sigma_{\omega_m} = 0.008$ is computed directly from micro data.

Parameter Value	Description/Notes
<u>Preferences</u>	
$\beta = 0.99$	Households' quarterly subjective discount factor
$\gamma = 0.99$	Firms' (additional) subjective discount factor
$\psi = 1.8$	Leisure calibrating parameter (calibrated in baseline model)
<u>Production Technology</u>	
$\alpha = 0.36$	Capital's share in production function
$\delta = 0.02$	Depreciation rate of capital
<u>Financial Markets and Agency Costs</u>	
$\mu = 0.15$	Per-unit monitoring cost
$\omega_m = 1$	Long-run mean of idiosyncratic productivity
$\bar{\sigma}^\omega = 0.156$	Long-run standard deviation of distribution of $\ln \omega$
$\rho_{\sigma^\omega} = 0.83$	Quarterly persistence of log firm risk process
$\sigma_{\sigma^\omega} = 0.0033$	Standard deviation of innovations to log firm risk
<u>Exogenous Process</u>	
$\rho_{\omega_m} = 0.83$	Quarterly persistence of log mean-TFP process
$\sigma_{\omega_m} = 0.0081$	Standard deviation of innovations to log mean-TFP

Table 4: Parameter values for baseline model.

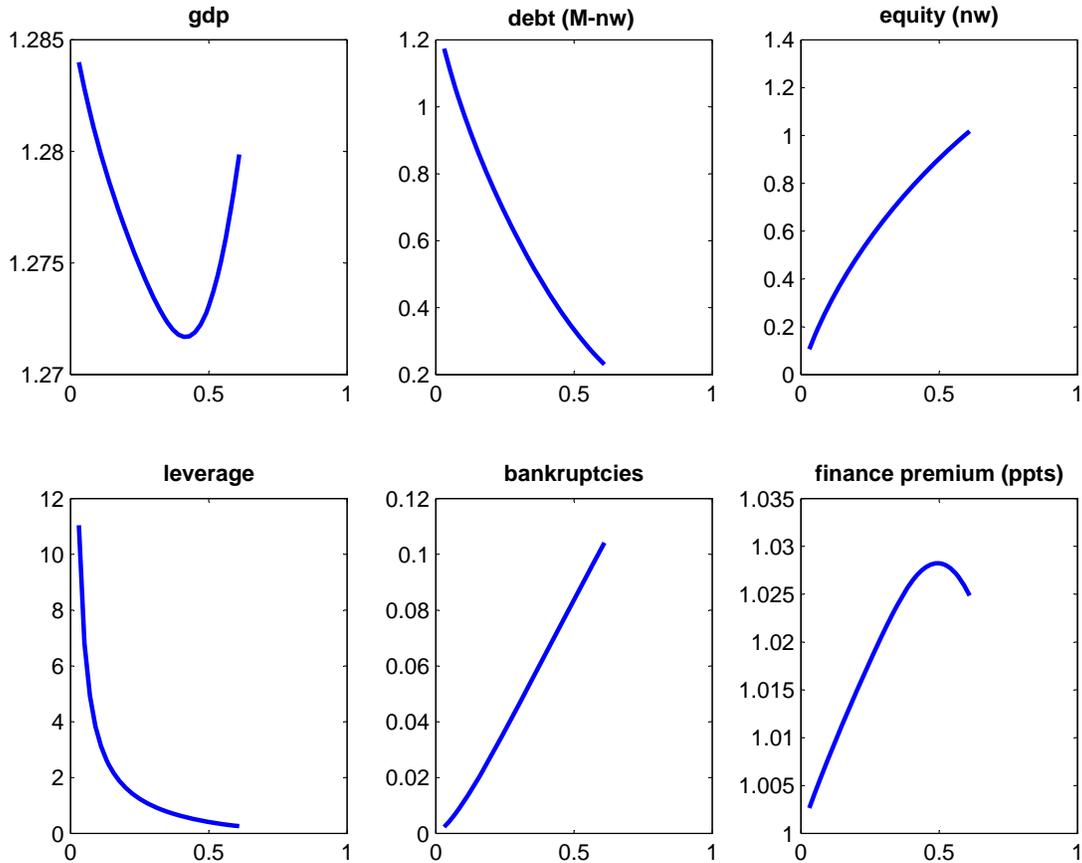


Figure 10: Long-run equilibrium as long-run standard deviation of idiosyncratic productivity distribution, $\bar{\sigma}^\omega$, varies; $\bar{\sigma}^\omega$ plotted on horizontal axis.

(lower middle panel) are strictly increasing in $\bar{\sigma}^\omega$. When σ_t^ω is allowed to fluctuate around the long-run dispersion $\bar{\sigma}^\omega = 0.156$ during simulations of the model, dispersion never reaches as high as 0.40, hence the model's dynamics do not cover the inflection point Figure 10 reveals.³³ I leave to future investigation further study of the nonmonotonicity.

