

Business Cycle Asymmetries and the Labor Market

Britta Kohlbrecher^a and Christian Merkl^{a,b}

^a Friedrich-Alexander-Universität (FAU) Erlangen-Nuremberg,

^b Kiel Institute for the World Economy (IfW)

Abstract

This paper shows two business cycle facts for the United States. First, the job-finding is very asymmetric over the business cycle and moves a lot more in recessions than in booms. Second, there is a positive correlation between the job-finding rate and the matching efficiency. We provide an explanation for both business cycle facts by enhancing a search and matching model with idiosyncratic shocks for match formation and by solving the full nonlinear structure of the model. Understanding the sources of these nonlinearities is very important. In our calibration, government interventions (such as wage subsidies and government spending) are three times more effective in a heavy recession than in a boom. In addition, shifts of the matching efficiency are not necessarily a sign for higher structural unemployment.

— **VERY PRELIMINARY DRAFT** —

JEL Classification: E24, E32, J63, J64

Key words: Business cycle asymmetries, matching function, job-finding rate, effectiveness of policy

1 Introduction

This paper shows two interesting business cycle facts for the US business cycle. First, the job-finding rate is very asymmetric over the business cycle. The decrease of the job-finding rate in recessions is a lot larger than the increase in booms. Thus, we isolate an important driver for asymmetric unemployment fluctuations (Abbritti and Fahr, 2013; McKay and Reis, 2008). Second, we show that the matching efficiency on the US labor market has a positive correlation with the job-finding rate. Thus, through the lens of a search and matching model it appears that the matching process is particularly inefficient in times of low job-finding rates. The decrease in the job-finding rate goes beyond the decline suggested by a drop of market tightness. This was not only the case during the Great Recession where the shift of the Beveridge curve was widely discussed. The procyclicality of the matching efficiency holds for long time episodes. This finding is connected to the observation that the Beveridge curve tends to shift outward in recessions (Diamond and Sahin, 2014).

We show that the two stylized facts can be explained by enhancing a search and matching model with a labor selection mechanism. After a contact is established between workers and firms (via the contact function¹), an idiosyncratic productivity shock is drawn. Employers choose a cutoff point of idiosyncratic productivity up to which they are willing to hire because applicants are sufficiently productive to generate an expected profit. The labor selection mechanism provides asymmetric reactions of the job-finding rate in response to symmetric aggregate shocks. In addition, the labor selection mechanism generates a positive correlation between the job-finding rate and the matching efficiency. What is the underlying intuition? Job creation is driven by two mechanisms, namely contact (through the contact function) and selection (through idiosyncratic shocks). The selection part generates strong business cycle asymmetries. In a recession, only workers with large idiosyncratic productivity are selected. The endogenous cutoff point of idiosyncratic productivity moves to a thicker part of the distribution. This amplifies the reaction of the job-finding rate in a recession and thereby generates the asymmetries of the job-finding rate that can be observed in the data. In addition, with labor selection firms anticipate that they will hire more productive workers in a recession and thereby vacancies fall by less than without labor selection. By looking at the resulting behavior of the job-finding rate and market tightness through the lens of a standard matching function (without selection), it appears that the job-finding rate has dropped too much relative to the drop in the market tightness. As a consequence, the backed-out matching efficiency, defined as the ratio between the actual job-finding rate and its prediction from a standard, time invariant matching function, would fall in a recession. Thus, it is positively correlated with the level of the job-finding rate. This is exactly what we observe in the data.

The two theoretical mechanisms are only at work if the steady state cutoff

¹In what follows, “contact function” refers to the theoretical function that establishes contacts between workers and firms.

point for idiosyncratic productivity is to the left of the peak of the corresponding density function. This is the same condition that is necessary to obtain a realistic elasticity of new matches with respect to vacancies in our model framework. Based on a survey of empirical studies, Petrongolo and Pissarides (2001) consider a weight on vacancies in the matching function of 0.5 as an upper bound. Our own estimations with different filtering techniques in Section 2 confirm this. Kohlbrecher et al. (2014) show that including idiosyncratic productivity for match formation in an otherwise standard search and matching model increases the elasticity of matches with respect to vacancies and hence the coefficient in an estimated matching function. The size of this effect depends on the position of the cutoff point for idiosyncratic productivity. A weight on vacancies smaller than 0.5 is only possible if the cutoff point is located to the left of the peak of idiosyncratic productivity. Thus, by targeting a data consistent elasticity of matches with respect to vacancies, our model generates the observed nonlinearities and the shift of the matching efficiency in recessions.

We argue that it is crucial to understand the driving forces of the described nonlinearities because they matter a lot for the effectiveness of policy interventions. We illustrate this by implementing a wage subsidy in a recession and in a boom. In the fully nonlinear model, the wage subsidy is roughly three times more effective in a recession than in a boom in terms of the effect on unemployment.

We have also started embedding our framework into a New Keynesian framework with sticky prices. This environment allows us to experiment in a meaningful way with traditional government spending (otherwise government spending would have no real effects). In a recession fiscal multipliers are also roughly three times larger than in a boom. Thus, our framework provides a new rationale for the empirical finding of time varying fiscal multipliers (Auerbach and Gorodnichenko, 2012).

Finally, our paper contributes to the policy debate on whether drops in the matching efficiency are a sign for a higher structural unemployment rate. Through the lens of our framework, the backed-out matching efficiency drops automatically in a recession. Only a decline that goes beyond this model induced drop would be indicative for an actual deterioration of the labor market performance and thereby an increase of structural unemployment. Thus, it is very important to distinguish these two components.

The rest of the paper is structured as follows. Section 2 presents stylized facts on business cycle asymmetries and the cyclicalities of the backed-out matching efficiency. Section 3 shows a search and matching model with labor selection. Section 4 provides analytical results. Section 5 outlays our calibration strategy and section 6 presents numerical results in the full nonlinear setting. Section 7 puts our results in perspective to the existing literature and Section 8 briefly concludes.

2 Stylized facts

Stylized Fact 1. *The job-finding rate shows a strong nonlinear pattern over the business cycle. The downward movement in recessions is a lot stronger than the upward movement in booms.*

Table 1 and 2 show the skewness for annual growth rates and Hodrick-Prescott filtered (smoothing parameter $\lambda = 1600$) variables (from 1951 to 2013) and the p-values for statistical significance with a one-sided test for skewness.

Variable	<i>JFR</i>	<i>U</i>	<i>N</i>	<i>V</i>	θ	<i>Y</i>
Skewness	-0.90	2.18	-0.69	0.05	0.25	-0.34
P-Value	0.01	0.05	0.03	0.38	0.17	0.02

Table 1: Business Cycle Skewness: Statistics are based on year-to-year growth rates for quarterly U.S. data from 1951 to 2013. Test statistics for skewness follow Bai and Ng (2005).

