

Health, Survival and Consumption over the Life Cycle: Individual versus Social Optimum and the Role of Externalities ^{*†}

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Abstract

This paper offers a framework that allows to compare the life-cycle allocation of consumption and health care that a social planner would choose when maximising the welfare of an age-structured population with the allocation that individuals would choose when maximising their own life-time utility. By curbing mortality health care spending affects individual life expectancy and population size. We derive the social versus private value of life and discuss how they can be used to identify inefficiencies in individual choice. The model is applied to study the effects of spillovers, where individual mortality is not only affected by individual health care expenditure but also by aggregate expenditure, e.g. due to learning-by-doing effects (positive) or due to congestion (negative). We derive the value of the externality and show how individual incentives can be aligned with the planner's by way of an optimal transfer scheme. Numerical analysis illustrates the workings of our model.

Keywords: demand for health, value of life, social planner, health related externality, optimal control

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1 Introduction

In recent years most industrialized countries have allocated increasing shares of their GDP to health. In the US the share of health expenditure in GDP increased from 5 % in 1970 to 15-16 % in 2000. Similar increases can be observed in Germany and Japan, where in Germany health expenditure increased from 6 % of GDP in 1970 to 11 % in 2000 and in Japan from 3 % in 1950 to 7-8 % in 2000 (see Bergheim [2]). At the same time life-expectancy has continued to increase. Whereas increasing life-expectancy is more than welcome as such, a debate is continuing on whether or not too much is being spent on health care. This begs two questions: (i) what motivates individuals to invest in reductions in mortality and to what effect? (ii) Do they get it right from a social welfare point of view? We seek to provide an answer to these questions by combining two models:

1. an age structured optimal control model, where a social planner maximizes welfare (i.e. individual utilities aggregated over time and age groups). This model determines the socially optimal pattern of consumption and health investments.
2. a life-cycle model, where an individual maximizes life-time utility. This model determines the individual pattern of consumption and health investment.

Solving and simulating models (1) and (2) and comparing the respective patterns of consumption and health investment we can deduce conclusions about the inefficiencies in individual behavior and where they arise.

Our model of individual behaviour (model 2) is closely related to the work of Ehrlich and Chuma [10] and Ehrlich [9]. As argued in Ehrlich and Chuma [10] (p. 762) the observed diversity in age specific life expectancy over time and across different population groups may be due not just to the influence of exogenous biological or technological factors but also to systematic variations in individuals' demand for longevity. To determine the demand for longevity an intertemporal setting is needed where the demand function for longevity can be modelled along with the demand for health care and consumption goods.¹ Similar to Ehrlich [9] we postulate that individuals maximize the discounted stream of utility obtained from consumption over their life cycle by choosing how much to spend on health care and consumption subject to their individual budget constraint. We assume that health care affects mortality but ignore the effect of health spending on the quality of life that also enters the objective function in Ehrlich [9].

In our model of aggregate behaviour (model 1), we consider a social planner who maximizes the discounted stream of utility of a population by choosing age

¹The seminal work on the life-cycle demand for health and health care is Grossman [15].

specific health care spending and consumption subject to an economy-wide budget constraint. In this case we need to model the evolution of the total population. We refer to the McKendrick equation in demography to model the evolution of population size over time and age. Similar to the individual optimisation problem, health spending enters negatively the age specific mortality function. Moreover, in the social planner model we introduce age in addition to time as a second dynamic variable. Health care spending is therefore age *and* time specific in model 1.² By applying the optimality conditions of age structured optimal control models (Feichtinger, Tragler and Veliov [11]) we can derive the socially optimal profiles (across age and time) of consumption and health expenditure. The social optimum is determined by equalizing the marginal consumption to the marginal social benefit of an increase in the population by one member through a reduction in mortality.

A measure commonly applied in life-cycle models is the willingness to pay for a small reduction in individual mortality (e.g. Shepard and Zeckhauser [34], Rosen [30], Johansson [17]), termed the Value of Life (VOL). Our framework allows us to derive the population equivalent, which we term the Social VOL (SVOL). The SVOL at age a and time t can be understood as the willingness to pay for lowering mortality (by a small amount) for all members of the population aged a at time t . It is thus given by the social value (in money terms) of one more individual at age a and time t multiplied by the size of this age group. We also offer an intuitive explanation of the value of one more individual at age a and time t and relate it to the value of population derived by Arrow et al. [1] within a macro-economic model. It goes without saying here that in line with most of the literature we take an ex-ante perspective on the saving of lives. Thus, what we consider is the expense of resources with a view to saving statistical lives.

For the individual choice model we can derive similar results. In the optimum the marginal utility of consumption is set equal to the marginal private benefit of health, i.e. the benefit from a decrease in mortality. Similar to the SVOL the private VOL (PVOL) then measures individual willingness to pay for a reduction in mortality. In optimum the PVOL equals the marginal costs (in terms of foregone consumption) of life. By deriving SVOL and PVOL within two closely connected models of social and individual behaviour we can thus bridge the gap between macro-economic models of population size and micro-economic models of individual mortality.

²Hall and Jones [16] derive a life-cycle allocation from a social planner's perspective in order to simulate the development of health care-spending in the US. Our model differs in that (i) we provide a continuous time formulation building on the McKendrick equation, and in that (ii) we allow wealth to be transferred across time. This allows us to compare the (steady-state) outcomes of the social planner model with those from a continuous time individual life-cycle model. Such a comparison is not part of Hall and Jones's [16] analysis. Neither do they assess the role of externalities.

Differences between the planner’s and an individual’s health spending and consumption pattern indicate a potential need for policy intervention. Discrepancies between individually and socially optimal behaviour may, of course, arise for many reasons. For instance, the absence of life insurance typically leads to suboptimal individual behaviour (Ehrlich [9]). In this study we focus, however, on the inefficiency that arises through spill-overs related to medical spending. More specifically, we assume that individual mortality not only depends on the level of individual health care expenditure but also on the average level of spending within the economy. The particular nature of these externalities requires the integration of a social planner and an individual life-cycle model as suggested in this paper. The spillovers we consider imply economic effects *across* different age-groups (or, indeed, cohorts) at any given point in time. By their very nature, the analysis and evaluation of such effects requires a model of the full population, such as given by model (1), and therefore stretches beyond what could be achieved within an individual life-cycle model alone. The latter allows to analyse the behaviour of a given cohort *along* the time path. It is thus fit to analyse the effects along the time path of imperfections in the insurance market referred to in Ehrlich [9]. However, by construction individual life-cycle models on their own are not amenable to an analysis of cross-cohort effects. Our approach provides a consistent and tractable way of analysing such effects.

We explicitly incorporate the scope for externalities, positive or negative, into our model. This allows us to derive the optimal solution from the planner’s perspective and to compare the outcome with that of individual behaviour.³ Generally, externalities, both positive and negative, will lead to different levels of mortality under individual decision making than would be optimal from a social point of view. Here the spending patterns diverge according to the nature of the externality. In the case of positive spillovers too little is expended on health care and too much is consumed. In the case of negative spillovers, too much is expended on care and consumption is too low. Numerical analysis reveals a number of more subtle differences between the forms of externality. In particular, while positive externalities allow for significant increases in life expectancy, negative externalities mostly result in offsetting effects, where individual health care efforts are neutralised by the negative spillovers. We also derive a tax-transfer-scheme which leads to an internalisation of the spillovers and an optimal expenditure pattern.

The remainder of the paper is organised as follows. Section 2 is devoted to the Social Welfare Model. We derive necessary conditions under which the social

³Bolin et al. [4] consider a setting where both an employee and an employer invest into the employee’s health, which in turn affects her productivity. The resulting Nash-equilibrium is also plagued by externalities. Their set-up differs from ours for a number of reasons. First, they consider only the interaction between a single employer and a single employee; thus population structure plays no role. Second, they take a game theoretic approach. Third, they do not derive an optimal policy.

optimum is obtained and introduce the social value of life. The individual choice model is briefly discussed in section 3 where we present the individual optimum and review the private value of life. In section 4 we introduce spillover effects of health related expenditures and consider the inefficiency introduced through such an externality. We propose a tax-transfer-scheme that restores the first best solution at the individual level. We present numerical simulations to support the analytical results in section 5. The final section offers a discussion and outlook for further research.

2 The Social Welfare Model

The dynamics of the population are described by the McKendrick equation (see Keyfitz [19])

$$N_a + N_t = -\mu(a, h(a, t))N(a, t) \quad N(0, t) = B(t), N(a, 0) = N_0(a) \quad (1)$$

The state variable $N(a, t)$ represents the number of a -year old individuals at time t . The age specific mortality rate $\mu(a, h(a, t))$ trivially depends on age a and can be reduced instantaneously by providing to the individual an age specific amount $h(a, t)$ of health care (or other health enhancing goods and services). Here, $h(a, t)$ is a distributed control variable in our model. We model the mortality rate according to the proportional hazard model (see Kalbfleisch and Prentice [18])

$$\mu(a, h(a, t)) = \tilde{\mu}(a)\phi(a, h(a, t)) \quad (2)$$

where $\tilde{\mu}(a)$ denotes the base mortality rate (effective in the absence of any health care) and $\phi(a, h(a, t))$ describes the impact of health spending. We assume that $\phi(a, h(a, t))$ is a strictly decreasing concave function satisfying the Inada conditions, i.e. $\phi_h < 0$, $\phi_{hh} > 0$, $\phi_{ha} > 0$, $\phi(a, 0) = 1$ ($\forall a$) and $\phi_h(a, 0) = -\infty$ ($\forall a$).⁴ Note, that the proportional hazard model implies that the effectiveness of health care in reducing mortality increases in the hazard rate of mortality.

$N_0(a)$ describes the initial age distribution of the population and $B(t)$ equals the number of newborns at time t defined as

$$B(t) = \int_0^\omega \nu(a)N(a, t) da \quad (3)$$

where $\nu(a)$ denotes the age specific fertility rate.

The second control variable is consumption $c(a, t)$. The nonnegativity assumption is trivially fulfilled, as we assume $\lim_{c \rightarrow 0^+} u_c(c) = +\infty$ for the utility function.

⁴Thus the usual assumption of nonnegative health investments is not necessary.

The objective of the social planner is to maximize social welfare, defined as the sum of the instantaneous utilities of all individuals (total utilitarianism)

$$\int_0^T \int_0^\omega e^{-\rho t} u(c(a, t)) N(a, t) da dt \quad (4)$$

where ω is the maximal age an individual can reach. This is no restriction to the model if ω is chosen big enough. The function $u(c(a, t))$ represents the per capita instantaneous utility, which depends only on consumption and is assumed to be concave in its argument. The parameter ρ denotes the rate of time preference. Note that we also allow for an infinite planning horizon $T = +\infty$ at this stage.

Finally we assume a budget constraint that is balanced for each cohort. This is expressed by the introduction of the total wealth $A(a, t)$ held by age-group a at time t and the following dynamics:

$$\begin{aligned} A_a + A_t &= rA(a, t) + (y(a) - c(a, t) - h(a, t))N(a, t) \\ A(0, t) &= A(\omega, t) = 0 \quad \forall t \\ A(a, 0) &= A_0(a), A(a, T) = A_T(a) \quad \forall a \end{aligned} \quad (5)$$

r denotes the interest rate, assumed to be exogenous to the economy, and $y(a)$ denotes the income/output (net of the returns to capital) accruing to an a -year old individual. We assume that health care is purchased at a relative price, which we normalize to one. Each cohort is assumed to hold zero assets at the time of birth and death.