For the baseline calibration, the model's long-run leverage ratio, at 1.77, is larger than the 1.03 long-run leverage ratio documented in Section 3. In the model, the conceptually most important determinant of long-run leverage is long-run dispersion, $\bar{\sigma}^\omega$. That is, as dispersion shrinks to zero, which means that lenders face no risk whatsoever on their loans, the leverage ratio grows unboundedly, independent of all other parameter values. This effect is shown in the lower left panel of Figure 10.³⁴ Apparently, the empirically-relevant $\bar{\sigma}^\omega = 0.156$ is small enough steady-state risk

³³As Table 4 shows, the calibrated value of the standard error of the shocks to the dispersion process is $\sigma_{\sigma^\omega} = 0.0027$, which is sufficiently small that during simulations, $\sigma_t^\omega = 0.40$ was never reached.

³⁴That is, as $\bar{\sigma}^\omega \rightarrow 0$, lenders are willing to lend ever larger quantities. Alternatively, one could say that leverage

Financial Measure	Long-Run Value
Leverage ratio, $\ell(\bar{\omega})$	1.77
External premium, $100(p - 1)$	2.10 percent
Bankruptcy rate, $100\Phi(\bar{\omega})$	1.04 percent

Table 5: Long-run financial variables for the baseline calibration of the model.

that the model overpredicts long-run leverage. To force the model to explain long-run leverage given the rest of the parameters requires $\bar{\sigma}^\omega = 0.24$. Indeed, $\bar{\sigma}^\omega = 0.24$ is closer to typical macro calibrations of this class of models, such as CF and BGG. However, the overprediction of long-run leverage here is not a shortcoming of the analysis. Instead of treating $\bar{\sigma}^\omega$ as a free parameter to match aggregate moments, as other agency-cost macro models do, it seems important to know that direct micro evidence on this parameter leads to perhaps substantially different long-run aggregate predictions.

It is useful to also highlight the long-run values implied by the model of two other financial variables of interest: the (annualized) finance premium and the bankruptcy rate. These are collected in Table 5. The long-run bankruptcy rate is substantially lower than in the Dun & Bradstreet evidence cited by CF (1998, p. 590), while the finance premium is in line with most of the measures of premia presented in DeGraeve (2008).³⁵ The former result is again a reflection of a relatively low level of long-run risk, while the latter is the calibration target at which γ was aimed.

6.4 Business Cycle Dynamics

I divide the presentation of the baseline model’s cyclical dynamics into three parts. First, to establish a baseline that can be directly compared to CF’s experiments, I document how macro as well as financial aggregates respond to standard shocks to average TFP, with cross-sectional dispersion of firm productivity held constant at $\bar{\sigma}^\omega$. Then, I document how the model behaves in response to only dispersion shocks, with average TFP held constant at $\omega_m = 1$. Finally, I allow both shocks to simultaneously drive the economy.

is undefined because financial frictions do not matter and the model technically pins down neither loan amounts nor leverage.

³⁵As discussed extensively by DeGraeve (2008), it is not clear what is the most relevant empirical counterpart to the model’s external finance premium. Many natural alternatives suggest themselves, such as the difference between the prime borrowing rate and the short-term T-bill rate, the interest spread between AAA-rated commercial paper and T-bills, the spread between BBB-commercial paper and T-bills, and so on. DeGraeve (2008) documents that these various empirical measures of “the external finance premium” behave differently enough over the business cycle that it remains an open question what the natural empirical counterpart of the model’s external finance premium is.

		GDP	C	I	leverage	debt	equity
Std. dev. (%)		2.42	0.83	9.05	3.23	4.53	3.08
Auto. corr.		0.80	0.98	0.78	0.01	0.66	0.81
Corr. matrix	GDP	1	0.50	0.97	0.57	0.98	0.82
	C		1	0.27	0.01	0.39	0.56
	I			1	0.63	0.97	0.75
	leverage				1	0.73	0.01
	debt					1	0.68
	equity						1
Annual std. dev (%) TFP		1.21	(Data: 1.26)				
Annual std. dev (%) σ^ω		0	(Data: 3.16)				
Annual correlation (GDP, σ^ω)		—	(Data: -0.83)				
Annual correlation (TFP, σ^ω)		—	(Data: -0.98)				

Table 6: Simulation-based business cycle statistics, only average TFP shocks. Upper panel from quarterly simulations, lower panel from annualization of quarterly simulations.

6.4.1 TFP Shocks

To establish a baseline, I first demonstrate that the model’s predictions are in line with those obtained by CF when $\sigma_t^\omega = \bar{\sigma}^\omega \forall t$ and it is only average TFP shocks that drive fluctuations. Figure 11 displays impulse responses to a one-time, one-standard deviation positive shock to average TFP, holding constant cross-sectional dispersion of firm productivity. The results are qualitatively in line with those documented in CF (1998, Figure 1) for their “output model,” although magnitudes differ due to different calibrations.

Figure 11 shows that leverage rises somewhat substantially, with the peak response about twice as large as the peak response of GDP. CF (1998, Figure 1) do not report the dynamics of leverage (nor is it reported in the related models of CF (1997) or BGG), so this result is somewhat new in the literature. Thus, in contrast to the conjecture in Carlstrom, Fuerst, and Paustian (2009, p. 8), the leverage ratio is not virtually constant, conditional on TFP shocks, in the basic agency-cost model.³⁶ However, a more informative metric may be the relative volatility of leverage with respect to GDP induced by TFP shocks.