Variable	<i>JFR</i>	<i>U</i>	<i>N</i>	<i>V</i>	θ	<i>Y</i>
Skewness	-0.37	0.34	-0.30	-0.53	-0.50	-0.39
P-Value	0.03	0.13	0.13	0.05	0.07	0.09

Table 2: Business Cycle Skewness: Statistics are based on log deviations from hp-trend with smoothing parameter $\lambda = 1600$ for quarterly U.S. data from 1951 to 2013. Test statistics for skewness follow Bai and Ng (2005).

The job-finding rate is significantly leftward skewed in both the growth rates and the deviation from HP-trend. It is somewhat weaker with the HP-filter because the filter absorbs part of the skewness. Figure 1 illustrates that the leftward skewness of the job-finding rate is a lot stronger than the skewness of real GDP (in terms of annual growth rates). Note that the skewness is driven by some severe recessions where the job-finding rate drops by 30 percent or more (in particular the Great Recession 2008/09), i.e. times of large negative aggregate shocks. By contrast, the job-finding rate rarely grew by more than 20 percent on a year-to-year-basis

Unemployment is generally rightward skewed, while employment and output are leftward skewed. This is well-known from Abbritti and Fahr (2013) and McKay and Reis (2008). However, in contrast to the job-finding rate, statistical significance depends on the employed filtering method. It is more difficult to establish skewness with a Hodrick-Prescott filter because the filter absorbs part of the skewness. However, based on annual growth rates these three variables

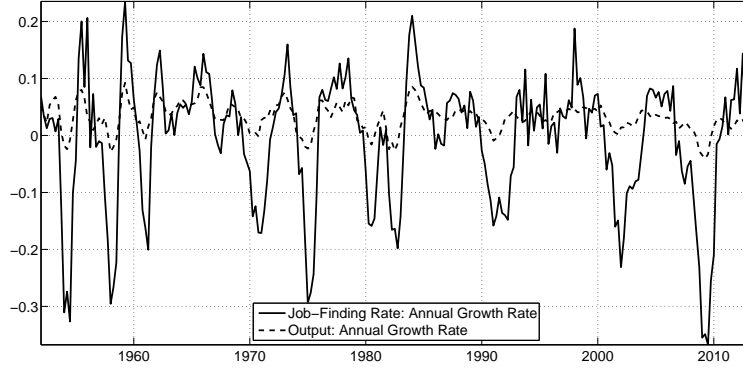


Figure 1: Growth rates of job-finding rate and real GDP

are all clearly skewed. Figure 2 provides an illustration for unemployment fluctuations based on year-to-year growth rates. The skewness of the job-finding rate is a clear driver for the skewness of unemployment and employment.

The picture is less clear for vacancies and market tightness. HP-filtering indicates some leftward skewness, while annual growth rate indicate no skewness at all.

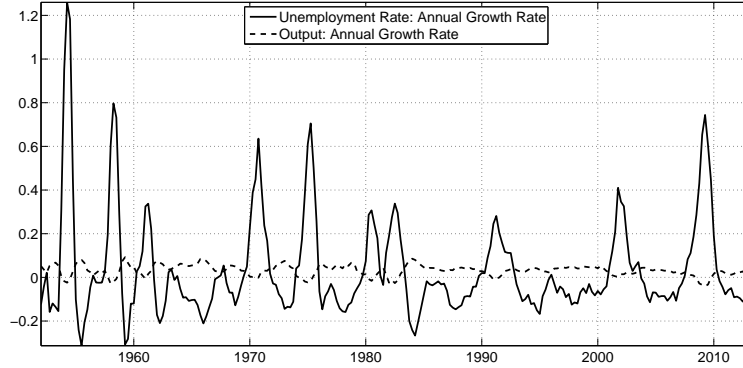


Figure 2: Growth rates of unemployment rate and real GDP

The robust leftward skewness of the job-finding rate shows that it is a driver for the rightward skewness of unemployment. This is particularly interesting against the background of a nonlinear employment equation, which drives part

of the skewness of (un)employment.²

Stylized Fact 2. *The backed-out matching efficiency has a positive comovement with the job-finding rate.*

To illustrate this point, we use the typical Cobb-Douglas constant returns matching function ($jfr_t = \mu\theta_t^\gamma$) and estimate γ via ordinary least squares.³ The first line in Table 3 shows the results for four different cases. In column I, the job-finding rate and market tightness are Hodrick-Prescott filtered with a smoothing parameter $\lambda = 1600$. In column II, a linear time trend is deducted. Columns III and IV show the results for non-detrended data, either with (column III) or without (column IV) a time trend in the regression. The weight on vacancies ranges in between 0.3 and 0.44, which is nicely in line with the surveyed studies by Petrongolo and Pissarides (2001).

	(I)	(II)	(III)	(IV)
Weight on vacancies in estimation	0.30***	0.36***	0.37***	0.44***
Correlation: matching efficiency-jfr	0.39	0.40	0.18	0.34
Coefficient on jfr (log-log-regression)	0.15***	0.19***	0.11***	0.22***

Table 3: Matching function estimates: (I) HP-filtered data ($\lambda = 1600$), (II) detrended with linear trend, (III) unfiltered data with time trend in estimation, (IV) unfiltered data without time trend in estimation. ***: Significance at 1%.

We then use the estimated matching function and the market tightness realizations to predict the job-finding rate. The ratio of the actual job-finding rate and the predicted job-finding rate provide the backed-out matching efficiency. Line 2 in Table 3 shows that this backed out matching efficiency has a positive correlation with the job-finding rate in all four cases.

Figure 3 illustrates this correlation for the HP-filtered time series. Visual inspection shows that the correlation between the job-finding rate and the matching efficiency is particularly strong in the second half of the sample, while it is not present in the first half. This can be explained by the fact that the elasticity of the matching function with respect to vacancies is much smaller in the first half of the sample (if estimated separately), i.e. there appears to be a structural break. When we do a sample split and estimate different elasticities, we obtain a similar positive correlation for both subsamples.

A simple log-log-regression⁴ for the entire sample period indicates that a 1 percent lower job-finding rate is associated with a 0.1 to 0.2 percent lower

²In steady state, $u = \frac{sr}{jfr+sr}$, where sr is the separation rate and jfr is the job-finding rate. Thus, even for a symmetric job-finding rate, unemployment would be rightward skewed. $\frac{\partial^2 u}{\partial jfr^2} > 0$

³Barnichon and Figura (2014) show that the results are very similar with an instrumental variable approach.