We introduce a cohort specific budget constraint for two reasons. By guaranteeing that each cohort spends precisely its own production, the cohort specific budget constraint allows us to compare steady-state allocations derived for the social planner model with allocations derived for the individual life-cycle model (see section 3). We thus rule out differences in total spending across cohorts which are due to discrepancies between the social rate of time preference and the interest rate. Consider a situation where budgets are pooled across all cohorts. If $\rho > r$, for instance, the social planner would then not only (i) shift consumption to the beginning of the planning horizon for each individual cohort, but also (ii) shift consumption from future cohorts to present cohorts. If $\rho < r$ the opposite would be true. Intuitively, we would like to account for shifts in consumption within cohorts (i) but not across cohorts (ii). Such an allocation rule also bears some intuitive appeal on equity grounds.

The formal problem of the social planner is then to choose the age specific schedule of consumption and health expenditure (health care) to maximize the sum of instantaneous utility of all individuals. Discounting the future at the rate ρ we come up with the following dynamic age-structured optimization problem with state variables $A(a, t)$ and $N(a, t)$ and control variables $c(a, t)$ and $h(a, t)$.

$$\begin{aligned}
& \max_{c,h} \int_0^T \int_0^\omega e^{-\rho t} u(c(a,t)) N(a,t) \, da \, dt \\
& \text{s.t.} \quad N_a + N_t = -\mu(a, h(a,t)) N(a,t) \\
& \quad \quad N(0,t) = B(t) = \int_0^\omega \nu(a) N(a,t) \, da \\
& \quad \quad N(a,0) = N_0(a) \\
& \quad \quad \mu(a, h(a,t)) = \tilde{\mu}(a) \phi(a, h(a,t)) \\
& \quad \quad A_a + A_t = rA(a,t) + (y(a) - c(a,t) - h(a,t)) N(a,t) \\
& \quad \quad A(0,t) = A(\omega,t) = 0 \quad \forall t \\
& \quad \quad A(a,0) = A_0(a), A(a,T) = A_T(a) \quad \forall a
\end{aligned} \tag{6}$$

Both parameters, time t and age a , are finite in our model, since in general no transversality conditions are available for age-structured optimal control models for infinite parameters. However, as discussed earlier this is no restriction to the model.

To clarify the domain for optimization we present a Lexis diagram in Figure 1. The Lexis diagram depicts the life experience of cohorts in time versus age. The 45-degree line represents cohort lines (e.g. a cohort born at $t - a$ is of age s at time $t - a + s$).

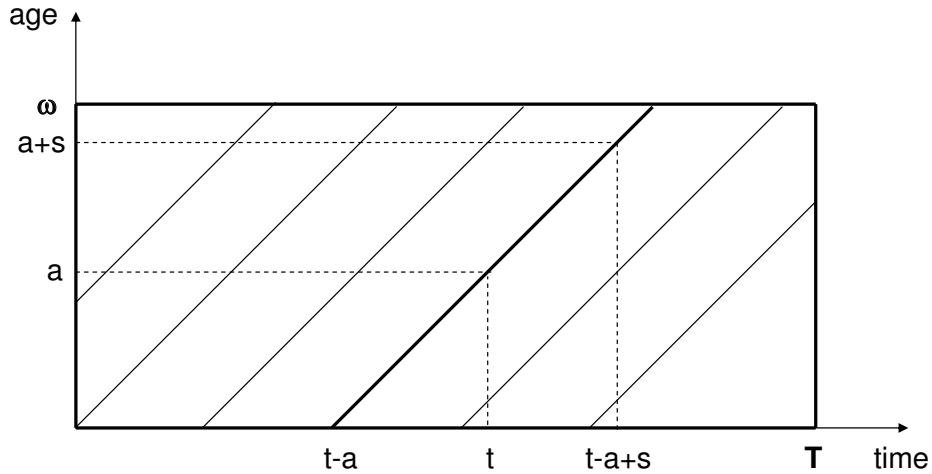


Figure 1: Lexis diagram

In the following section we derive the necessary optimality conditions of the above age specific control problem. Further we provide economic interpretations of important expressions.

2.1 The social optimum

To obtain necessary optimality conditions we apply the maximum principle for age-structured control models as recently derived in Feichtinger, Tragler and Veliov [11].

We define the Hamiltonian of the social welfare problem as follows:⁵

$$\mathcal{H} = u(c)N - \xi^N \mu(a, h)N + \xi^A(rA + (y - c - h)N) + \eta^B \nu N, \quad (7)$$

where we denote the adjoint variables that correspond to the state variables as follows:

- $\xi^N(a, t)$... population
- $\xi^A(a, t)$... assets
- $\eta^B(t)$... newborns,

such that the following system is satisfied:

$$\begin{aligned} \xi_a^N + \xi_t^N &= (\rho + \mu(a, h))\xi^N - u(c) - \xi^A(y - c - h) - \eta^B \nu \\ \xi_a^A + \xi_t^A &= (\rho - r)\xi^A \\ \eta^B &= \xi^N(0, t) \end{aligned} \quad (8)$$

together with

$$\xi^N(\omega, t) = 0 \quad (9)$$

From now on, we assume $T < +\infty$, which further implies $\xi^N(a, T) = 0$.

In order to obtain transversality conditions for ξ^A , we have to consider the conditions $A(a, 0) = A_0(a)$, $A(a, T) = A_T(a)$, $A(0, t) = 0$ and $A(\omega, t) = 0$. For age-specific optimal control models with initial and end state conditions there are no transversality conditions. Thus we ignore $A(\omega, t) = 0$ and $A(a, T) = A_T(a)$ and add the terms $-\lambda \int_0^T e^{-rt} A(\omega, t)^2 dt$ and $-\lambda \int_0^\omega e^{-rT} (A(a, T) - A_T(a))^2 da$ to the objective function. Thus we obtain $\xi^A(\omega, t) = -2\lambda A(\omega, t) > 0$ and $\xi^A(a, T) = -2\lambda(A(a, T) - A_T(a))$ as transversality conditions (implying $\xi^A(a, t) > 0$ for $\forall (a, t)$).

The necessary first order conditions are

$$\mathcal{H}_c = u_c(c)N - \xi^A N = 0 \quad (10)$$

$$\mathcal{H}_h = -\xi^N \mu_h(a, h)N - \xi^A N = 0 \quad (11)$$

Combining them we obtain

⁵From now on we omit a and t if they are not of particular importance.

$$u_c(c)N = -\xi^N \mu_h(a, h)N \quad (12)$$

This condition can be interpreted in a straightforward manner. The LHS corresponds to the aggregate marginal utility of consumption for age-group a at time t and gives the foregone welfare if health spending is increased by one unit for each member of this age-group. As usual, ξ^N can be interpreted as a shadow price, indicating the increase of the value function (i.e. social welfare) for a small (marginal) increase of $N(a, t)$. In other words, ξ^N gives the social value in utility terms of an individual. The term $-\mu_h(a, h)N$ equals the number of lives saved through a marginal increase of $h(a, t)$. Therefore the RHS represents the increase of social welfare if health expenditure is increased at the margin, which in optimum has to equal the utility loss due to foregone consumption. Alternatively, we can write the condition as $\frac{-u_c(c)}{\mu_h(a, h)} = \xi^N$. Here, the marginal cost (in utility terms) of saving one individual of age-group a at time t equals the social value of this individual.⁶

The change in consumption of a cohort born at $t - a$ can be expressed by the following formula (obtained by calculating the directional derivative of (12))

$$c_a + c_t = \frac{u_c(c)}{u_{cc}(c)}(\rho - r) \quad (13)$$

If the discount rate, ρ , equals the interest rate, r , the right hand side (RHS) is zero implying consumption smoothing over the whole life for each cohort. If $\rho > r$ the RHS has a negative sign, because of the concavity of the utility function. Thus, for each cohort, consumption will decrease over the life-cycle, which reflects that the impatience of the individuals is greater than the interest rate. In the case of $r > \rho$ the interpretation is the other way around, i.e. consumption increases over the life cycle for each cohort. Therefore, in general consumption will not be smoothed neither over the life-cycle of a cohort nor within one period across all ages.

It can be shown that the system reaches a steady state under specific assumptions on the birth trajectory and other parameters of the model.

Proposition: Assume $B(t) = B$ and $y(a), \rho, r$ are exogenous and constant with respect to t . Then for any nonnegative initial data $N(a, t), A(a, t)$ any optimal solution of (6) corresponds to an optimal steady state within the time interval $t \in [\omega, T - \omega]$.

The proof is delegated to appendix A. In case that $B(t)$ is only exogenous and not constant, the population level $N(a, t)$ will not reach a steady state while the consumption and health investments remain in their steady state (i.e. they only

⁶Note that the marginal cost of saving one individual is the higher the weaker the effect of health spending on mortality, i.e. the lower the value of $\mu_h(a, h)$.

change over the life-cycle but not across cohorts.) If, in addition $\rho = r$ holds, the social planner smoothes the consumption over the life cycle of each cohort and across all cohorts within $t \in [\omega, T - \omega]$.

Define

$$v(a, t) = \frac{u(c(a, t))}{u_c(c(a, t))} + (y(a) - c(a, t) - h(a, t)) + \frac{\xi^N(0, t)\nu(a)}{u_c(c(a, t))} \quad (14)$$

as the net social value attached to an individual of age a in a year t . It consists of (i) the individual's monetary benefit of consumption $\frac{u(c(a, t))}{u_c(c(a, t))}$; (ii) the individual's net savings, $y(a) - c(a, t) - h(a, t)$; and (iii) the monetary value of the individual's fertility, $\frac{\xi^N(0, t)\nu(a)}{u_c(c(a, t))}$, where $\frac{\xi^N(0, t)}{u_c(c(a, t))}$ is the monetary value of a newborn in time t . We can then write the change of health investments over time and age for a cohort as

$$h_a + h_t = -\frac{\mu_h(a, h)}{\mu_{hh}(a, h)} \left[r + \mu(a, h) + \mu_h(a, h)v(a, t) \right] - \frac{\mu_{ha}(a, h)}{\mu_{hh}(a, h)}. \quad (15)$$

Recall that $\mu_{hh}(a, h) = \tilde{\mu}(a)\phi_{hh} > 0$, implying decreasing returns to health care spending at any given age. Furthermore, it appears plausible to assume that $\mu_{ha}(a, h) = \tilde{\mu}(a)\phi_{ha} + \tilde{\mu}_a\phi_h > 0$, at least for high ages, implying that health care becomes less effective in curbing mortality as the individual ages. In this case, $-\frac{\mu_{ha}(a, h)}{\mu_{hh}(a, h)} < 0$, implying that health expenditure tends to decrease at least for old individuals. However, this purely technological effect is modified by a number of economic effects which are summarised in the square brackets. Note that $-\frac{\mu_h(a, h)}{\mu_{hh}(a, h)} > 0$. Hence, consider the first couple of terms, $r + \mu(a, h)$. The demand for health care, h , tends to increase over time/age as it is easier to finance at a later stage, where the stock of assets increases with an effective rate $r + \mu(a, h)$. Next, consider the term $\mu_h(a, h)v(a, t)$, which is positive if and only if the net social value of an age- a individual in year t is negative. Thus, health investments tend to decrease for age-groups who have currently a positive net social value. For these individuals health care should have been purchased earlier on, thus leading to a tendency towards lower future spending.