³⁶More precisely, leverage is not virtually constant, conditional on TFP shocks, in the CF output model. This paper does not test the dynamics of leverage in the CF (1997) investment model.

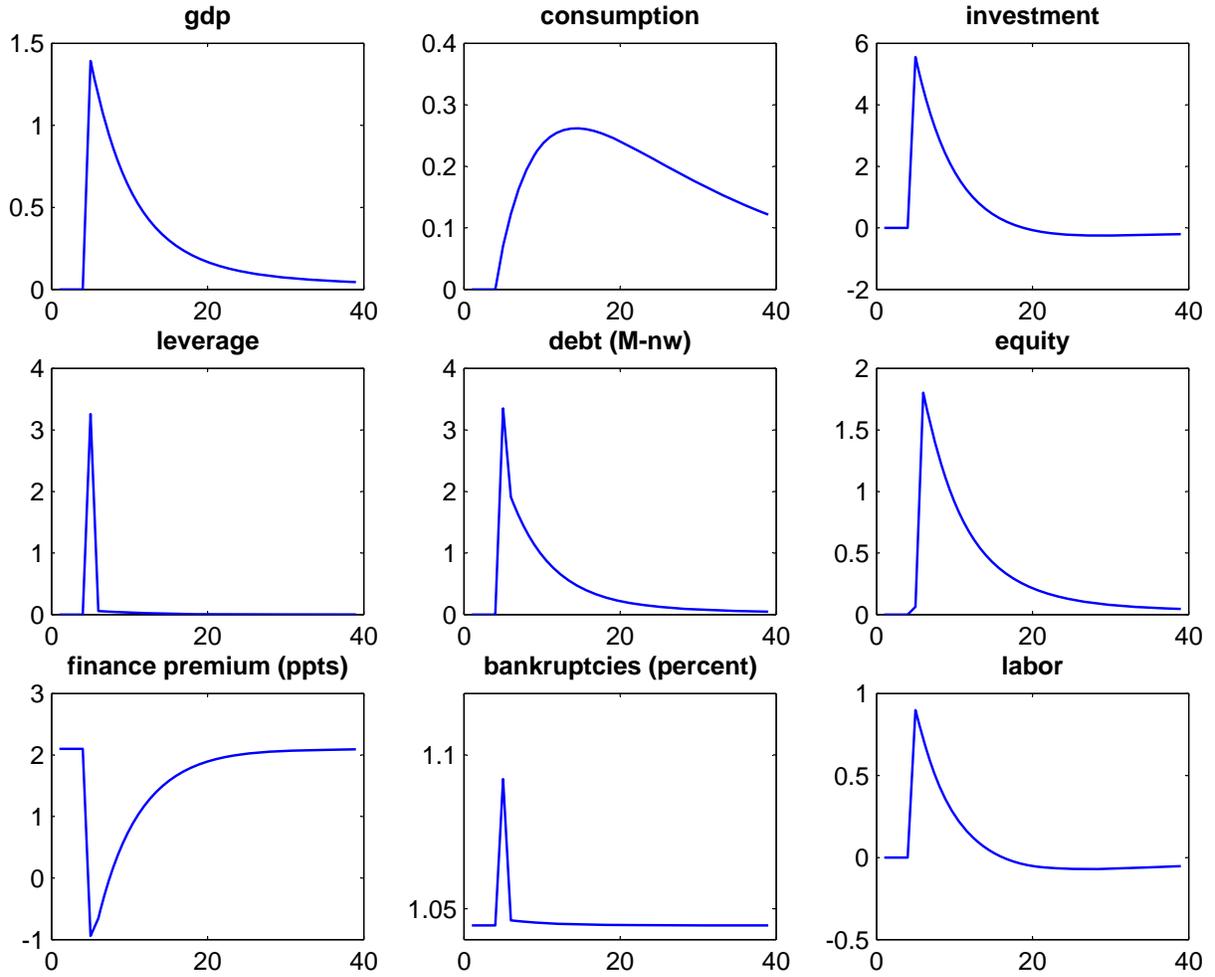


Figure 11: Impulse response to a one-standard-deviation exogenous increase in average TFP, holding constant the dispersion σ^ω of firm productivity. Except where noted, scale is percentage point deviation from steady state.

To this end, Table 6 presents business cycle statistics from model simulations when the only exogenous process is fluctuations in average TFP.³⁷ While CF do not report simulation-based moments, the model reproduces basic business cycle stylized facts: for example, gross investment is nearly four times as volatile as GDP, consumption is less volatile than GDP, and GDP, consumption, and investment are all highly persistent.

On financial measures, leverage, debt, and equity are all more volatile than GDP. In a relative volatility sense, though, none is as volatile as the evidence documented in Table 1. On balance, then, the view seems mixed on whether or not the basic agency-cost model predicts meaningful fluctuations in leverage conditional on shocks to average TFP. Leverage fluctuations are certainly not miniscule, which is the impression left by Carlstrom, Fuerst, and Paustian (2009, p. 8). Relative to the volatility of GDP, though, leverage fluctuations in the model are over three times smaller than in the empirical evidence presented in Table 1.