⁴We estimate $me_t = \beta_0 + \beta_1 jfr_t + \varepsilon_t$, where me_t is the backed out matching efficiency.

backed-out matching efficiency (statistically significant at the 1% level). Obviously, this simple regression does not allow any causal interpretation. However, the statistical significance shows that there is an important comovement between these two variables. Through the lens of the matching function, it appears that matching gets more efficient in booms (or more precisely in times of high job-finding rates) and less efficient in recessions (or more precisely in times of low job-finding rates). There are two simple possible interpretations for this phenomenon: Either exogenous matching efficiency shocks are a strong driver of the US business cycle. Or there is an endogenous mechanism that leads to a deterioration of the matching efficiency in recessions. In our theoretical part, we propose a mechanism for the latter.

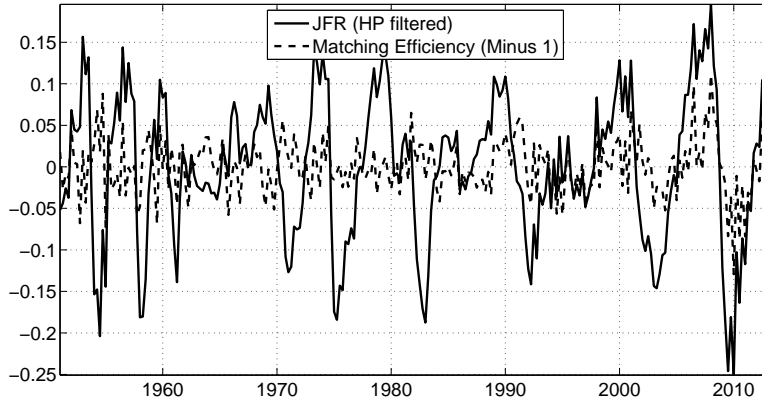


Figure 3: Backed-out matching efficiency and job-finding rate

Changes in the matching efficiency are associated with shifts of the Beveridge curve. Diamond and Sahin (2014) have emphasized that a shift of the Beveridge curve is a stylized fact of most past recessions in the United States. We complement their finding by showing a systematic correlation between matching efficiency and the job-finding rate.

3 The Model

We use a version of the Diamond-Mortensen-Pissarides (DMP) model (e.g. Mortensen and Pissarides, 1994) in discrete time and enrich it with one simple mechanism: idiosyncratic productivity for newly created jobs.

There is a continuum of workers on the unit interval who can either be employed or unemployed. Unemployed workers search for jobs and receive unemployment compensation b , employed workers can lose their job with constant probability ϕ . Firms have to post vacancies in order to get in contact with a

worker and pay vacancy posting costs κ per vacancy. We assume free-entry of vacancies. Contacts between searching workers and firms are established via a standard Cobb-Douglas contact function. In contrast to the basic DMP model not every contact between vacancies and workers ends in a hire. Upon contact, firms and workers draw from an idiosyncratic productivity distribution determining the match specific productivity, ε_{it} , for the first period of production. Only contacts that are productive enough, $\varepsilon_{it} \geq \tilde{\varepsilon}_t$, will result in a job, where $\tilde{\varepsilon}_t$ is the cutoff productivity that makes a firm indifferent between hiring and not hiring a worker. Our model is the same as in Kohlbrecher et al. (2014) and similar to the stochastic job matching model (Pissarides, 2000, chapter 6) or many of the endogenous separation models (e.g. Krause and Lubik, 2007). See also Brown et al. (2015) and Lechthaler et al. (2010) for labor selection models with idiosyncratic productivity (but without contact function). Our model is different from the latter in assuming that only new hires vary in their productivity. We choose this simplifying assumption to emphasize the role of idiosyncratic shocks for match formation.

3.1 Contacts

Contacts between searching workers and firms are established via a Cobb-Douglas, constant returns to scale (CRS) contact function,

$$c_t = \mu v_t^\gamma u_t^{1-\gamma}, \quad (1)$$

where u_t and v_t are the beginning of period t unemployment and vacancy stocks, μ is the contact efficiency and c_t is the overall number of contacts in period t . We define the contact probability for a worker as $p_t = \mu \theta_t^\gamma$ and the contact probability for a firm as $q_t = \mu \theta_t^{\gamma-1}$ with $\theta_t = v_t/u_t$.

3.2 The Selection Decision

Upon contact, firm and worker gain information about their match specific productivity at the start of the match. Technically, they draw a shock, ε_{it} , from an idiosyncratic productivity distribution, which is *iid* across workers and time, with density $f(\varepsilon)$ and cumulative distribution $F(\varepsilon)$.⁵ The expected discounted profit of a firm hiring a new worker (denoted with E for entrant) with match-specific productivity ε_t is given by

$$\pi_t^E(\varepsilon_t) = a_t + \varepsilon_t - w_t^E(\varepsilon_t) + \delta(1 - \phi) E_t(\pi_{t+1}^I), \quad (2)$$

where a_t is the aggregate productivity in the economy at time t , $w_t(\varepsilon_t)$ is the match specific wage, δ is the discount factor, and π_t^I is the expected discounted profit of an existing match (denoted with I for incumbent). Existing matches are not subject to shocks and thus all produce with the same productivity. The profits are thus given by:

⁵Due to the *iid* assumption, we abstract from the worker-firm pair specific index i from here onward.

$$\pi_t^I = a_t - w_t^I + \delta (1 - \phi) E_t(\pi_{t+1}^I). \quad (3)$$

A firm will hire a worker whenever the expected discounted profit is positive. The cutoff productivity that makes a firm indifferent between hiring and not hiring the worker is thus

$$\tilde{\varepsilon}_t = w_t^E(\varepsilon_t) - a_t - \delta (1 - \phi) E_t(\pi_{t+1}^I). \quad (4)$$

The ex-ante probability that a contact is selected into a job is thus

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f(\varepsilon_t) d\varepsilon_t. \quad (5)$$

3.3 Vacancies

In order to make a contact, firms have to post vacancies and pay vacancy posting costs κ . The value of a vacancy is

$$\Psi_t = -\kappa + q_t \eta_t E_t[\pi_t^E | \varepsilon_t \geq \tilde{\varepsilon}_t] + (1 - q_t \eta_t) \Psi_t, \quad (6)$$

where $q_t \eta_t$ is the overall probability that a vacancy leads to a productive match. We assume free entry for vacancies such that the value of a vacancy will be driven to zero. The vacancy condition thus simplifies to

$$\frac{\kappa}{q_t \eta_t} = E_t[\pi_t^E | \varepsilon_t \geq \tilde{\varepsilon}_t] \quad (7)$$

$$= a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty} (\varepsilon_t - w_t^E(\varepsilon_t)) f(\varepsilon_t) d\varepsilon_t}{\eta_t} + \delta (1 - \phi) E_t(\pi_{t+1}^I). \quad (8)$$