2.2 The social value of life

Finally we calculate the willingness to pay for a small reduction of the mortality rate for age a at time t . To our knowledge this concept was firstly developed in a formal manner by Shepard and Zeckhauser [34] (see also Rosen [30] and Johansson [17]) who apply the *value of life* (VOL) concept to a single individual in a life cycle model. As our approach uses a macro economic setting we term it consequently *social value of life* (SVOL). Before we discuss the differences between the VOL and the SVOL we derive an analytic expression for the SVOL. Analogously to Rosen [30] the SVOL, denoted by $\Psi^S(a, t)$, equals

$$\Psi^S(a, t) = -\frac{\partial V/\partial \mu}{\partial V/\partial A} \quad (16)$$

where V denotes the value function, i.e. maximised social welfare (see (6)). This formula expresses the marginal rate of substitution (MRS) between mortality and social wealth, which describes the slope of the indifference curve including all combinations of $\mu(a, h(a, t))$ and $A(t)$ yielding the same level of social welfare. The denominator equals the shadow price of wealth ξ^A . For the numerator we obtain

$$\frac{\partial V}{\partial \mu} = \frac{\partial V}{\partial N} \frac{\partial N}{\partial \mu} = -\xi^N(a)N(a, t) \quad (17)$$

The second equality can be verified by solving the partial differential equation for N by the method of characteristics. Putting the above two expressions together we obtain

$$\Psi^S(a, t) = \frac{\xi^N(a, t)N(a, t)}{\xi^A(a, t)} = \frac{\xi^N(a, t)}{u_c(c)}N(a, t) \quad (18)$$

Here, $\frac{\xi^N(a, t)}{u_c(c)}$ denotes the monetary value of an individual aged a at time t . The SVOL for age-group a at time t (i.e. the monetary value of a small reduction in the mortality for this age group) then follows as the product of the monetary value of an individual and the size of the age-group $N(a, t)$. In the following, we will denote the social value of an individual life (SVIL) by

$$\psi^S(a, t) := \frac{\xi^N(a, t)}{u_c(c)}. \quad (19)$$

We can derive an explicit expression of the SVIL - again to be understood in statistical terms - for a cohort born in $t - a$

$$\psi^S(a, t) = \int_a^\omega v(s, t - a + s) e^{-(s-a)r - \int_a^s \mu(s', h)} ds' ds,$$

where $v(s, t - a + s)$ as defined in (14) corresponds to the social evaluation of an individual's life year at time $t - a + s$, when the individual is aged s (see Appendix B for a derivation of the above result). Hence, $\psi^S(a, t)$ gives the expected discounted sum of the social value of the individual's remaining life years.⁷

We can then decompose the change of the SVOL within one cohort born at time $t - a$ as follows

⁷Hall and Jones [16] [eq (21)] derive a similar expression in a discrete time context. Note, however, that in their formulation the value of future births is not taken into account. This is because they assume exogenous births.

$$\begin{aligned}
\Psi_a^S + \Psi_t^S &= (\psi_a^S + \psi_t^S) N(a, t) - \mu(a, t) N(a, t) \psi(a, t) \\
&= [(\psi_a^S + \psi_t^S) - \mu(a, t) \psi^S(a, t)] N(a, t).
\end{aligned} \tag{20}$$

Hence, changes in the SVOL can be decomposed into the change of the SVIL of each member of the cohort (the first term in the expression) and the the loss of SVIL due to mortality (the second term in the expression). For the time path of SVIL we obtain

$$\psi_a^S + \psi_t^S = (r + \mu(a, h))\psi^S - v(a, t) \tag{21}$$

or

$$\psi_a^S + \psi_t^S = (r + \mu(a, h)) \int_a^\omega (v(s, t - a + s) - v(a, t)) e^{-(s-a)r - \int_a^s \mu(s', h) ds'} ds + v(a, t) R_a(a) \tag{22}$$

with

$$\begin{aligned}
R(a) &= \int_a^\omega e^{-(s-a)r - \int_a^s \mu(s', h) ds'} ds \\
R_a(a) &= (r + \mu(a, h))R(a) - 1.
\end{aligned}$$

The first expression (21) corresponds to the typical time path for the value of some asset, i.e. the value increases with the gross interest (here: including the mortality) and decreases in the current return (i.e. the social value of the current life year of an individual aged a). According to the second expression (22) the SVIL for the cohort under examination develops (along the Lexis Diagram) under two effects: SVIL tends to increase as long as the average social value of future life years exceeds the social value of the current life year [the first summand on the RHS of (22)]. Observing that $R(a)$ is a (discounted) measure of remaining life expectancy and noting that $R_a(a) < 0$ is generally true at least for high ages, it follows that SVIL will eventually decrease with age due to a fall in the (expected) remaining life span.

It is instructive to compare our SVIL to the value of population as derived by Arrow et al. [1] within a neo-classical growth model. Expanding the RHS of (21) to

$$\psi_a^S + \psi_t^S = (r + \mu(a, h))\psi^S - \left[\frac{u(c(a, t))}{u_c(\cdot)} + y(a) - c(a, t) - h(a, t) + \frac{\xi^N(0, t) \nu(a)}{u_c(\cdot)} \right]$$

and substituting $\frac{1}{u_c(c)} = \frac{\psi^S}{\xi^N(a, t)}$ we obtain

$$\psi_a^S + \psi_t^S = \left[r + \mu(a, h(a, t)) - \frac{\xi^N(0, t)}{\xi^N(a, t)} \nu(a) \right] \psi^S - \left[\frac{u(c(a, t))}{u_c(\cdot)} + y(a) - c(a, t) - h(a, t) \right],$$

where $-\frac{\xi^N(0,t)}{\xi^N(a,t)}\nu(a)$ gives a weighted expression of fertility. Compare this to the time derivative of the value of population in Arrow et al. ([1]), which in our notation can be expressed as⁸

$$\dot{\psi}^S = (r - \nu) \psi^S - \left[\frac{u(c)}{u_c} + p - c \right].$$

Here, ν is the constant net growth rate of a population without age structure, and p is the marginal product of (homogeneous) labour. The value of population thus evolves more or less equivalently to our SVIL with the following distinctions: (i) Arrow et al. [1] do not consider the scope for health care, h , to reduce mortality (ii) Arrow et al. [1] measure the net growth rate, where we have distinct rates of fertility and mortality; (iii) Arrow et al. [1] consider a homogeneous population; thus in their model $\frac{\xi^N(0,t)}{\xi^N(a,t)} \equiv 1$. (iv) Arrow et al. [1] establish $\psi^S + k$ as the discounted value of the total value of life for the entire population, where k is the capital stock per capita. Thus, in their model, in which capital acts as a factor of production, the value of life at aggregate level is not only determined by the value of an individual but also by the economy's capital intensity. Finally, (v) a correspondence of our SVIL with the value of population requires that the individual's income $y(a)$ equals the age-dependent productivity $p(a)$.⁹

3 Individual Choice Model

In this section we provide a brief representation of the individual choice model over the life-cycle, as we aim at a comparison between socially and individually optimal paths of consumption and health care spending. Within the individual choice model

⁸Consider the equation following equation (24) in Arrow et al. ([1], p. 224)

$$\dot{q} = (F_K - \phi') q - \left[\frac{U(c)}{U'(c)} + F_N - c \right].$$

In our notation, $q = \psi^S$, $U(c) = u(c)$ and $F_N = p$. Furthermore, it is true that in equilibrium the marginal product of capital F_K equals the interest rate, r . Arrow et al. [1] model neither age structure nor (explicitly) mortality. In their model the development of the homogeneous population is thus described by the differential equation $\dot{N} = \phi(N)$. If we posit $\phi(N) = \nu(N)N$ (see their equation (11)) and assume $\nu(N)$ to be a constant, then we have $\phi' = \nu$ as the net growth rate. The expression we report follows immediately.

⁹While this last requirement is satisfied in a competitive labour market, we should take note that in such a case individuals are systematically valued by the planner according to their productivity. In the presence of individual heterogeneity, it is obvious that this may lead to ethically unpalatable solutions.

we only consider index a which stands for both age and time. All variables have the analogous meaning as in the social welfare problem.

Each individual earns income $y(a)$. Assets over time are allowed to be positive and negative. We follow Yaari [37] and Ehrlich [9] in considering a set-up in which fair life-insurance is available to the individuals. Thus, they can fully annuitise their wealth by buying actuarial notes from an insurer, who in turn, pays them an interest in excess of the market rate r . More specifically, the individual's risk of leaving positive wealth to the insurer due to mortality is compensated by the insurer through payment of a risk premium equal to $\mu(a, h)$. Thus, gross interest is given by $r + \mu(a, h)$. The same applies if an individual takes out a credit from the insurer by selling actuarial notes. In this case, the risk is borne by the insurer that an individual may die in debt. Again, the mortality risk is factored into the gross interest rate.¹⁰ Hence, individual wealth develops according to

$$\dot{A}(a) = (r + \mu(a, h(a)))A(a) + y(a) - c(a) - h(a) \quad A(0) = 0. \quad (23)$$

Disregarding planned-for bequests, we obtain $A(\omega) = 0$. The probability of surviving to age a (modelled analogously to the social planner problem) equals

$$M(a) := \exp\left(-\int_0^a \mu(s, h) ds\right) \quad (24)$$

with $\mu(a, h(a)) = \tilde{\mu}(a)\phi(a, h(a))$. As in the social welfare model we assume Inada conditions for $\phi(a, h)$ and $u(c)$. The individual then maximizes utility by choice of consumption and the procurement of health care according to

$$\begin{aligned} \max_{c, h} \quad & \int_0^\omega e^{-\rho a} u(c(a)) M(a) da \\ \text{s.t.} \quad & \dot{M}(a) = -\mu(a, h(a))M(a) \\ & \dot{A}(a) = (r + \mu(a, h(a)))A(a) + y(a) - c(a) - h(a) \\ & M(0) = 1, A(0) = 0, A(\omega) = 0 \\ & \mu(a, h) = \tilde{\mu}(a)\phi(a, h(a)) \end{aligned} \quad (25)$$

Note that in contrast to the work known to us, we have modelled the survival probability as a state $M(a)$. We can now derive the first-order conditions for optimal individual choices (section 3.1) and develop the according private value of life (section 3.2).

¹⁰Note that these arrangements correspond well to real world life-insurance contracts, typically paying a return in excess of the market interest rate, as well as to real world credit contracts, with banks typically requiring creditors to purchase life-insurance, the payment of which serving as a collateral in the case of death.

3.1 The individual optimum

The Hamiltonian of the individual problem reads (again omitting a if it is not of particular importance)

$$\mathcal{H} = u(c)M - \lambda_M \mu(a, h)M + \lambda_A ((r + \mu(a, h))A + y - c - h) \quad (26)$$

where λ_M and λ_A denote the adjoint variables of the survival probability and individual assets respectively. From the necessary optimality conditions we can derive the following system of adjoint variables:

$$\begin{aligned} \dot{\lambda}_M &= (\rho + \mu(a, h))\lambda_M - u(c) \\ \dot{\lambda}_A &= (\rho - r - \mu(a, h))\lambda_A \end{aligned} \quad (27)$$

with the transversality conditions $\lambda_M(\omega) = 0$ and $\lambda_A(\omega) = -2\lambda A(\omega)$, since we implement the terminal condition $A = 0$ in the same way as in the social welfare problem. Thus both adjoint variables are always positive. The necessary first order conditions are

$$\begin{aligned} \mathcal{H}_c &= u_c(c)M - \lambda_A = 0 \\ \mathcal{H}_h &= -\lambda_M M \mu_h(a, h) + \lambda_A A \mu_h(a, h) - \lambda_A = 0 \end{aligned} \quad (28)$$

We combine them and obtain

$$u_c(c)M = -(\lambda_M M - \lambda_A A) \mu_h(a, h) \quad (29)$$

The LHS gives marginal utility from consumption conditioned on the individual's survival. The RHS represents the increase in life-time utility if health investments are increased marginally. While the overall effect turns on the reduction in mortality, it now falls into two distinct sub-effects, one driven by the 'utility' value for the individual's survival and the other driven by the impact on the individual's wealth. The first effect $-\lambda_M M \mu_h(a, h)$ is always positive and captures the expected flow of future utility if the individual can survive. The second effect, $\lambda_A A \mu_h(a, h)$ is negative if the individual holds positive wealth $A > 0$ and positive if the individual is in debt, i.e. if $A < 0$. By curbing mortality, health expenditure lowers the risk premium included in the gross interest rate. If the individual holds positive wealth, this lowers its returns on assets and thus implies an indirect cost of health expenditure. In contrast, if the individual is in debt, a reduction in the risk premium constitutes an indirect return to health care expenditure. Note that the wealth and 'utility' effects are complementary if $A < 0$ and offsetting each other if $A > 0$.¹¹

¹¹Note that $\lambda_M M - \lambda_A A$ is always non-negative. This can be seen by inserting $A = \frac{1}{\mu_h} + \frac{\lambda_M M}{\lambda_A}$ (obtained by the FOC for h), which yields $\lambda_M M - \lambda_A A = -\frac{\lambda_A}{\mu_h} > 0$ since λ_A is always positive.