6.4.2 Risk Shocks

With the baseline dynamics of the model established, I now present the main set of experiments conducted in the model, namely dynamics in the face of pure risk shocks. Figure 12 presents impulse responses to a one-time, one-standard deviation positive shock to the cross-sectional dispersion of firm productivity, holding constant average TFP. Complementing this impulse-response analysis are the simulated business cycle statistics reported in Table 7, in which it is only risk shocks that generate business cycles.

Comparing Figure 12 with Figure 11 shows that a pure risk shock induces virtually no GDP response — the peak response of GDP in Figure 12 is two orders of magnitude smaller than the peak response of GDP in Figure 11! This is one of the main results of the model analysis: empirically-relevant risk shocks seem to play little role as an independent driver of aggregate quantity fluctuations. This is one of the main messages of the theoretical model of BB, as well, even though their model does not situate financial frictions as part of the potential transmission channel for risk shocks. Examining just the role of financial frictions in the transmission mechanism leads to a broadly similar conclusion as BB. The result here is even starker than in BB, though, because I found innovations in firm risk to be about five times larger than found by BB, as discussed in Section 2. Thus, despite much larger risk shocks, the pass-through of risk shocks to quantity fluctuations is still minor.

However, comparing Figure 12 with Figure 11 shows that financial variables do react much

³⁷These business cycle statistics are generated by simulating the model 1000 times around the deterministic steady state equilibrium, with each simulation 1000 periods in length, and then computing the medians across simulations of standard deviations, correlations, etc.

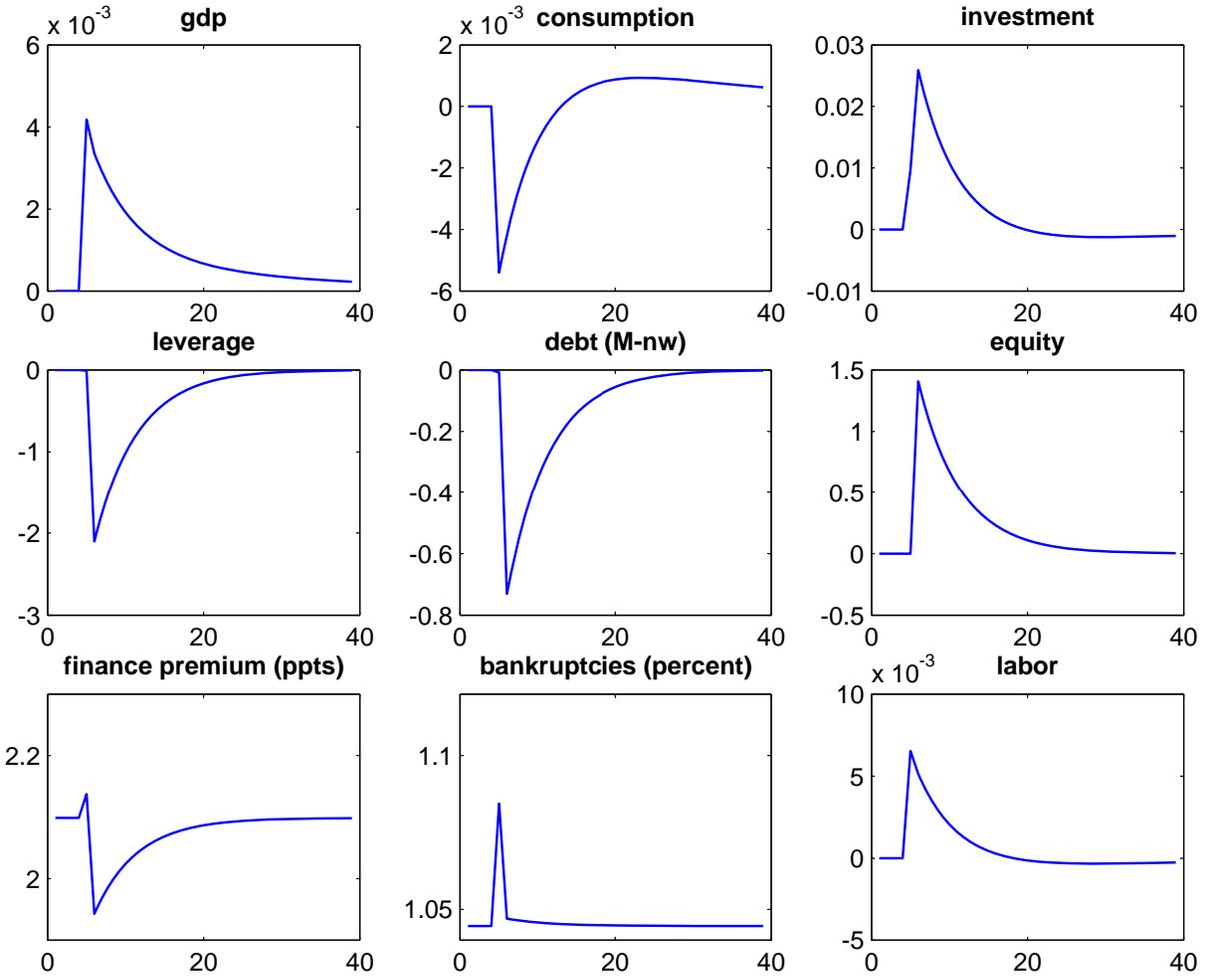


Figure 12: Impulse response to a one-standard-deviation exogenous increase in the dispersion σ^ω of firm productivity, holding constant average TFP. Except where noted, scale is percentage point deviation from steady state.