3.4 Wages

We assume Nash bargaining for both new and existing matches. Workers have linear utility over consumption. Let V_t^U , V_t^E , and V_t^I denote the value of unemployment, the value of a job for an entrant worker and the value of a job for an incumbent worker.

$$V_t^U = b + \delta E_t(c \eta_{t+1} V_{t+1}^E + (1 - c \eta_{t+1}) V_{t+1}^U), \quad (9)$$

$$V_t^E(\varepsilon_t) = w_t^E(\varepsilon_t) + \delta E_t((1 - \phi) V_{t+1}^I + \phi V_{t+1}^U), \quad (10)$$

$$V_t^I = w_t^I + \delta E_t((1 - \phi) V_{t+1}^I + \phi V_{t+1}^U). \quad (11)$$

The wage for an entrant and the wage for an incumbent worker are thus determined by the following maximization problems:

$$w_t^E(\varepsilon_t) \in \arg \max (V_t^E(\varepsilon_t) - V_t^U)^\alpha (\pi_t^E(\varepsilon_t))^{1-\alpha}, \quad (12)$$

$$w_t^I \in \arg \max (V_t^I - V_t^U)^\alpha (\pi_t^I)^{1-\alpha}. \quad (13)$$

3.5 Employment

The law of motion for employment is

$$n_{t+1} = (1 - \phi)n_t + c_t \eta_t u_t \quad (14)$$

with

$$u_t = 1 - n_t. \quad (15)$$

3.6 Labor market equilibrium

Given an initial condition for employment n_0 and a stochastic process for technology $\{a_t\}_{t=0}^{+\infty}$, the labor market equilibrium is a sequence of allocations $\{u_{t+1}, n_{t+1}, c_t, \pi_t, \eta_t, v_t, \tilde{\varepsilon}_t, w_t, w_t^E\}_{t=0}^{+\infty}$ satisfying equations (1), (3), (4), (5), (8), (12), (13), the law of motion for employment (14), and the definition of unemployment (15).

4 Analytical Results

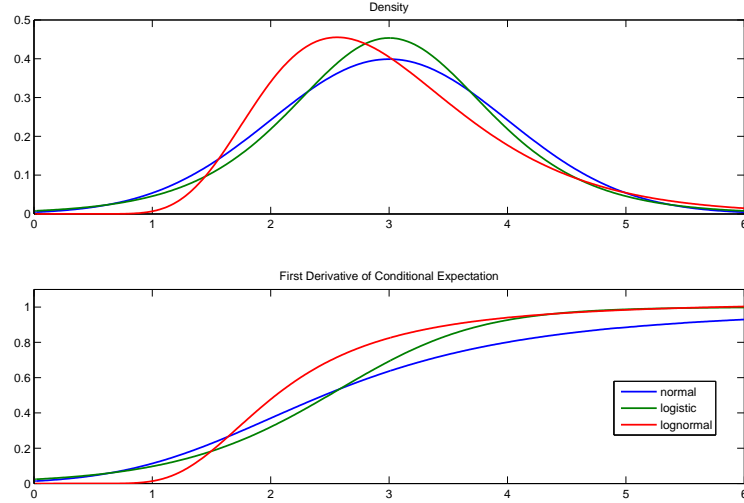
The idiosyncratic shocks (i.e. the selection mechanism) in the search and matching model lead to powerful extra-effects. To illustrate this, we show analytical steady state results for two extreme cases of the model in the Appendix. First, we switch off the selection mechanism, such that the job-finding rate is entirely determined by the contact function. This corresponds to the most simple search and matching model. Second, we switch off the contact margin by setting $\gamma = 0$. In this case, the job-finding rate is only driven by the selection mechanism. In the case without selection, there are only asymmetries with some wage rigidity. By contrast, in the case with a degenerate contact function (i.e. $\gamma = 0$), the model can generate labor market asymmetries even when wages comove one to one with productivity.

Interestingly, the version of the model with selection only also generates a positive co-movement between the job-finding rate and market tightness (see Kohlbrecher et al. (2014)). The job-finding rate increases in a boom because, due to a larger productivity, firms have an incentive to hire workers with a lower idiosyncratic productivity. Although in aggregate more vacancies do not lead to more jobs in this case, firms have an incentive to post additional vacancies in a boom (and thereby push up the market tightness). This is the case because with larger aggregate productivity, the expected profits of posting a vacancy increase.

Kohlbrecher et al. (2014) show that in the steady state version of the model with selection only, the elasticity of the job-finding rate with respect to market tightness (i.e. the coefficient on vacancies in a standard matching function)

depends on the position of the cutoff point in the idiosyncratic productivity distribution. Figure 4 illustrates this point. The upper panel shows three arbitrary density functions. The lower panel shows the corresponding weight of vacancies in an estimated Cobb-Douglas constant returns matching function. We can prove analytically that the weight of vacancies corresponds to the first derivative of the conditional expectations of the idiosyncratic shock (see Appendix for details).

Figure 4: Predicted matching coefficients for standard distributions



Notes: Density function and first derivative of conditional expectation for different standard distributions (namely, normal, logistic, and lognormal). For comparability reasons, the variance is normalized to 1 and the mean is set to 3 (the lognormal distribution requires a positive mean).

The steady state version of the selection model allows us to prove two propositions in order to isolate the driving forces of the labor market asymmetries and the shifts of the matching efficiency analytically. This is helpful for a better understanding of the numerical results in Section 6.

Proposition 1. *If the cutoff point is to the left of the peak of the idiosyncratic productivity density function, the job-finding rate reacts more in recessions than in booms.*

Proof. $\frac{\partial^2 \eta}{\partial a^2} < 0$ if $f'(\bar{\varepsilon}) > 0$. For details see Appendix C.2.1.

Intuition. When the probability density function is upward sloping at the cutoff point, negative productivity shocks lead to larger movements of the selection

rate than positive productivity shocks. The cutoff point moves to a thicker part of the density function. Thus, a given movement in the cutoff point generates larger effects. In this case, the selection mechanism generates business cycle asymmetries (as observed in the data).

Proposition 2. *With $f'(\bar{\varepsilon}) > 0$, a positive productivity shock generates a shift of the matching efficiency and thus the Beveridge curve.*

Proof. See Appendix C.2.2 for a proof.

Intuition. Due to the nonlinearities, the job-finding rate reacts more in a recession than in a boom. In addition, firms anticipate that they will hire more productive workers in a recession and thereby vacancies fall by less than without labor selection. Thus, the job-finding rate drops very strongly in a recession, while the market tightness reacts by less. Overall, this leads to a drop of the backed-out matching efficiency and to an outward shift of the Beveridge curve in a recession.