Lemma 1: Assume $B(t) = B$ and $y(a), \rho, r$ are exogenous and constant with respect to t . The steady-state of the individual choice model is then equal to that of the social welfare model for $t \in [\omega, T - \omega]$. Thus the individual behaves socially optimal if a 'perfect' life-insurance is available¹².

Proof: Referring to Appendix A, the optimization of the parametric family $J_1[\tau](q[\tau], z[\tau])$ results in an optimization problem for a cohort born at time τ . As $B(t)$ is exogenous, the objective function and system dynamics depend only multiplicatively on the the number of births. Hence we obtain $B(\tau)$ individual choice models.

From the first order conditions we can derive the change in consumption over the life cycle

$$\dot{c} = \frac{u_c(c)}{u_{cc}(c)}(\rho - r) \quad (30)$$

In case of $\rho = r$, the individual can smooth consumption over the life cycle similarly to the planner in the social welfare model. This is because in the presence of a fair life insurance (and credit market) the individual has complete control over the interest payments on his assets even when facing a mortality risk. Define

$$\tilde{v}(a) = \frac{u(c)}{u_c(c)} + (y - c - h) \quad (31)$$

as the net private value attached to an individual at age a . Similar to the net social value it consists of (i) the individual's monetary benefit of consumption $\frac{u(c)}{u_c(c)}$ and (ii) the individual's net contribution $y - c - h$. Compared to the net social value the monetary value of a newborn is not included.

The change in the health expenditure over the life cycle is given by

$$\dot{h} = -\frac{\mu_h(a, h)}{\mu_{hh}(a, h)} \left[r + \mu(a, h) + \mu_h(a, h)\tilde{v}(a) \right] - \frac{\mu_{ha}(a, h)}{\mu_{hh}(a, h)} \quad (32)$$

As with consumption, the time path of h develops similarly as in the social optimum. Thus, health expenditure tends to decrease if age reduces the productivity of investments to a sufficient degree. In contrast, health expenditure tend to be shifted to advanced ages in line with high risk adjusted interest rates. Finally, health expenditure tends to increase (decrease) when the current net value of an individual life year, $\frac{u(c)}{u_c(c)} + y - c - h$, is negative (positive).

¹²Due to the initial and boundary conditions this equivalence may not hold at the beginning and end of the planning horizon.

By comparison with (15) we see that the only difference to the social welfare model is that individuals do not consider the value of their own fertility, $\frac{\xi^N(0,t)\nu(a)}{u_c(c(a,t))}$, in their optimization. Of course this difference would be mitigated or vanish if altruistic behaviour were to be included in the individual model.

3.2 The private value of life

Similar to Schelling [33], Shepard and Zeckhauser [34], Rosen [30], Johansson [17] and Murphy and Topel [24] we can apply the value of life concept to the individual choice model, which we now call the private value of life (PVOL). The derivation is analogous to the SVOL and yields

$$\Psi^P = \frac{1}{u_c} \left(\lambda_M - \lambda_A \frac{A}{M} \right) \quad (33)$$

As in the first order condition we obtain an expression which hinges both on the 'utility' and on the 'asset' value of life. These values work in the opposite ($A > 0$) or in the same direction ($A < 0$). The marginal utility in the denominator transforms the PVOL into monetary terms. The time path of the PVOL equals

$$\dot{\Psi}^P = (r + \mu(a, h))\Psi^P - \tilde{v}(a) \quad (34)$$

or alternatively

$$\dot{\Psi}^P = (r + \mu(a, h)) \int_a^\omega (\tilde{v}(s) - \tilde{v}(a)) e^{-(s-a)r - \int_a^s \mu(s,h) ds'} + \tilde{v}(a) R_a(a) \quad (35)$$

with

$$R_a(a) = (r + \mu(a, h)) \int_a^\omega e^{-(s-a)r - \int_a^s \mu(s,h) ds'} ds - 1 \quad (36)$$

and $R(a) = \int_a^\omega e^{-(s-a)r - \int_a^s \mu(s,h) ds'} ds$. This corresponds exactly to the findings by Murphy and Topel ([24], equation (10)). The interpretation is similar to the one established for the SVIL. The PVOL tends to increase if the average net value of future life years exceeds the value of the current life year $\tilde{v}(a)$. Any reduction in remaining life-expectancy with age for $R_a(a) < 0$ will exert a negative impact on PVOL. For high ages a , it is plausible that $\tilde{v}(s) < \tilde{v}(a)$, for any future year $s > a$, in particular if age is associated with low levels of income net of interest and/or high levels of health expenditure. In this case, PVOL decreases unambiguously.

Lemma 2: Assume $B(t) = B$ and $y(a), \rho, r$ are exogenous and constant with respect to t . Then the PVOL and the SVIL are equal within the steady state interval $[\omega, T - \omega]$.¹³

Proof: By applying the FOCs both the PVOL and the SVIL can be transformed easily into $-\frac{1}{\mu_h(a,h)}$, which is equal if $B(t)$ is exogenous. Furthermore note, that if $B(t)$ is exogenous, the value of fertility $\frac{\xi^N(0,t)\nu(a)}{u_c(c(a,t))}$ drops out from the SVIL and individual and social behaviour coincide.

In the presence of a perfect capital market and fair life-insurance and in the absence of any relevant externalities the results in Lemma 1 and Lemma 2 are hardly surprising. Nevertheless, they provide an important benchmark against which one can model the effects of imperfections in the economy that would lead to discrepancies between individual and social behaviour. Following on from this, the benchmark can also be used in the analysis of policy towards improving the outcomes from suboptimal individual choices. Indeed, it would appear that one requirement of first-best policy-making is that the PVOL after the policy measures have been undertaken equals the SVIL. The following section illustrates the issue by providing an analysis of spillovers related to the demand for health care.

4 Spillovers in the provision of health care

From now on we assume an exogenous number of newborns. Our base model is then void of external effects and, unsurprisingly, individual choices lead to the first best outcome. In this section we sketch an extension where individual mortality is not only affected by individual health care spending but also by the aggregate health care activity, as measured by per capita spending averaged across the whole population. In the Lexis-diagram in figure 1 individual mortality develops along one of the 45 degree lines (e.g. the one for the cohort born at $t - a$) not only according to individual health spending $h(a)$ but also according to the health spending $h(\hat{a})$, $\hat{a} \neq a$ realised by other cohorts at each point in time, t . It is easy to see then that the first best result cannot be attained by individuals without correcting policy-measures. We analyse the discrepancies between individual and social choice in the presence of spillovers and derive an age-specific tax/subsidy on health investments by which the planner can induce individuals to behave in the first best way. Before we present the formal models – at the aggregate (section 4.2) and individual (section 4.3) level – we provide some motivation of the externality in the next section (4.1).

¹³As in Lemma 1 this does not hold at the beginning and end of the planning horizon due to the initial and boundary conditions.

4.1 Motivation

One could think of a number of ways in which aggregate health care activity (as measured by health care expenditure per capita averaged across all age groups) may affect individual mortality. First, medical research has identified a positive relationship between volume of (surgical) activity and outcomes, frequently measured by (lower) mortality (for an overview see Phillips and Luft [27]). This volume-outcome relationship (VOR) reflects learning-by-doing effects, usually at physician and/or hospital level.¹⁴ While, admittedly, it is unknown to us whether learning-by-doing is also effective at the level of the health care system, we do not see why this should not be the case, at least when care is provided locally in a large number of 'small' hospitals. It is then plausible that learning-by-doing occurs in each individual hospital and thus on aggregate.¹⁵

Second, higher levels of aggregate health care spending may translate into greater scope for medical R&D or other quality enhancing activities that would not be lucrative in 'low spending' health care systems. Murphy and Topel [25], for instance, model an R&D race for a pharmaceutical innovation and show that the overall probability of innovation increases in the share of the social value that the winning firm is able to capture. In our model the prize for innovation would correspond to the winner's share of aggregate health expenditure, thus establishing a link between aggregate health expenditure and individual mortality.¹⁶ Third, higher aggregate spending may lead to a denser spatial provision of medical capacity (e.g. a denser net of hospitals, physician practices and ambulance services) or to the provision of a capacity of higher quality, again translating into a lower mortality risk at individual level.¹⁷

Fourth, spillovers may arise in the context of preventive activities. The most obvious example relates to vaccination: A negative effect of aggregate activity on individual mortality arises when individual mortality is negatively related to the degree to which the population is vaccinated against an infectious disease (for an overview see Philipson [28]).¹⁸ The same applies to antimicrobial treatment of infec-

¹⁴One could object that the causality runs the other way: Hospitals with better outcomes attract larger number of patients. Gaynor [14] report a number of recent studies that confirm VOR under endogenous patient choice.

¹⁵As far as higher aggregate health expenditure merely reflects a larger population, one should, of course, not expect VOR effects at health care system level. This is because greater total activity is likely to be spread across a greater number of hospitals. By considering expenditure per capita we provide a scale-independent measure of total activity which can be related to VOR.

¹⁶Indeed, in this case proper scale effects may be relevant. As we take into account expenditure per capita, this would give a conservative estimate of the degree of spillovers.

¹⁷Our model only partially captures this in that we do not model medical capacity as a stock variable.

¹⁸In our model we do not measure the share of the population vaccinated (every one engages in medical spending) but rather the degree of vaccination, perhaps as reflected by different quality

tious disease. Fifth, one could think about the prevention of mortality in terms of safety measures and public hygiene. For instance, adoption of the anti-lock breaking system helps to reduce individual accidental mortality but also contributes to the safety of other road-users. The same applies to the installation of smoke-detectors in the individual flats of tenement buildings. Finally, we may think of measures related to public health such as the cleaning of sewerage, proper disposal of household waste or the reduction of air pollution. Cutler and Miller [6], for instance, show that in the early 20th century nearly half of the total mortality reductions in major US cities can be attributed to the introduction of clean-water technologies, i.e. the filtration and chlorination of water supplies.¹⁹ 'Pure' public health measures constitute a polar case of our model, where there is no effect on mortality of individual health care expenditure and all mortality reductions are due to cumulative expenditure.²⁰ Our model is then equivalent to a public goods problem. But even in less extreme cases, the problem of private underprovision arises as long as a part of private health expenditure flows towards a public good (i.e. communal reductions in mortality).²¹

While all of the above examples suggest 'positive' spillovers (i.e. higher total spending translating into lower mortality), we also envisage negative spillovers. These could arise from congestion effects or from microbial resistance against antibiotics. In as far as higher aggregate medical spending reflects greater demand for health services, this would lead to congestion in the presence of capacity constraints. Congestion could either lead to a direct reduction in the efficacy of medical care or it could result in some form of rationing.²² A prominent example for a direct impact of congestion on mortality relates to increased infection rates in over-crowded hospitals. Furthermore, in the absence of an explicit rationing scheme providers may only be able to cope with excess demand by reducing the quality of care across the board (e.g. by providing less time-intensive care to everyone, or by reducing the effort

grades of vaccines where the more expensive ones are more effective or by the number of diseases against which vaccination is obtained.