		GDP	C	I	leverage	debt	equity
Std. dev. (%)		0.02	0.01	0.04	3.60	1.23	2.38
Auto. corr.		0.81	0.77	0.87	0.80	0.80	0.79
Corr. matrix	GDP	1	-0.79	0.87	-0.79	-0.79	0.79
	C		1	-0.82	0.62	0.62	-0.62
	I			1	-0.96	-0.96	0.96
	leverage				1	1	-1
	debt					1	-1
	equity						1
Annual std. dev (%) TFP		0	(Data: 1.26)				
Annual std. dev (%) σ^ω		3.16	(Data: 3.16)				
Annual correlation (GDP, σ^ω)		0.98	(Data: -0.83)				
Annual correlation (TFP, σ^ω)		—	(Data: -0.98)				

Table 7: Simulation-based business cycle statistics, only risk shocks. Upper panel from quarterly simulations, lower panel from annualization of quarterly simulations.

more strongly to a risk shock than to a shock to average TFP. This is the second main result of the model analysis. In line with the intuitive discussion in Section 5, debt and hence leverage fall (sharply) in response to a rise in firm risk, as lenders pull back on extending credit. In terms of relative volatilities of fluctuations in financial variables compared to aggregate macro quantities, risk shocks are quantitatively very powerful.

The quantitative power of pure risk shocks on financial outcomes is more clearly revealed by the simulation-based results reported in Table 7. The cyclical volatilities of leverage, debt, and equity are broadly similar in magnitude to their volatilities conditional on only average-TFP shocks (Table 6), even though the volatility of GDP is less than two percent as large.³⁸ In terms of cyclicity of the leverage ratio, it is strongly countercyclical (-0.79) with respect to GDP, at odds with the virtual acyclicity documented in Section 3. Note, however, that this result arises in the face of only risk shocks — below, I show that if the economy is hit by both shocks to average TFP and risk shocks, the cyclicity of leverage is much more consistent with the data.

Somewhat counterintuitively, and clearly counterfactually, an increase in cross-sectional dispersion induces an *increase* in GDP. As shown in Figure 2 and as also documented by BB and BFJ, firm-level dispersion is clearly countercyclical. The reason this result seems to arise in the model is due to the “Hartman-Abel effect,” which also arises in the simplest version of the BFJ model that features a minimum of adjustment costs for capital and labor. The idea, as described by BFJ (p. 20), is that absent sufficient adjustment costs, a higher variance of productivity increases output because marginal revenue products are convex in productivity. While I do not model “adjustment costs” in the way the firm-level literature typically does, the entire agency cost/financial friction mechanism can be viewed broadly as a type of “adjustment cost.” However, it apparently is not strong enough to overturn the Hartman-Abel effect. In Section 7, I modify the model in a simple way to deliver countercyclical firm risk. The leverage volatility result in this baseline model, though, carries over to the modified model, hence it is useful to understand how the baseline model works, both its successes and shortcomings.

6.4.3 Both First-Moment Shocks and Second-Moment Shocks

Although risk shocks cause great amplification of financial variables compared to shocks to average TFP, the correlation of leverage with macro quantities does not line up with its empirical counterpart. In particular, the strong negative contemporaneous correlation of leverage with GDP is not in line with the empirical evidence presented in Section 3. However, even conditional on just a productivity-driven view of business cycles, it seems reasonable to think of fluctuations as being due to *both* first-moment shocks *and* second-moment shocks.

³⁸The first row of Table 7 reports GDP volatility of 0.02 percent; without rounding, the volatility is 0.01733 percent.

		GDP	C	I	leverage	debt	equity
Std. dev. (%)		2.42	0.83	9.05	4.88	4.72	3.87
Auto. corr.		0.81	0.99	0.78	0.44	0.67	0.80
Corr. matrix	GDP	1	0.50	0.97	0.38	0.93	0.64
	C		1	0.27	0.01	0.37	0.44
	I			1	0.42	0.93	0.59
	leverage				1	0.67	-0.44
	debt					1	0.35
	equity						1
Annual std. dev (%) TFP		1.21	(Data: 1.26)				
Annual std. dev (%) σ^ω		3.16	(Data: 3.16)				
Annual correlation (GDP, σ^ω)		-0.01	(Data: -0.83)				
Annual correlation (TFP, σ^ω)		0.07	(Data: -0.98)				

Table 8: Simulation-based business cycle statistics, with average TFP shocks and risk shocks. Upper panel from quarterly simulations, lower panel from annualization of quarterly simulations.