These two propositions show that we require $f'(\bar{\varepsilon}) > 0$ in order to generate nonlinearities and shifts of the matching efficiency. The density function needs to be upward sloping or put differently, the selection cutoff point needs to be to the left of the peak of idiosyncratic productivity.

As Figure 4 illustrates, this is the case as long as the weight on vacancies in the matching function is smaller than 0.5. Interestingly, both the survey of Petrongolo and Pissarides (2001) and our own estimations in Section 2 provide strong support for this condition to hold. Thus, our analytical exercise shows that when we calibrate our dynamic model to a realistic matching function, business cycle asymmetries and shift of the matching efficiency will show up.⁶ We will illustrate this below.

5 Calibration

We calibrate the model on a monthly frequency and then aggregate to obtain quarterly series. We target an overall job-finding rate, $p \cdot \eta$, of 0.45 as in Shimer (2005) and set the separation rate to 3% per month. Market tightness is normalized to one. As in Shimer (2005) we set the elasticity in the contact function $\gamma = 0.28$. In line with Hosios (1990) rule we assign the same value to the bargaining power of workers α . The discount factor is $\delta = 0.99^{1/3}$. Unemployment compensation is 70% of steady state productivity. This is an intermediate value between those used in Shimer (2005) and Hagedorn and Manovskii (2008).

We use a normal distribution for our match specific shocks. We do not have any direct evidence on the relative size of the contact and selection rate. We therefore choose the contact rate such that the ex-post estimated elasticity of the

⁶We have done the analytics for a model with degenerate contact function. However, if we have $\gamma > 0$, this will increase the weight in the ex-post estimated matching function. Thus, the cutoff point has to be even further to the left in the density function to make sure that the estimated coefficient is smaller than 0.5. Thus, our argument is without loss of generality.

job-finding rate with respect to market tightness is 0.45. Following Petrongolo and Pissarides (2001) this seems to be a plausible value. This implies a steady state contact rate of 0.5 and a selection rate of 0.9. It follows that the cutoff point is located to the left of the peak of the distribution, which is a necessary condition for our mechanism to work. We believe that this is a valid assumption for two reasons. First, having a cutoff point to the right of the peak of the distribution would imply an unrealistically high weight on vacancies in matching function estimations (see Section 4).⁷ Second, we show in Appendix B the distribution of wages of newly hired workers for a homogeneous reference group, i.e. we control for observed heterogeneity. These wages provide indirect evidence on the distribution of idiosyncratic productivity. The cutoff point is clearly to the left of the peak.

We choose the mean and variance of the normal distribution in order to meet two targets: The steady state selection rate and the standard deviation of the job-finding rate relative to productivity in the data, which is 9.1.⁸ We obtain a mean of -7.3992 and a standard deviation of $\sigma = 0.104$.

Chugh and Merkl (2014) show that the standard deviation of the idiosyncratic shock distribution is tightly connected to the ability of selection models to generate strong amplification effects. A smaller standard deviation generates larger amplification effects because more density is centered around the idiosyncratic cutoff point. *Ceteris paribus* this leads to larger fluctuations of the selection cutoff point and thereby the job-finding rate. In our paper, we do not want to discuss the ability of the selection part to generate amplification. By contrast, we want to analyze the business cycle properties of a search and matching model with realistically volatile selection. Thus, we have chosen a strategy to pick a sufficiently low σ for this purpose. In future versions of this paper, we will also experiment with other ways of generating strong amplification effects and thereby check for the robustness of results.

With steady state market tightness equal to one, we obtain a contact efficiency $\mu = 0.5$. Finally, we set the vacancy posting cost to satisfy the zero profit condition.

Labor productivity follows an AR(1) process with mean 1. We calibrate the process as follows. We interpolate quarterly labor productivity with monthly data on industrial production in order to obtain a monthly series. We hp-filter with smoothing parameter⁹ $1600 * 3^4$ and estimate an AR(1) process from the data. We obtain a correlation coefficient of $\rho = 0.9249$. In the dynamic

⁷Kohlbrecher et al. (2014) have shown that introducing idiosyncratic shocks into a search and matching model will increase the ex-post estimated elasticity of the job-finding rate with respect to vacancies. The further to the right the cutoff point is located in the distribution, the higher the weight on vacancies.

⁸We calculated the standard deviations of the hp-filtered quarterly job finding rate and labor productivity from 1952-2013. We used a smoothing parameter of $\lambda = 1600$. Labor productivity is defined as real GDP over employment. Our relative volatility is higher as e.g. in Shimer (2005) as we use a smoothing parameter of 1600 and not 100,000 and because our time span includes the Great Recession.

⁹This corresponds to a smoothing parameter of 1600 for quarterly series. See Ravn and Uhlig (2002).

simulation in section 6.1 we use the residuals from the AR(1) process as our shock series, i.e. we feed the actual labor productivity series into our model.

6 Results

6.1 Dynamic Model Simulation

In a first step we show that our dynamic model can replicate the stylized facts from section 2. As we are interested in the nonlinear dynamics of the model we solve the model nonlinearly using the Fair-Taylor extended path method as implemented in Dynare (Fair and Taylor, 1983; Adjemian et al., 2011). We make one modification: Instead of feeding a random shock series into the model we use the actual realizations of labor productivity between 1952 and 2013 as our productivity series (as described in section 5). This allows us to compare our simulated series directly to the data. The simulated data are detrended using an hp-filter with smoothing parameter $\lambda = 1600$. Table 4 and 5 show the skewness and associated p-values for both the levels and the detrended series.

Variable	a	jfr	u	n	v	θ	y
Skewness	-0.23	-0.73	1.20	-1.20	-0.32	-0.13	-0.80
P-Value	0.16	0.01	0.01	0.01	0.07	0.29	0.01

Table 4: Business Cycle Skewness in the Dynamic Model: Statistics are based on levels of simulated quarterly series using actual labor productivity and Fair-Taylor method.

Variable	a	jfr	u	n	v	θ	y
Skewness	-0.17	-0.85	0.71	-0.97	-0.66	-0.61	-0.71
P-Value	0.22	0.01	0.01	0.01	0.01	0.01	0.01

Table 5: Business Cycle Skewness in the Dynamic Model: Statistics are based on log deviations from hp-trend (smoothing parameter $\lambda = 1600$) for simulated quarterly series using actual labor productivity and Fair-Taylor method.