¹⁹Watson [36] studies the impact of public sanitation interventions in US Indian Reservations on the child mortality of Native Americans in the US as opposed to White infants. She finds that they were quite effective in reducing the mortality gap despite a sizeable externality on the health of neighbouring White children.

²⁰Easterlin [8] argues that, indeed, most of the historical reductions in mortality due to preventive measures, vaccination and antimicrobials are not attributable to the market for reason of various forms of externalities.

²¹An alternative but analogous interpretation is one in which health care is a good with (positive) network externalities.

²²By considering health care activity per capita we assume that capacity is increased in line with population size but not in line with per capita expenditure. Black and Pearson [3] discuss the problems related with a recent bout in hospital congestion in the UK. Naylor et al [26] examine the management of a spell of serious excess demand for coronary surgery in Ontario (1987-1988). They find wide discrepancies in management processes across hospitals and identify large inefficiencies. Koizumi et al [20] simulate congestion in a system of mental care.

taken in the administration of care). Such quality reductions may well be implicit when over-stretched staff are more likely to commit medical errors. At either rate, if sufficiently severe these quality-reductions translate into higher mortality.

If a rationing scheme is in place, it very much depends on its nature as to whether individual demand causes negative spillovers. Indeed, negative spillovers should not arise when rationing is efficient, either through prices or through waiting times. Theoretical and empirical studies suggest, however, that rationing is generally not fully efficient. For instance, in a fundholding system, where a provider of care receives a budget per period in order to allocate it to health care for a population, people who require medical care at a late date (perhaps because they have contracted an illness late within a period) may find themselves unduly rationed (Glazer and Shmueli [13]). First-comers then clearly impose a negative externality on late-comers. The same goes if rationing is organised according to a lottery. In this case, too many people are willing to sign-up, ignoring the worsening of the odds of obtaining care for their fellow patients.²³

Finally, it is well-known that microbes (bacteria and viruses) tend to develop resistance against antimicrobial treatments. The probability that a resistant microbial strain develops increases in the level of exposure. Thus, assuming that health expenditure flows into the purchase of antimicrobial treatments, then individual use of antibiotics tends to curb individual mortality but may, in aggregate, lead to an increased mortality risk due to microbial resistance.²⁴

4.2 Social optimum in the presence of spillovers

In the following, we assume that the mortality rate also depends on the average health expenditure (across the full population) at time t , which we denote by $\bar{H}(t)$.

²³For a survey on waiting as a rationing device see Cullis et al. [5] who include a review of empirical studies on the cost of waiting. While health policy-makers (at system level), hospitals (at institutional level) and physicians (at individual level) usually apply some form of prioritisation, the problem is that there is no agreement on a best-rule for prioritisation (see Mooney [23] for a discussion of rules and Ryynaenen et al [32] for empirical evidence on the attitudes towards prioritisation). Prioritisation thus appears to follow an eclectic mix of evidence-based rules, rules of thumb and individual incentives. This suggests the presence of inefficiencies as were, indeed, identified by Naylor et al [26] in their case study of waiting for heart surgery in Ontario.

²⁴Easterlin [8] discusses evidence on the excessive use of antimicrobial treatments in a number of developing countries. Laxminarayan and Weitzman [22], Rudholm [31] and Lasserre et al [21] provide economic models covering various aspects of microbial resistance. The first paper shows that if the risk of resistance can be reduced by the prescription of a variety of drugs then this provides a rationale for the inclusion of some cost-ineffective drugs in the prescription portfolio. The second paper shows that the outcome of a dynamic treatment game between two countries with negative spillovers in terms of resistance involves excessive use of antibiotics. It also derives the Pigouvian tax as an optimal policy-response. The third paper studies optimal explorative use of antiviral treatments in a sequential treatment game in the presence of learning and resistance externalities (arising between the first and second period).

Hence, mortality is given by $\mu = \mu(a, h(a, t), \bar{H}(t))$, implying spillover effects from individual health expenditure. To fix ideas, consider learning by doing as an example. In our model, private care is purchased at a price equal to unity. Thus, h and $\bar{H}(t)$ are measures of individual consumption of health care and of aggregate health care activity (per capita), respectively. Higher levels of aggregate activity per capita imply that physicians become more effective in providing care and mortality rates fall. This can be represented by a negative relation between the mortality rate and per-capita total health expenditure, i.e. $\mu_{\bar{H}}(a, h, \bar{H}) := \frac{\partial \mu(a, h(a, t), \bar{H}(t))}{\partial H(t)} < 0$.

We can write per capita health expenditure $\bar{H}(t) := \frac{H(t)}{\tilde{N}(t)}$, with

$$\begin{aligned} H(t) &= \int_0^\omega h(a, t)N(a, t) da \\ \tilde{N}(t) &= \int_0^\omega N(a, t) da \end{aligned} \quad (37)$$

denoting total health investments and total population at time t respectively. Aggregate health expenditure for age-group a at time t is given by the sum of individual expenditure, $h(a, t)N(a, t) = \int_1^{N(a, t)} h_i(a, t) di$. The impact of individual spending on average health expenditure is then given by $\frac{\partial \bar{H}(t)}{\partial h_i(a, t)} = \frac{1}{N(t)}$ which tends to zero for large populations. Thus, individuals rationally anticipate that they are unable to influence aggregate spending and an externality arises. For our example this externality is positive, in the sense that, individual spending also raises the expected life-time utility of the rest of the population.

Allowing for the spillovers implies that the social welfare model (6) has to be augmented by the two integral states $H(t)$ and $\tilde{N}(t)$ as defined in (37).

Similar to section 2.1 we can obtain the necessary optimality conditions by applying the maximum principle for age-structured control models. The first order conditions are similar, however the inclusion of two integral states implies that we have to introduce the shadow price corresponding to total health expenditure and total population, respectively:

$$\begin{aligned} \eta^H(t) &= - \int_0^\omega \xi^N \mu_{\bar{H}}(a, h, \bar{H}) \frac{N}{\tilde{N}} da \\ \eta^{\tilde{N}}(t) &= \int_0^\omega \xi^N \mu_{\bar{H}}(a, h, \bar{H}) \frac{HN}{\tilde{N}^2} da \end{aligned} \quad (38)$$

For a positive externality ($\mu_{\bar{H}}(\cdot) < 0$), mortality is decreasing in the aggregate health expenditure and increasing in the size of the total population. The shadow price of health investments is then positive while the shadow price of total population is negative. The opposite applies in case of a negative externality.

The necessary first order condition now reads

$$u_c(c) = -\xi^N \mu_h(a, h, \bar{H}) + \eta^H \quad (39)$$

Noting that $\eta^H > 0$ in case of a positive externality, the planner will expend additional resources on health care so that the individual benefit of care $-\xi^N \mu_h(a, h, \bar{H})$ falls short of the marginal utility of consumption. Again, the converse is true for negative externalities.

The time path of consumption is similar as in the social welfare model derived in section 2. However, the positive externality has an effect on the time path of the health investments, whose change over time and age for a cohort we shall investigate numerically in section 5.

While the forms - but not values - for the SVOL and the SVIL remain unchanged, we can derive an adjustment of the SVIL expression that takes into account the externality. For this, we rewrite the first-order condition (39) as

$$\begin{aligned} \frac{-1}{\mu_h(a, h, \bar{H})} &= \psi^S(a, t) - \frac{\eta^H(t)}{u_c(c(a, t)) \mu_h(a, h(a, t), \bar{H})} \\ &= \psi^S(a, t) + \Theta(a, t) \\ \Theta(a, t) &: = \int_0^\omega \frac{u_c(c(\hat{a}, t))}{u_c(c(a, t))} \psi^S(\hat{a}, t) \frac{N(\hat{a}, t)}{\tilde{N}(t)} \frac{\mu_{\bar{H}}(\hat{a}, h, \bar{H})}{\mu_h(a, h, \bar{H})} d\hat{a}. \end{aligned} \quad (40)$$

According to this reinterpretation of the first-order condition, the marginal cost (in money) of saving the life of an individual aged a at time t has to equal the SVIL plus the value of the externality, $\Theta(a, t)$, related to the provision of care to an individual at age a and time t . The value of the externality is determined by the following factors: (i) the weighted sum across age-groups of the SVIL's for (\hat{a}, t) individuals, where the population share $\frac{N(\hat{a}, t)}{\tilde{N}(t)}$ of these individuals is used as weight; (ii) the relative effectiveness in reducing mortality of aggregate spending for (\hat{a}, t) individuals as given by $\frac{\mu_{\bar{H}}(\hat{a}, h, \bar{H})}{\mu_h(a, h, \bar{H})}$; and (iii) a conversion factor $\frac{u_c(c(\hat{a}, t))}{u_c(c(a, t))}$.²⁵ Hence, the value of a (positive) externality is large if the spillovers are particularly effective for large age groups composed of members with a high SVIL.

Let us now turn to the individual choice model.

4.3 Individual choice and correcting policy

Using the identity $t \equiv t_0 + a$, where t_0 stands for the year of birth, we can express the individual life-cycle problem in the presence of externalities as follows

²⁵If consumption is fully smoothed across age-groups then the conversion factor equals one.

$$\begin{aligned}
& \max_{c,h} && \int_0^\omega e^{-\rho a} u(c(a)) M(a, t_0 + a) da \\
& \text{s.t.} && \dot{M}(a) = -\mu(a, h(a)) M(a, t_0 + a) \\
& && \dot{A}(a) = (r + \mu(a, h(a))) A(a, t_0 + a) + y(a) - c(a) - h(a) \\
& && M(0, t_0) = 1, A(0, t_0) = 0, A(\omega, t_0 + \omega) = 0 \\
& && \mu(a, h) = \tilde{\mu}(a) \phi(a, h(a), \bar{H}(t_0 + a))
\end{aligned}$$

We have shown in the previous section that individuals do not expect to affect health care spending per capita (as averaged across each age-group) and therefore take $\bar{H}(t_0 + a)$ as given at each point in time. In the absence of any correcting policies, the necessary FOCs are then similar to those derived in section 3.1, and can be expressed by $1 = -\Psi^P(a) \mu_h(a, h, \bar{H})$. The externality has a bearing on the effectiveness of individual health care spending and is therefore prone to alter individual spending patterns. However, individuals do not take into account the benefit (or harm) they bestow on others in the case of positive (negative) spillovers.