In Table 8, I report business cycle statistics when the model economy is hit by independent shocks to both average-TFP and cross-firm dispersion. The contemporaneous correlation between leverage and GDP is now only modestly positive, which at least brings the model’s prediction closer to the acyclical/mildly countercyclical pattern of leverage documented in Section 3. The combination of first-moment and second-moment shocks implies volatilities of leverage, debt, and equity remarkably close to the evidence presented in Table 1. The volatilities of consumption and investment relative to that of GDP are unchanged from the TFP-shock-only case.

However, no matter which configuration of shocks is considered — first-moment shocks alone, second-moment shocks alone, or both in tandem — none of the experiments conducted in the baseline model lead to countercyclicality of firm risk. For example, the combined-shock model features zero correlation between risk fluctuations and GDP, as the next-to-last row of the lower panel of Table 8 shows. As documented in Section 2, this is opposite the empirical evidence, which uniformly reveals strong *countercyclicality* of firm risk. In Section 7, I modify the model to accommodate this.

7 Bundled Aggregate Shocks: TFP-Induced Risk Fluctuations

I model countercyclicality of firm risk by linking time-variation in average TFP directly to fluctuations in firm-level risk. Specifically, the cross-sectional dispersion of productivity across firms is now assumed to decline when average TFP improves. First-moment shocks are thus assumed to be bundled with second-moment shocks, and I refer to the entire bundle as an “aggregate shock.” The two processes are assumed to be linked according to

$$\sigma_t^\omega = \bar{\sigma}^\omega + \varphi \ln \omega_{mt}. \quad (34)$$

This condition replaces the exogenous law of motion (1) for σ_t^ω , and the evolution of ω_{mt} is still described by (2). The rest of the model is exactly the same as above. The parameter φ is clearly the key parameter of this version of the model, with $\varphi < 0$ implying countercyclicality of firm-level risk.³⁹ In terms of correlation between average TFP and dispersion of TFP, $\varphi < 0$ obviously implies a perfect negative correlation between the two, but this portrayal is not counterfactually stark compared to the data; recall from Section 2 that the contemporaneous cyclical correlation between average TFP and dispersion of TFP is -0.98.

Figure 13 illustrates why $\varphi < 0$ leads to countercyclical firm risk. A positive shift in average TFP will, all else equal, increase GDP. If at the same time cross-sectional dispersion declines due to $\varphi < 0$, and supposing initially that the bankruptcy threshold $\bar{\omega}$ were fixed, fewer firms would be

³⁹Clearly, $\varphi > 0$ would deliver procyclical firm-level risk, and $\varphi = 0$ would recover the baseline CF model in which there are never any changes in firm risk.

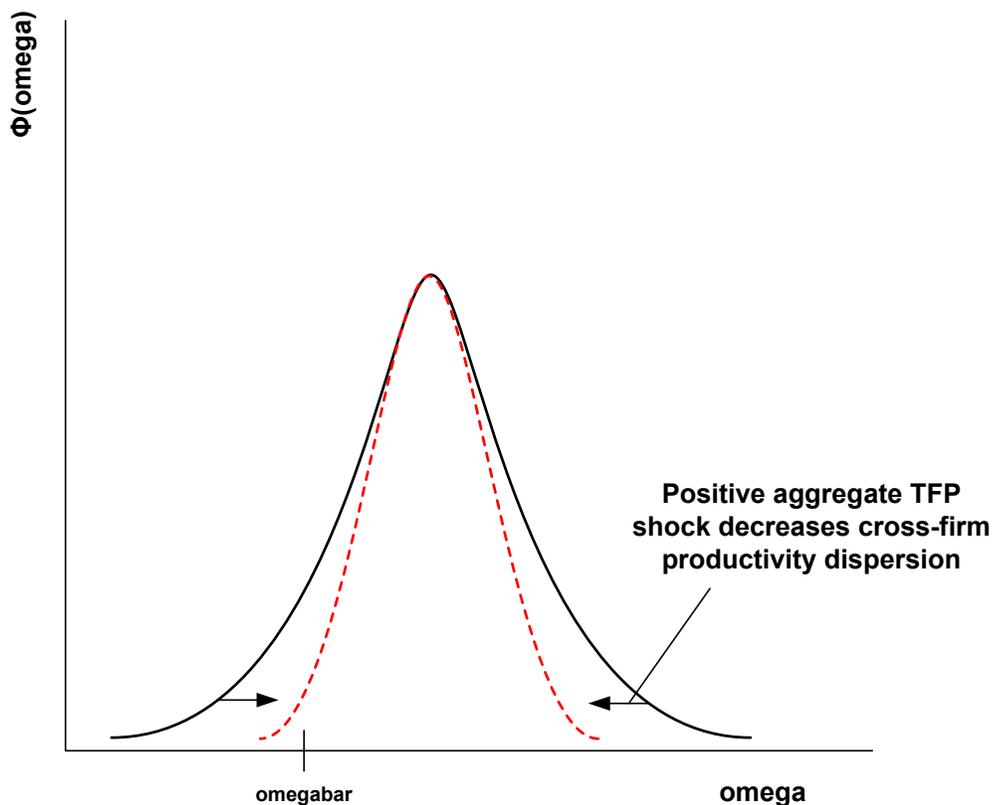


Figure 13: A positive shock to the mean of aggregate TFP causes a decrease in the dispersion of productivity across firms. The bankruptcy threshold $\bar{\omega}$ shown is for the original distribution; if the threshold were to remain unchanged, fewer firms would be expected to go bankrupt, which in turn would make lenders willing to allow larger leverage ratios.