First of all note that productivity is not significantly skewed, i.e. our results are not driven by a skewed underlying productivity process. As expected, the job-finding rate is significantly leftward skewed in both the levels and the detrended series. Unemployment is significantly rightward skewed. Interestingly, vacancies and market tightness seem to be relatively symmetric in the levels, although the filtering method induces some leftward skewness. Output is leftward skewed as in the data.

Figure 5 compares the simulated job-finding rate to the real job-finding rate. The fit is remarkably good. Except for the last 10 years the simulated series captures almost all of the peaks and troughs. The simulated job-finding rate leads the real series by one quarter. This is not surprising, as the job-finding rate in the model can react instantaneously to changes in productivity. The correlation between the simulated job-finding rate and the real job-finding rate lagged by one quarter is 0.5. So far the only source of shocks in our model is productivity. This might explain why we are not very successful in capturing the period around the Great Recession.

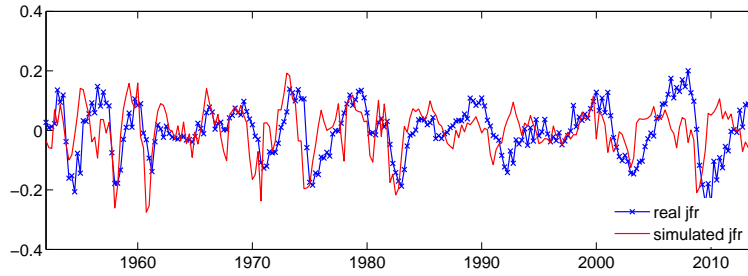


Figure 5: Real versus simulated job-finding rate. Both series are log deviations from hp-trend with $\lambda = 1600$.

Figure 6 plots the simulated job-finding rate against the backed-out matching efficiency from the simulation. Again, the backed-out matching efficiency is defined as the ratio between the simulated job-finding rate and the prediction of the job-finding rate based on a standard Cobb-Douglas matching function, i.e. $\theta^{0.45}$. The coefficient 0.45 is the total elasticity of the job-finding rate with respect to market tightness in the model (see calibration section). We observe an interesting phenomenon. In bad times, both the job-finding rate and the matching efficiency go down. The intuition is straightforward: In times of low productivity, firms decrease hiring and the job-finding rate goes down. Only part of the decrease in hiring is achieved through vacancy posting, in addition firms select less workers for a given number of contacts. This second effect creates the impression that matching efficiency decreases in times of recessions. In deep recessions, the backed-out matching efficiency drops by 4 to 6 percent. This mechanism could therefore explain why the Beveridge curve seems to shift in recessions. In contrast to an exogenous shift in the matching efficiency, our mechanism would not imply a structural deterioration of the labor market. Only downward movements that go beyond those 4 to 6 percent would indicate a decline of the exogenous matching efficiency.

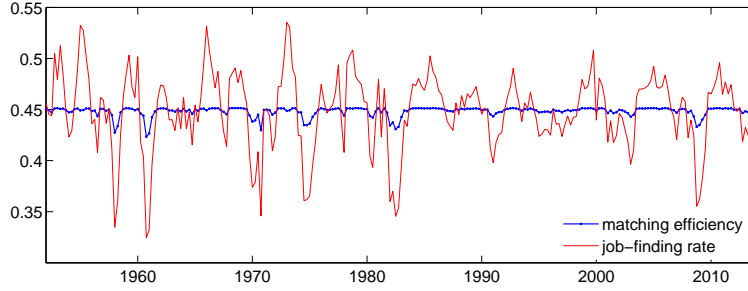


Figure 6: Simulated job-finding rate and backed-out matching efficiency.

6.2 Inspecting the Mechanism

To illustrate the driving forces for business cycle asymmetries and the shift of the matching efficiency, Figure 7 shows the fully nonlinear deterministic impulse response function of the model economy to a 3% positive and a 3% negative productivity shock. These exercises are meant to replicate severe recession and boom scenarios.

Although the aggregate shocks are symmetric, the job-finding rate moves by almost 50% more in case of the negative productivity shock (amplitude reaction of 28 versus 20 percent). Thus, unemployment rises by roughly 25% in case of the negative shock at maximum, while it only drops by 13% in case of a positive shock.

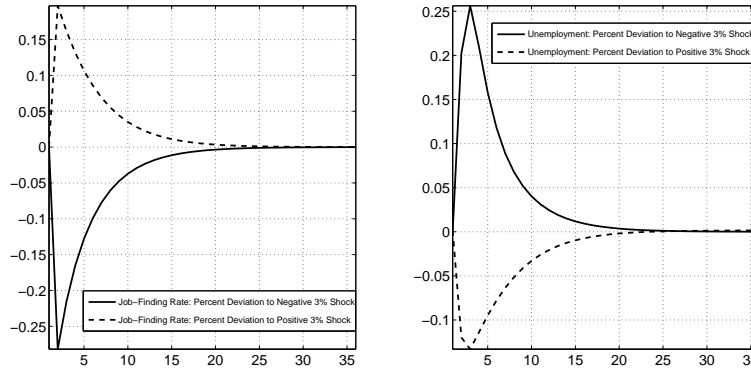


Figure 7: Impulse responses of job-finding rate and unemployment to positive and negative productivity shock.

This exercise illustrates how powerful the nonlinearities in the selection part of the model are. A negative aggregate shock moves the selection cutoff point to

a thicker part of the idiosyncratic productivity density function. Thus, a given movement of the cutoff point (via the aggregate productivity shock) exerts a larger effect on the job-finding rate.

Figure 8 shows how these two aggregate shocks affect the backed-out matching efficiency. The backed-out matching efficiency drops sharply by about 6 percent in the recession scenario. Thus, through the lens of search and matching it appears that the ability of the matching function to create new jobs has deteriorated substantially. However, the real driving force is again the nonlinearity of the selection part of the model. Since the economy moved to a thicker part of the density distribution, the job-finding rate drops more in a recession.

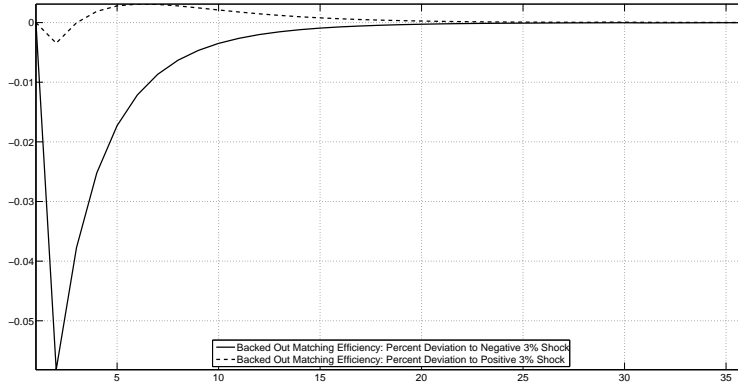


Figure 8: Impulse response of backed-out matching efficiency to positive and negative productivity shock.