In order to attain the first best result the social planner can introduce the following tax/subsidy scheme. Let $\tau(a, t_0 + a)$ denote a (net) subsidy on each unit of private health care spending or, equivalently, on each unit of private health care consumed. Hence, for each unit of care, the individual only spends an amount of $1 - \tau(a, t_0 + a)$. In order to balance the budget in expected terms and to deprive the individual from any transfer income in expected terms, the government levies a (net) lump-sum tax equal to the amount $\tau(a, t_0 + a) h^S(a, t_0 + a)$, where $h^S(a, t_0 + a)$ corresponds to the socially optimal level of health expenditure for an individual aged a at time $t_0 + a$. Note that the lump-sum transfer is entirely exogenous to individual decision making. The individual asset dynamics under the transfer scheme can then be written as follows

$$\begin{aligned}
\dot{A}(a) &= (r + \mu(a, h, \bar{H}(t_0 + a))) A(a, t_0 + a) + y(a) - c(a) - h(a) + \\
&\quad \tau(a, t_0 + a) (h(a) - h^S(a, t_0 + a))
\end{aligned} \tag{41}$$

The transfer that leads to the first best result can then be derived as follows. For the sake of simplifying notation, let us assume a steady state, where under our assumption that $B(t) = B$, $y(a)$, ρ and r are constant with respect to t , it follows that the variables will solely depend on age a but not on the date of birth t_0 .²⁶ From the FOCs of the social welfare problem (SW) and of the individual choice problem (IC) we then obtain straightforwardly

²⁶Obviously, outside a steady-state all of the relevant variables would also depend on t_0 .

$$\begin{aligned}
\text{SW:} \quad & 1 = -\psi^S(a) \mu_h(a, h^S(a), \bar{H}^S) + \frac{\eta^H}{u_c(c^S(a))} \\
\text{IC:} \quad & 1 = -\Psi^P(a) \mu_h(a, h, \bar{H}) + \tau(a),
\end{aligned} \tag{42}$$

respectively. Here, variables superscribed with 'S' correspond to the social planner's optimum. Combining the two first-order conditions yields²⁷

$$\begin{aligned}
\tau^*(a) &= \Psi^P(a) \mu_h(a, h(a), \bar{H}) - \psi^S(a) \mu_h(a, h^S(a), \bar{H}^S) + \frac{\eta^H}{u_c(c^S(a))} \\
&= \frac{\eta^H}{u_c(c^S(a))} = \frac{\Theta(a)}{\psi^S(a) + \Theta(a)}
\end{aligned} \tag{43}$$

When the transfer induces the individual of all ages to spend optimally on health care, mortality rates and consumption levels correspond to the socially optimal levels as well. It follows that SVIL and PVOL are equalised within each age-group. The optimal transfer is then an increasing function of the value of the externality $\Theta(a)$. It is positive if and only if $\Theta(a) > 0$. Hence, in the presence of positive externalities, subsidies increase in the value of the externality. In the limiting case where mortality can only be reduced through collective expenditure \bar{H} , it is true that $\Theta(a) \rightarrow \infty$. In this case, $\tau^*(a) = 1$, implying that individuals receive health care free of charge at the point of use but have to pay a lump-sum tax $\tau^*(a) h^S(a)$. Note that we can also interpret the transfer scheme in the context of health insurance: insurance premia amount to $\tau^*(a) h^S(a)$ and care is then provided subject to a co-payment equal to $1 - \tau^*(a)$. In the presence of negative externalities, the optimal transfer is negative. Here, individual expenditure should be taxed.

Finally, note that when consumption is fully smoothed across all age groups for $\rho = r$, the transfer becomes entirely independent from the individual's age. This is readily verified as $\frac{\eta^H}{u_c(c^S(a))}$ is then constant with respect to age. It follows that in our setting the transfer is age-dependent only in as far as it adjusts for age-related differences in the propensity to consume. On second thought this is not surprising as the marginal contribution towards the externality (in terms of an additional unit of health expenditure) is independent of the individual's age. However, this also suggests that transfers should be age-dependent - even in the presence of consumption smoothing - whenever some age-groups are generating spillovers more effectively than others.

²⁷When the transfer induces the individual of all ages to spend optimally on health care, we obtain $h(a) - h^S(a) = 0$. In this case, $\mu_h(a, h, \bar{H}) = \mu_h(a, h^S, \bar{H}^S)$. As consumption levels will all equal the socially optimal levels, it follows that $\Psi^P(a) = \psi^S(a)$ for all age-groups. Hence, $\tau^*(a) = \frac{\eta^H}{u_c(c^S(a))}$. Substituting in turn from (40) and from IC in (42) we obtain $\tau^*(a) = -\mu_h(a, h^S, \bar{H}^S)\Theta(a) = (1 - \tau^*(a)) \frac{\Theta(a)}{\Psi^P(a)}$. This solves to $\tau^*(a) = \frac{\Theta(a)}{\Psi^P(a) + \Theta(a)} = \frac{\Theta(a)}{\psi^S(a) + \Theta(a)}$.

5 Numerical Results

In this section we numerically illustrate the results derived in sections two through four. We apply the following functional specification

$$u(c(a, t)) = b + \frac{c(a, t)^{1-\sigma}}{1-\sigma} \quad (44)$$

where $b = 5$, $\sigma = 2.5$ and $\alpha = 0.5$. For simplicity we assume that the time preference rate equals the interest rate $r = \rho = 0.03$. The maximal life-span ω is set equal to 110. Mortality data have been taken from the human mortality data base [7] for the years 1990-2000. Furthermore, we assume that individual income (net of interest payments) $y(a)$ is proxied by individual wages which, in turn, are assumed to equal the age-specific marginal product of labour. Data on age-specific productivity have been taken from Skirbekk [35], who bases the productivity estimates on a weighted average over 6 age-dependent abilities (numerical ability, managerial ability, clerical perception, finger dexterity, manual dexterity, experience). Consequently the productivity profile does not represent the productivity for a special profession, but the average over (more or less) all of them. All our results are calculated for a steady state with a stable population.

We distinguish between two functional forms for the effect of health spending on mortality. In the first case we assume no externality of aggregate health expenditure on mortality:

$$\phi_1(a, h(a, t)) = 1 - \sqrt{\frac{h(a, t)}{z} \left(a \frac{1-d}{1-\omega} + \frac{d-\omega}{1-\omega} \right)} \quad (45)$$

where we set $z = 3$ and $d = 0$. The latter implies that health expenditure becomes entirely ineffective for $a = \omega$. In the second case we assume that aggregate health expenditure may positively or negatively affect mortality:

$$\phi_2(a, h(a, t), \bar{H}(t)) = 1 - \sqrt{\frac{h(a, t)}{z} \left(a \frac{1-d}{1-\omega} + \frac{d-\omega}{1-\omega} \right)} \pm \sqrt{\frac{\bar{H}(t)}{z'}} \quad (46)$$

with $z = 3$, $z' = 10$ and $d = 0$.

In the first two figures we summarize the results obtained in the base model where we assume no externality (i.e. where we apply the functional form ϕ_1). Figure 2 (left panel) plots the consumption path together with the age-specific productivity profile (the bell shaped curve) and the age specific health investment (right panel). Obviously social choice and individual choice result in the same time paths. Since we assume $\rho = r$ it is optimal to smooth consumption over all ages for each cohort. Health investments (right panel) are determined by the mortality rate and initially increase over age. As health investments become less effective in curbing mortality at higher ages, the optimal strategy is to reduce health investment in old age (cf. our

discussion in section 2.1). As Figure 3 indicates health investments reduce the base mortality (left panel) over the whole age range while the value of life decreases over age. Referring to our discussion in section 2.2 and 3.2 our numerical setting obviously implies that the negative impact as caused by a fall in the expected remaining life span dominates.

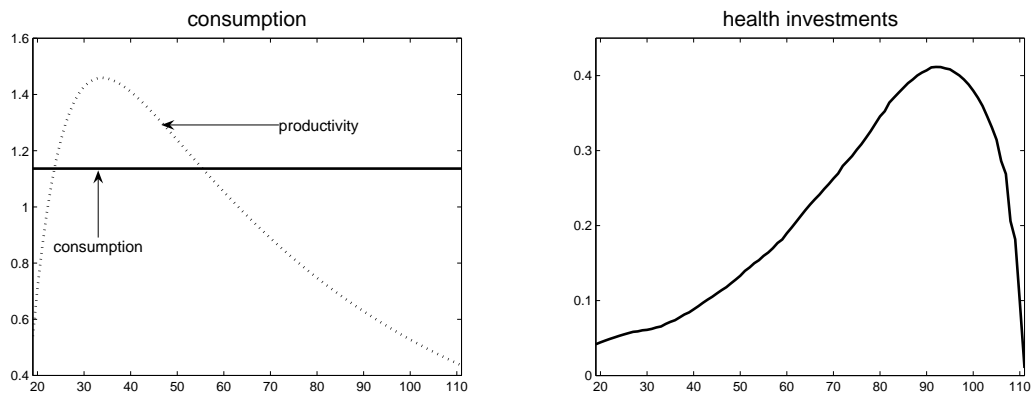


Figure 2: Consumption (left) and health investments (right) in the base model

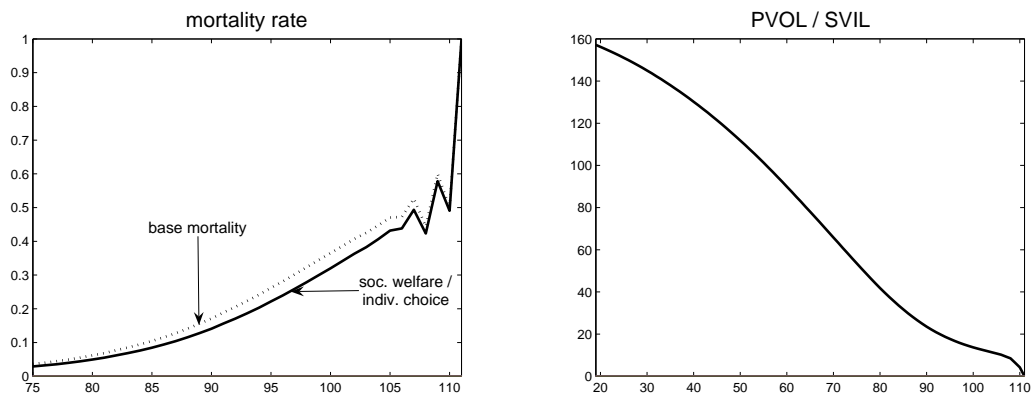


Figure 3: mortality (left) and PVOL/SVIL (right) in the base model

We can now turn to the impact of the spillover effects. Figure 4 plots individually optimal consumption against the social optimum for the case of positive (left panel) and negative (right panel) externalities, respectively. Figure 5 plots individually optimal health expenditure against the social optimum for the same two cases. As we would expect, individual consumption is too high (low) in the case of positive

(negative) externalities. Correspondingly, too little (much) is spent on health care in the case of positive (negative) spillovers. This is, indeed, what one would expect given the nature of the externality.

Figure 6 plots the mortality levels corresponding to the individual and social optimum, respectively, for the case of positive externalities (the left panel covering ages 19-60 and the right panel ages 60+). As expected, mortality rates are higher under individual choice than they should optimally be. Interestingly, however, this applies mainly to ages up to 60, whereas for old ages the difference is smaller. This corresponds with the observation that individually and socially optimal health care spending converge over age. For young ages, the effect of own spending h on an individual's mortality is weak, thus, leading to low individual spending levels. From a social point of view, however, there is a social interest in increasing individual health care expenditure in particular for the young age-groups. As they are large in number $N(a, t)$, increases in individual spending within these groups has a strong effect on aggregate expenditure and thus on external benefits.

Figure 7 depicts comparative mortality in the case of negative spillovers for the ages 80+. We notice that the effects on mortality of health investments are very weak - indeed for ages below 80, they are almost absent. This suggests that the impact on mortality from individual health spending is neutralised by the negative impact on mortality of aggregate expenditure. Whereas social and individual spending tends to lower mortality (by modest amounts) below the baseline for ages up to 100, mortality somewhat increases above the baseline for the highest ages. Compared with the social optimum, mortality tends to be somewhat higher under individual choice. While this reflects the problem of significant over-spending in all age-groups (see figure 5 right panel), the small discrepancy between the mortality rates suggests that the predominant effect of excessive individual health spending is to neutralise the negative externality. This represents a form of tread-mill effect.