expected to go bankrupt. This in turn would induce lenders to extend more credit, hence leverage rises for given net worth. Indeed, the second part of the intuitive argument is exactly the same as that underlying Figure 9. What is different from the baseline model is the event that now induces the change in dispersion. In the baseline model, the change in dispersion itself was the exogenous event, whereas here it is a positive shock to average TFP.

This bundled aggregate shock is of course a reduced-form construct. However, I bring the same empirical evidence presented in Section 6.2 to bear on the calibration of the crucial elasticity parameter φ . The calibration approach is to choose φ so that the model matches the observed time-series variation in cross-sectional dispersion. Section 6.2 documented that the time-series volatility in annual cross-sectional dispersion is 3.15 percent. Given this target and holding fixed all parameters in Table 4, this simulated-method-of-moments procedure (with average TFP fluctuations now as the sole truly exogenous driving process) leads to $\varphi = -1.43$.

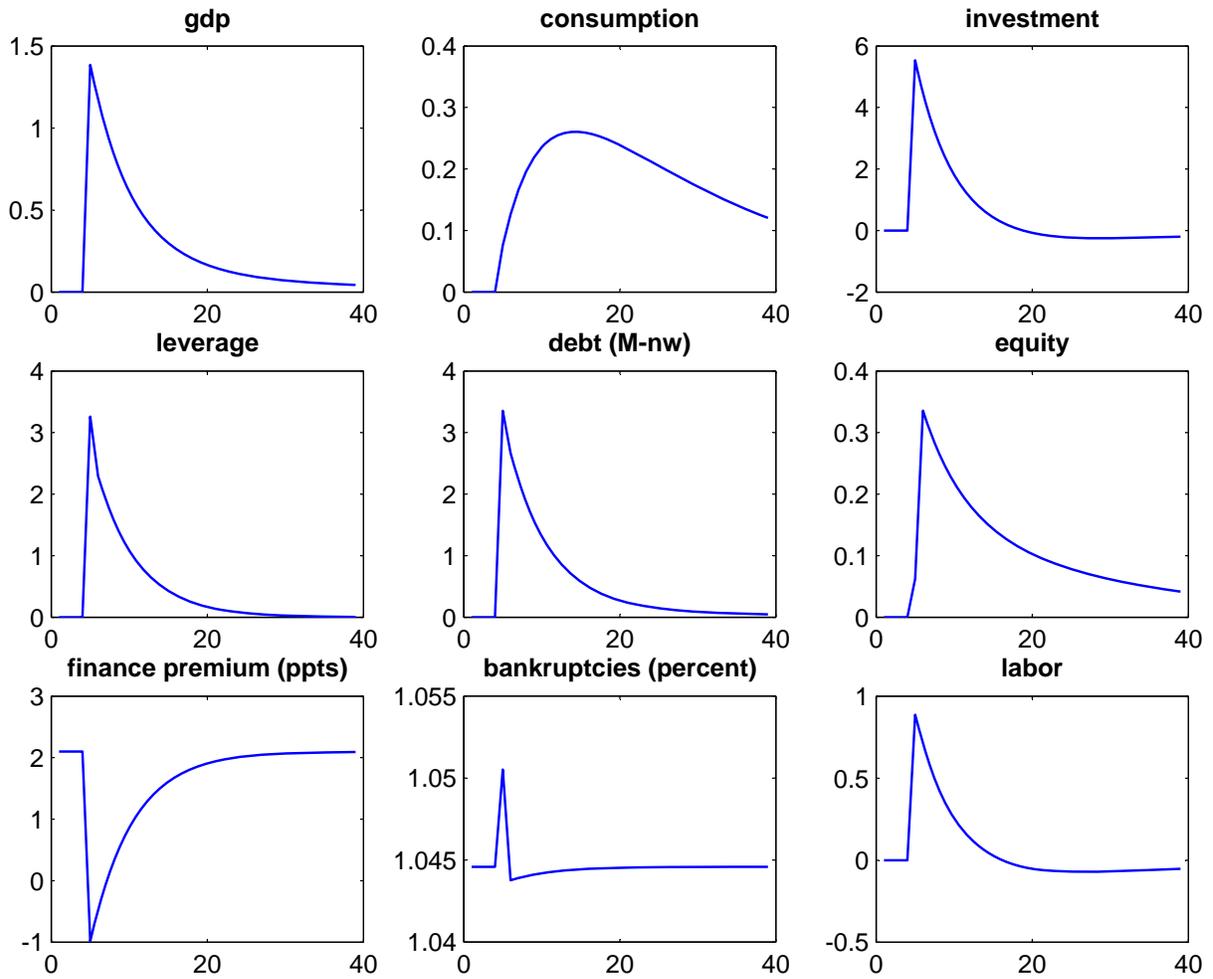


Figure 14: Impulse response to a positive “bundled aggregate shock,” in which a one-standard-deviation exogenous increase in average TFP induces a decrease in cross-sectional dispersion. Except where noted, scale is percentage point deviation from steady state.