6.3 Why these Nonlinearities Matter?

The nonlinearities due to the idiosyncratic shock distribution do not only create asymmetric business cycles, but they also matter a lot for the effectiveness of policy interventions. To illustrate this point, we assume that the government implements a subsidy of 1% of productivity for all existing worker-firm pairs to stimulate the economy. The subsidy program exists for half a year and is abandoned afterwards.

When the government implements the subsidy, the present value of workers increases (although part of the subsidy is transferred to workers via wage bargaining), vacancy posting rises and the selection rate increases. To illustrate the nonlinearities, we show how the effects of the subsidy differ when it is implemented at the beginning of a recession and at the beginning of a boom (corresponding to the 3% productivity shocks from above).

Figure 9 shows the effects of the wage subsidy in the boom and recession scenario. Interestingly, the effects of the subsidy on unemployment is roughly

three times as large during the recession than during the boom. The intuition is straightforward: The selection cutoff point is at a thicker part of the idiosyncratic density function in the recession. Thus, the government intervention has a larger effect because a given change in the present value moves the selection rate and thereby unemployment by more.

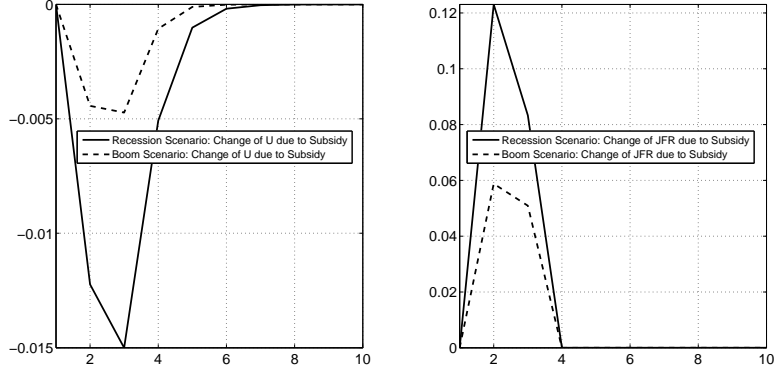


Figure 9: Subsidy

We have also experimented with a New Keynesian version of our model. In contrast to our baseline model, government spending would have non-zero effects on output in a New Keynesian setting (due to sticky prices). When the government implements the fiscal stimulus in a recession, government spending multipliers are also roughly three times larger than in a boom. Thus, our model framework provides a rationale for time varying fiscal multipliers.

7 Connection to the Literature

Our paper is connected to the literature on business cycle asymmetries and to the literature on shifts of the Beveridge curve. This section provides a brief praise of the most relevant papers and puts our contribution in context.

McKay and Reis (2008) show that contractions in the United States are briefer and more violent than expansions. They propose a model with asymmetric employment adjustment costs and a choice when to replace old technologies to account for these facts. Abbritti and Fahr (2013) show for various OECD countries that unemployment is skewed over the business cycle, while vacancies are not. They are able to explain the unemployment asymmetries with a model of asymmetric wage adjustment costs (i.e. nominal wage cuts are more costly than wage increases). While the asymmetry of (un)employment is well-known, we document that the job-finding rate is very skewed in the United States. We are the first to propose idiosyncratic productivity shocks to be the driving source for this phenomenon.

Search and matching models without idiosyncratic shock for match formation are also able to generate asymmetries of the job-finding rate. However, they either need to rely on some form of wage rigidity (as e.g. Petrosky-Nadeau and Zhang (2013) who use a Hagedorn and Manovskii (2008) calibration that generates mildly rigid wages) or asymmetric wage adjustment (as Abbritti and Fahr (2013)). Our paper is the first to propose a way of generating asymmetries of the job-finding rate without any wage rigidity. The asymmetry of the job-finding rate is driven by the curvature of the idiosyncratic productivity shock. Thus, we provide an additional complementary mechanism which is useful given that the debate on wage rigidities is still not resolved (Haefke et al., 2013).

The debate on shifts of the matching efficiency was spurred by the Great Recession. Diamond and Sahin (2014) show that Beveridge curve shifts occurred during most post-war recessions in the United States. Barnichon and Figura (2014) discuss reasons for the procyclicality of the matching efficiency (in particular during the Great Recession). In contrast to us, Barnichon and Figura (2014) do not point towards the positive correlation between the job-finding rate and matching efficiency (but they discuss the systematic correlation between the business cycle and the matching efficiency). Barnichon and Figura (2014) also use heterogeneities as an explanation for shifts in the Beveridge Curve. However, in contrast to us they use systematic differences in search efficiency and labor market segmentation as driving forces, while we use idiosyncratic shocks for match formation. Thus, their theoretical explanation is complementary to ours.

Sedláček (2014) is closest to ours in terms of the shifts of the matching efficiency. He also shows that match efficiency is procyclical and proposes a model with idiosyncratic productivity shocks for both new matches and incumbent workers and firing costs to account for this. Our paper is different in several dimensions: First, we solve the full nonlinear structure of the model, while Sedláček (2014) linearizes the model in order to estimate it. This allows us to also discuss business cycle asymmetries and connect them to shifts of the matching efficiency. Sedláček (2014) estimates a model without idiosyncratic shocks and then calibrates the model with idiosyncratic shocks. Kohlbrecher et al. (2014) show that idiosyncratic shocks for match formation change the elasticity on vacancies/market tightness in the estimated matching function. In other words, if idiosyncratic shocks matter for match formation, the elasticity of matches with respect to vacancies is not the same as the coefficient in the theoretical Cobb-Douglas matching function. We use this knowledge in our paper and show that even if this is taken into account, there are additional shifts of the matching efficiency that are due to nonlinearities. The connection between the shape of the idiosyncratic shock at the cutoff point and the shape of the estimated matching function is very useful in our context. Empirically, the elasticity of the matching function with respect to vacancies is smaller than 0.5 (see our estimations in Section 2 and Petrongolo and Pissarides (2001)). This is only the case in a model with idiosyncratic productivity shocks if the cutoff point is to the left of the peak of the density function. Interestingly, this is exactly the condition we require in order to generate the business cycle asymmetries as

found in the data (see Section 4).

8 Conclusion

This paper shows that the US labor market reacts in asymmetric fashion to business cycle fluctuations. The job-finding rate moves a lot more in recessions than in booms. In addition, through the lens of a standard search and matching model, the matching efficiency appears to drop in recessions (i.e. the Beveridge curve shifts). We provide an explanation for both phenomena by enhancing a standard search and matching model with a labor selection mechanism and by solving for the full nonlinear structure. In recessions, the model economy moves to a thicker part of the idiosyncratic productivity density function and thereby generates strong nonlinear effects. In this nonlinear environment, policy interventions have quantitatively very different effects in booms and recessions.