The differences in mortality patterns in the case of positive and negative externalities are reflected in figure 8, which plots the net increase in remaining life-expectancy at age a attainable if individuals were to spend optimally. For instance, in the presence of positive externalities (the upper graph) the life expectancy at age 20 would increase by a little more than one year if it were possible to induce individuals to spend optimally. This stands in contrast to the case of negative externalities, where socially optimal behaviour leads to only modest increases in the remaining life expectancy for ages above 40.²⁸ The impact of differential health spending on mortality in the presence of positive externalities - and its absence under negative

²⁸Indeed, a scrutinous look at the data shows that for ages below 30, the remaining life expectancy in the social optimum lies (by a very small amount) below the life expectancy in the case of individually optimal spending. This is the case because from a social point of view, it is optimal to spend close to nothing for the very young ages. The resulting increase in mortality for these age groups is more than compensated for by the reductions in mortality for the ages 30+.

externalities - is also evident in the resulting consumption patterns. Consulting figure 4, we see that the gap between socially and individually optimal consumption is larger in the case of positive externality. Here, individually optimal consumption is higher both due to under-investment in health *and* due to the fact that consumption is spread across a significantly shorter length of life (thus allowing higher levels of per-period consumption). In the presence of negative spillovers, the effect through changes in life-expectancy is virtually absent, so that the gap in consumption levels predominantly reflects excessive individual spending on health care.

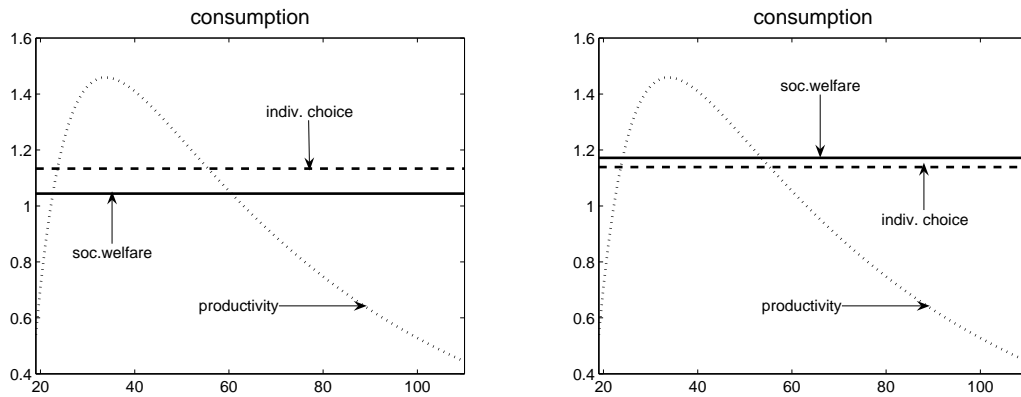


Figure 4: Consumption in the model positive (left) and negative (right) externalities

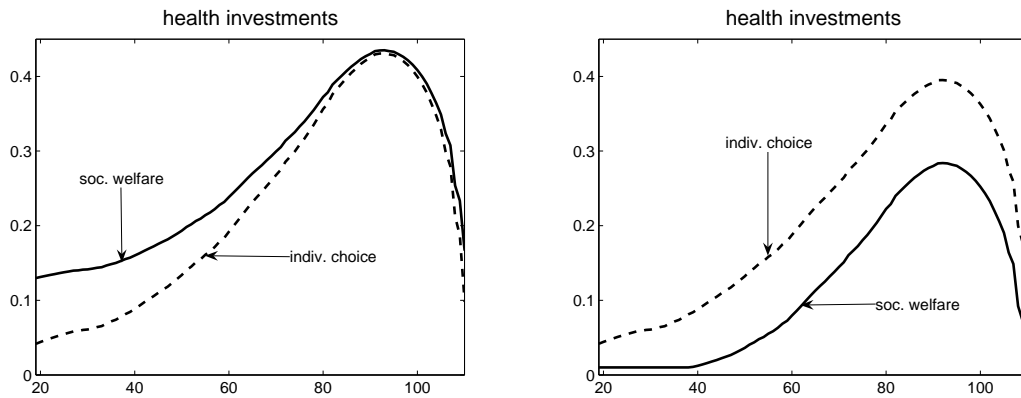


Figure 5: Health investments in the model positive (left) and negative (right) externalities

Figure 9 shows how the private value of life (PVOL) develops against the social value of individual life (SVIL) again for the case of positive spillovers (left panel)

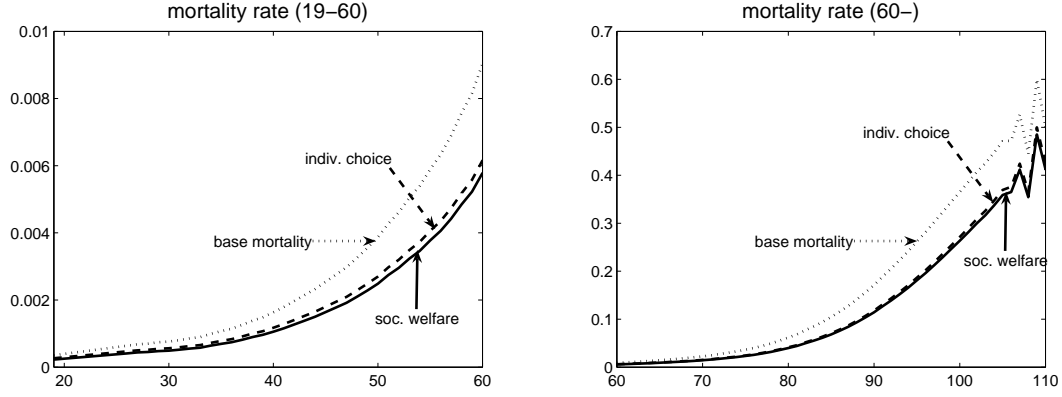


Figure 6: Mortality in the model of positive externalities

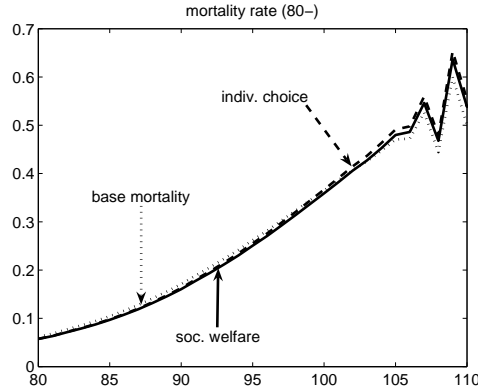


Figure 7: Mortality in the model of negative externalities

and negative spillovers (right panel). PVOL exceeds SVIL in the case of positive spillovers but falls short of it in the case of negative spillovers. Setting $\tau(a) \equiv 0$ in the planner's and the individual's first-order condition as reported in (42) and rearranging terms we obtain $\Theta(a) = [\Psi^P(a) - \psi^S(a)] - \varepsilon(a)\Psi^P(a)$, with $\varepsilon := 1 - \frac{\mu_h(a, h, \bar{H})}{\mu_h(a, h^S, \bar{H}^S)}$. In the case of positive externalities $h < h^S$ for all ages. It follows that $\mu_h(a, h, \bar{H}) > \mu_h(a, h^S, \bar{H}^S)$ for our model and therefore $\varepsilon(a) < 0$. The distance $\Psi^P(a) - \psi^S(a) > 0$ therefore gives a lower bound on the value of the externality. The same applies for negative externalities, where $\Theta(a) < 0$ and $\varepsilon(a) > 0$. We can also infer that the absolute value of the externality $|\Theta(a)|$ decreases with age.²⁹

²⁹Consider the derivative $\Theta_a = (\Psi^P - \psi^S)_a - (\varepsilon\Psi_a^P - \varepsilon_a\Psi^P)$. For the case of positive externalities inspection of Figure 8 (left panel) shows that $(\Psi^P - \psi^S)_a < 0$ and $\Psi_a^P < 0$. Moreover, $\varepsilon_a > 0$

Apart from the highest ages, this is due to the fact that the relative effectiveness of aggregate spending $\frac{\mu_{\bar{H}}(\hat{a}, h, \bar{H})}{\mu_h(a, h, H)}$ tends to decrease with age a [see (40)].

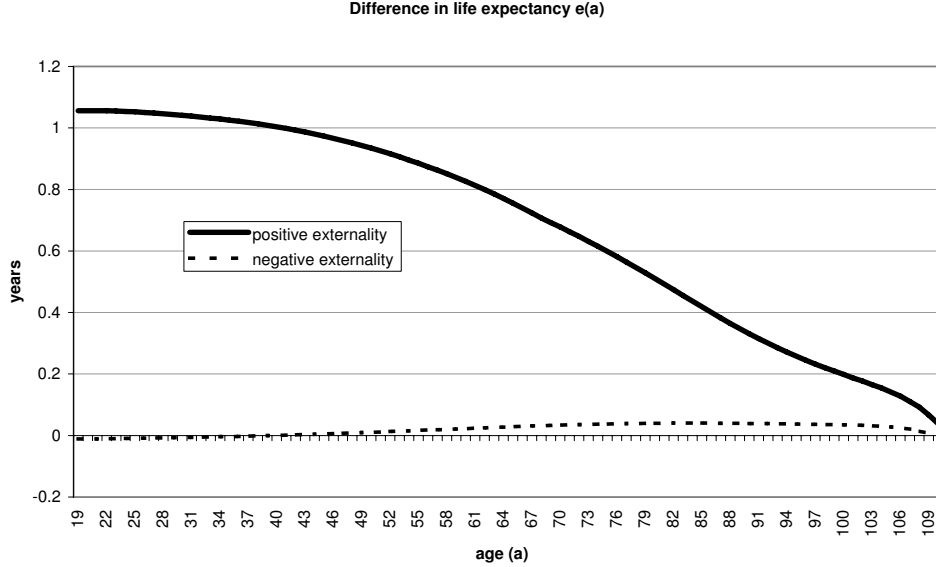


Figure 8: Differences in life expectancy at age a ($e(a)$) between social welfare and individual choice model for positive and negative externalities.

6 Conclusions

We provide a framework for assessing the efficiency of individual choices within a continuous-time life-cycle framework by modelling and comparing the health expenditure / consumption paths chosen by a social planner with those chosen by an individual. The social planner has in mind a whole population and maximises over two dimensions: age-structure and time. The individual, in contrast, has in mind its

follows from the fact that in the case of positive spillovers h and h^S converge with rising age (see Figure 5, left panel). But then, $\varepsilon \Psi_a^P - \varepsilon_a \Psi^P > 0$ implying that $\Theta_a < 0$. In the case of negative externalities, we see from Figure 8 (right panel) that $(\Psi^P - \psi^S)_a > 0$ and $\Psi_a^P < 0$. Furthermore, inspection of Figure 5 (right panel) shows that h and h^S diverge up to age 95, implying $\varepsilon_a > 0$ for ages up to age 95. For ages 95+ $\Theta_a > 0$ is likely to be true. While convergence of h and h^S implies $\varepsilon_a < 0$ for these ages, Ψ^P is already rather low. Therefore, $\varepsilon \Psi_a^P - \varepsilon_a \Psi^P < 0$ is still likely to hold. Hence, $\Theta_a > 0$ for the case of negative externality. It follows that in both cases the (absolute) value of the externality decreases with age.