		GDP	C	I	leverage	debt	equity
Std. dev. (%)		2.35	0.80	8.86	4.82	5.38	0.66
Auto. corr.		0.80	0.99	0.78	0.73	0.77	0.89
Corr. matrix	GDP	1	0.49	0.97	0.98	0.99	0.82
	C		1	0.26	0.37	0.43	0.76
	I			1	0.99	0.98	0.69
	leverage				1	0.99	0.72
	debt					1	0.78
	equity						1
Annual std. dev (%) TFP		1.16	(Data: 1.26)				
Annual std. dev (%) σ^ω		3.16	(Data: 3.16)				
Annual correlation (GDP, σ^ω)		-0.99	(Data: -0.83)				
Annual correlation (TFP, σ^ω)		-1	(Data: -0.98)				

Table 9: Simulation-based business cycle statistics for bundled aggregate shocks, in which average TFP (first-moment) shocks induce changes in cross-sectional dispersion. Parameter $\varphi = -1.43$. Upper panel from quarterly simulations, lower panel from annualization of quarterly simulations.

Figure 14 presents impulse responses to a positive bundled aggregate shock. The most salient comparison for these impulse responses are those presented in Figure 11, in which the same size first-moment shock is also the exogenous impulse except with no change in cross-firm dispersion. Comparing Figure 14 with Figure 11 shows that the bundled aggregate shock induces very similar dynamics in most variables as does the unbundled first-moment shock alone. The only difference compared to Figure 11 is that equity rises by much less in response to the bundled shock.

Finally, Table 9 presents simulation-based business cycle statistics. The first row shows that the high volatility of leverage (and debt) carries over from the baseline model's results presented in Table 8. However, a shortcoming of the bundled-shock model is that the leverage ratio is extremely procyclical, at odds with the evidence presented in Section 3.

To summarize, the bundled-shock model by construction is consistent with the empirically-observed countercyclicality of cross-sectional firm risk (see the last two rows of the lower panel of Table 9), and it retains the volatility predictions of the baseline model driven by independent first-moment and second-moment shocks. However, it fails to predict empirically-relevant (a)cyclicality of leverage. On the other hand, the baseline model driven by a complete set of independent,

“unbundled,” shocks performed well on the volatility dimension, reasonably well on the cyclical dimension, but failed to capture the countercyclicality of firm-level risk. Although I do not take up this extension here, a conjecture is that a combination of bundled shocks along with independent, exogenous, shocks to firm risk may help in capturing all these dimensions of the data.⁴⁰

8 Conclusion

This paper documented the business-cycle properties of firm risk based on micro data, and of aggregate leverage, debt, and equity in the non-financial business sector. Using a baseline quantitative financial accelerator model, the main theoretical question was to assess the extent to which the latter can be explained by the former. Empirically-relevant risk shocks turn out to explain quite well the observed volatility of these financial measures. However, in the model, the financial fluctuations that risk shocks induce lead to only very small fluctuations of real activity — GDP volatility conditional on risk shocks alone is less than two percent of GDP volatility conditional on shocks to average TFP alone.

These results perhaps pose a challenge for the emerging literature studying the joint dynamics of financial and real activity. The agency-cost framework has become a common building block, especially of late, in medium-scale and large-scale DSGE models, including for practical policy analysis. At the same time, the idea of risk shocks — or “financial shocks” more broadly defined — has begun appearing in a growing number of DSGE models. The results of this paper show that, when calibrated in a way consistent with micro evidence, the effects of risk shocks on real activity are small. More optimistically, the results suggest that in richer agency-cost models that *do* find important linkages between financial fluctuations and real fluctuations, the linkages are not driven by the basic agency-cost friction *per se*, but rather by other features of the model that interact with the friction. This sort of model parsing of results seems important to understand as the profession’s interest in the joint modeling of financial and real dynamics grows.

A broader idea that emerges is that understanding changes directly in the distribution of micro-level risk may be important for guiding the further development of business-cycle models featuring financial frictions. This paper has exploited second-moment disturbances. As noted by LNZ (2004, p. 33), fluctuations in third- or higher-order moments may also need to be considered for understanding some aspects of the financial data. This requires moving away from the symmetry of normally-distributed (log) productivity standard in macro models. Given the robust evidence that firm-level outcomes are distributed non-normally, there seems reason to think that skewness and higher moments of the firm productivity distribution may be time-varying. Such “higher-moment

⁴⁰Of course, there are a host of other model features and/or shocks one could consider introducing to the model. Such analysis is left to future work.

shocks” would also be expected to affect leverage and so possibly real activity; the quantitative degree to which they do may be an interesting question.

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