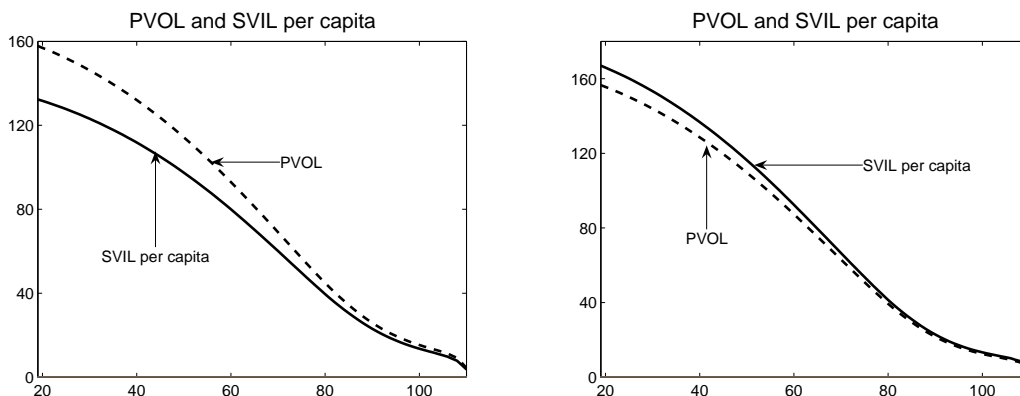


Figure 9: PVOL compared to the SVIL in the model positive (left) and negative (right) externalities

own mortality and maximises over one dimension only, where time measures individual age. We derive the optimal time paths and show how they can be compared. A summary comparison is best based on the social value of an individual life (SVIL) as compared to the private value of life (PVOL). The latter is well established in individual life-cycle modelling. We show that the former can be constructed from the planner's problem in an analogous way. We also illustrate how the SVIL corresponds to the value of population as derived in macro-economic models. We thus provide a bridge between the hitherto unrelated micro-models with a focus on individual mortality and macro-models with a focus on a population.

We apply our model to the examination of spillovers related to individual health care spending - or equivalently: individual use of health care. Spillovers may arise for a number of reasons. Positive spillovers may be due to learning-by-doing-effects or due to the fact that higher aggregate spending levels boost R&D or the provision of 'high quality' capacities. They also arise, of course, in the context of vaccination against infectious disease. Negative spillovers may arise due to congestion of the health care system, resulting either in a reduction of the quality of care or in inefficient forms of rationing, such as medically undue waiting. We show analytically how the planner (but not the individual) incorporates in her decision making the value of the externality. Furthermore, we derive a transfer scheme that induces optimal expenditure and consumption plans on the part of the individual. We illustrate our model(s) by way of numerical analysis. The analysis of positive and negative spillovers reveals the expected distortions from the optimum in individual consumption and health care spending. More interestingly, the nature of the externality has rather distinct consequences for the pattern of mortality. In the presence of positive spillovers mortality can be reduced significantly below its baseline with correspond-

ing increases in life-expectancy. However, owing to their under-spending individuals fail to realise a significant share of these gains in life-expectancy. In contrast, when spillovers are negative, no substantial reductions in mortality below the baseline can be attained. Here, the inefficiency of individual behaviour is manifest in a treadmill effect, where individuals over-spend on health care without great effect and, thereby, forego consumption. Finally, we show how the value of the externality can be inferred from a comparison between SVIL and PVOL.

It has been our main objective to provide a modelling framework to analyse the efficiency of individual life-cycle behaviour, to present the critical elements of such an analysis and to illustrate the channels of transmission by which direct period effects and effects through changes in the life-expectancy impact on life-cycle choice. In order to facilitate the representation as much as possible we have therefore adopted a number of simplifying assumptions regarding the nature of the externalities. In particular, by assuming that the spillovers flow through aggregate health care expenditure, we assume that all age-groups contribute in a symmetric way. For our numerical analysis, we impose additional assumptions, namely that the marginal productivity of individual health expenditure is unaffected by the externality and that the impact of the externality on mortality is independent of age. Clearly, these assumptions are unrealistic and rule out an application of our results to the various forms of real-world externalities that were discussed earlier. There is clear scope for drawing up more realistic models of life-cycle externalities; but we leave this to future research.

Finally, our model lends itself to the analysis of other imperfections in individual behaviour. The cross-cohort spillovers that give rise to inefficiency are clearly not only present in the health care sector but also - and perhaps more prominently - in the production and/or consumption of goods. Externalities in production arise with regard to saving towards the accumulation of a (common) capital stock that affects the productivity of (everyone's) labour; and with regard to health or educational investments that increase individual productivity but also the productivity of co-workers. Externalities with regard to consumption arise for many modes of unhealthy consumption (see Forster [12] for a life-cycle-model of unhealthy consumption without spillovers). Most prominently this relates to smoking which not only raises individual mortality but also the mortality of others. Similar arguments apply, however, to other consumption goods, such as cars, that directly or indirectly lead to the emission of pollutants. As should have become evident from our analysis, such externalities will lead to distortions both due to period effects and through effects in overall life-expectancy. We would thus envisage a number of interesting applications.

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A Appendix: Proof of steady state-relation in the case of exogenous $B(t)$

Within this section we prove the existence of a steady state of the social planner model in the time interval $t \in [\omega, T - \omega]$ if the number of newborns $B(t)$ is exogenous. As major parts of the proof are analogous to the proof of proposition 2 of Prskawetz and Veliov [29] we sketch the method and discuss only the differences in detail.

In order to write our social planner problem in a more compact form we write our controls $c(a, t)$ and $h(a, t)$ and states $N(a, t)$ and $A(a, t)$ as column vectors

$$\begin{aligned} m(a, t) &:= (c(a, t), h(a, t))' \\ x(a, t) &:= (N(a, t), A(a, t))'. \end{aligned} \quad (47)$$

Then we can write (6) as

$$\begin{aligned} \max \quad & \int_0^T \int_0^\omega e^{-\rho t} g(a, x(a, t), m(a, t)) \, da \, dt + \int_0^T e^{-\rho t} g'(\omega, x(\omega, t)) \, dt + \\ & + \int_0^\omega e^{-\rho T} g''(a, x(a, T)) \, da \\ \text{s.t.} \quad & x_a + x_t = f(a, x, m) \\ & x(a, 0) = x_0(a) \\ & x(0, t) = \varphi(t) \end{aligned} \quad (48)$$

where $g(\cdot)$ and $f(\cdot)$ denote the objective functional and the system dynamics resp. $g'(\cdot)$ and $g''(\cdot)$ denote the additional terms for the end state constraints. The initial and boundary constraints are denoted by $x_0(a)$ and $\varphi(t)$.

Then we define the following functions for a given control-trajectory pair³⁰

$$\begin{aligned} z[\kappa](s) &= x(s, \kappa + s), & q[\kappa](s) &= m(s, \kappa + s) \\ y[\zeta](t) &= x(\zeta + t, t), & p[\zeta](t) &= m(\zeta + t, t) \\ v[\theta](a) &= x(a, \theta + a), & k[\theta](a) &= m(a, \theta + a) \end{aligned} \quad (49)$$

where

$$\kappa \in [0, T - \omega], \, s \in [0, \omega], \, \zeta \in [0, \omega], \, t \in [0, \omega - \zeta], \, \theta \in [T - \omega, T] \text{ and } a \in [0, T - \theta].$$

³⁰ y, z and v denote the state variables (population, cohort savings) and p, q and k denote the controls (consumption, health expenditures) for cohorts that have been already alive at the beginning of the planning horizon, for cohorts that are born and die within the planning horizon and for cohorts that are still alive at the end of the planning horizon resp.

After changing the order of integration and changing the variables the above optimization problem can be represented as

$$\begin{aligned} \max \quad & \int_0^\omega J_0[\zeta](p[\zeta], y[\zeta]) \, d\zeta + \int_0^{T-\omega} e^{-\rho\kappa} J_1[\kappa](q[\kappa], z[\kappa]) \, d\kappa + \\ & + \int_{T-\omega}^T e^{-\rho\theta} J_2[\theta](k[\theta], v[\theta]) \, d\theta \end{aligned} \quad (50)$$

with

$$\begin{aligned} J_0[\zeta](p[\zeta], y[\zeta]) &= \int_0^{\omega-\zeta} e^{-\rho t} g(\zeta + t, y[\zeta](t), p[\zeta](t)) \, dt + e^{-\rho(\omega-\zeta)} g'(\omega, x(\omega, \omega - \zeta)) \\ J_1[\kappa](q[\kappa], z[\kappa]) &= \int_0^\omega e^{-\rho s} g(s, z[\kappa](s), q[\kappa](s)) \, ds + e^{-\rho\omega} g'(\omega, x(\omega, \kappa + \omega)) \\ J_2[\theta](k[\theta], v[\theta]) &= \int_0^{T-\theta} e^{-\rho a} g(a, v[\theta](a), k[\theta](a)) \, da + e^{-\rho(T-\theta)} g''(T - \theta, x(T - \theta, \mathbb{B})) \end{aligned}$$

and

$$\begin{aligned} \frac{dy[\zeta](t)}{dt} &= f(\zeta + t, y[\zeta](t), p[\zeta](t)), \quad y[\zeta](0) = x_0(\zeta) \\ \frac{dz[\kappa](s)}{ds} &= f(s, z[\kappa](s), q[\kappa](s)), \quad z[\kappa](0) = \varphi(\kappa) \\ \frac{dv[\theta](a)}{da} &= f(a, v[\theta](a), k[\theta](a)), \quad v[\theta](0) = \varphi(\theta) \end{aligned} \quad (52)$$

Thus the parametric family $J_1[\kappa](q[\kappa], z[\kappa])$ for $\kappa \in [0, T - \omega]$ reaches the same results for all κ . Following the arguments in Prskawetz and Veliov [29] the system reaches a steady state for³¹ $[\omega, T - \omega]$.

Note that the optimization of the parametric family $J_0\cdot$, $J_1\cdot$ and $J_2\cdot$ results in an optimization over cohorts not influencing each other (further all cohorts within the parametric family $J_1\cdot$ have the identical conditions). Thus the proof does not work when the number of newborns is endogenous, as then the boundary condition changes and the cohorts may have different conditions.

B Appendix: Derivation of SVIL

From solving the adjoint equation for $\xi^N(a, t)$ with the method of characteristics we obtain

³¹More general, the system reaches a steady state in the parallelogram defined by the four corner points $(0, 0)$, (ω, ω) , (ω, T) and $(0, T - \omega)$ of the Lexis diagram.

$$\begin{aligned} \xi^N(a, t) = & \int_a^\omega (u(c(s, t - a + s)) + \xi^A(s, t - a + s)(p(s) - c(s, t - a + s) - h(s, t - a + s)) + \\ & + \xi^N(0, t - a + s)\nu(s))e^{-(s-a)\rho - \int_a^s \mu(s', h(s', t - a + s')) ds'} ds \end{aligned} \quad (53)$$

since $\xi^N(\omega, t) = 0$ holds for every t . Inserting the above expression into (19) and applying (14) yields

$$\psi^S(a, t) = \int_a^\omega \frac{u_c(c(s, t - a + s))}{u_c(c(a, t))} v(s, t - a + s) e^{-(s-a)\rho - \int_a^s \mu(s', h(s', t - a + s')) ds'} ds \quad (54)$$

As the marginal utility of consumption $u_c(c(a, t))$ only depends on consumption and not on t the total derivative with respect to time equals

$$\left(\frac{\partial}{\partial a} + \frac{\partial}{\partial t} \right) u_c(a, t) = u_{cc}(c_a + c_t) \quad (55)$$

Again applying the method of characteristics we obtain

$$c(a, t) = c(s, t - a + s) e^{-(s-a)(\rho-r)} \quad (56)$$