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A Data

Table A.1: Data

Variable	Description	Source
Output	Real Gross Domestic Product in 2005 Dollar	NIPA-tables (FRED) ¹
Job-finding rate	Job finding probability for unemployed workers	BLS (FRED) – Calculation as in Shimer (2012)
Unemployment	Unemployed	BLS (FRED)
Employment	Civilian Employment	BLS (FRED)
Vacancies	Composite Help-wanted Index	from Barnichon (2010) ²
Separation rate	Exit probability for employed workers	BLS (FRED) – Calculation as in Shimer (2012)
Industrial Production	Industrial Production Index	Federal Reserve Board (FRED)

¹ FRED: Federal Reserve Bank of St. Louis Economic Database
(<http://research.stlouisfed.org/fred2/>)

² Barnichon’s Composite Help-wanted Index
(<https://sites.google.com/site/regisbarnichon/research>)

B US Wage Distribution

Figure B.1 shows the distribution of real wages for a homogeneous group of new hires. The nominal wages of new hires are obtained from Haefke et al. (2013) and span from April 1989 to March 2004. They are deflated using the CPI and adjusted for year and month fixed effects. As we are not interested in wage differentials due to observables we restrict our attention to a group of new hires with the following characteristics: male, white, full time worker (> 35 hours), age 25-55, and more than 12 years of education.

The wage distribution shows that there is a substantial fraction of wages to the left of the peak of the density function. Thus, if the wage distribution (controlling for observables) is a good proxy for idiosyncratic shocks (which would be the case under standard Nash bargaining), this provides indirect evidence in favor of a cutoff point to the left of the peak of the idiosyncratic productivity function.



Figure B.1: Real wages of new hires.

C Analytical Appendix

We have derived a search and matching model with labor selection in the main part. To illustrate that the nonlinearities are driven by the selection part, we separate our model into two analytical parts. First, we analyze the role of nonlinearities and the matching efficiency in a search and matching model without labor selection. Second, we do the same for a selection model with degenerate contact function.

C.1 Search and Matching Model

In steady state, the search and matching model can be expressed in terms of the job creation condition

$$p = \left(\frac{a - w}{\kappa (1 - \delta (1 - \phi))} \right)^{\frac{\gamma}{1-\gamma}}, \quad (16)$$

a wage equation and the employment equation.

C.1.1 Business Cycle Asymmetries

The ability of a pure search and matching model to generate nonlinearities depends crucially on the wage formation mechanism. To illustrate this, let's assume that wages and productivity comove proportionally. In this case:

$$p = \left(\frac{(1 - \alpha) a}{\kappa (1 - \delta (1 - \phi))} \right)^{\frac{\gamma}{1-\gamma}} \quad (17)$$

Thus:

$$\frac{\partial \ln p}{\partial \ln a} = \frac{\gamma}{1 - \gamma}. \quad (18)$$

In this case, the second derivative of the elasticity of the job-finding rate with respect to productivity is zero

$$\frac{\partial \ln p}{\partial \ln a \partial a} = 0, \quad (19)$$

i.e. the search and matching model reacts in symmetric fashion to positive and negative productivity shocks. Under wage rigidity, search and matching models are able to generate asymmetric reactions to productivity (unless $\gamma = 0.5$, which would again create symmetric reactions of the job-finding rate).

C.1.2 Shifts of the Matching Efficiency

By definition, in a pure search and matching model the economy operates along a stable contact function. Thus, shifts in the matching efficiency are exogenous events that signal a change of the ability of the labor market to create jobs, which can be interpreted as a different level of structural unemployment.

C.2 Selection Model

In steady state, a selection model can be described by four equations (the wage equation¹⁰, the cutoff point, the selection rate, and the vacancy free entry condition) and the employment equation.

$$w(\varepsilon_i) = \bar{w} + \alpha \varepsilon_i \quad (20)$$

with

$$\bar{w} = \omega(a, \eta, \theta, x), \quad (21)$$

$$\tilde{\varepsilon} = \frac{\bar{w} - a}{(1 - \delta)(1 - \phi)} + \alpha \tilde{\varepsilon}, \quad (22)$$

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) d\varepsilon, \quad (23)$$

$$\theta = \frac{p\eta}{\kappa} \left(\frac{a - \bar{w}}{1 - \delta(1 - \phi)} + \frac{(1 - \alpha) \int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} \right). \quad (24)$$

As shown by Kohlbrecher et al. (2014), in this model environment the comovement between job-finding rate and market tightness can be expressed as

¹⁰We assume that the wage has one part which does not depend on idiosyncratic components $\bar{w} = \omega(a, \eta, \theta, x)$ and which is a function of aggregate variables. In addition, the wage depends proportionally on idiosyncratic productivity shocks ($\alpha \varepsilon_i$). This allows us to show general results and to nest the Nash bargained wage as used in the main part of the paper.

$$\begin{aligned}
\frac{\partial \ln(p\eta)}{\partial \ln \theta} &= \frac{\frac{-f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} a}{\eta}}{\frac{\frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a}{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}}{\eta} - \tilde{\varepsilon}} \\
&= \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon} \right) \\
&= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}.
\end{aligned}$$

C.2.1 Business Cycle Asymmetries

The job-finding rate is

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) d\varepsilon \quad (25)$$

Thus, the first order derivative is

$$\frac{\partial \eta}{\partial a} = -f(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} > 0 \quad (26)$$

Given that $\frac{\partial \tilde{\varepsilon}}{\partial a} < 0$, this expression is larger than zero, i.e. larger productivity leads to more hiring.

Business cycle asymmetries can be expressed as

$$\frac{\partial^2 \eta}{\partial a^2} = -f'(\tilde{\varepsilon}) \frac{\partial \tilde{\varepsilon}}{\partial a} \frac{\partial \tilde{\varepsilon}}{\partial a} - f(\tilde{\varepsilon}) \frac{\partial^2 \tilde{\varepsilon}}{\partial a^2} \quad (27)$$

Let's assume the standard wage formation mechanism: $\bar{w} = \alpha(a + \kappa\theta) + (1 - \alpha)b$. In this case, $\tilde{\varepsilon} = \frac{\alpha(a + \kappa\theta) + (1 - \alpha)b - a}{(1 - \delta(1 - \phi))(1 - \alpha)}$, $\frac{\partial \tilde{\varepsilon}}{\partial a} = \frac{\alpha(1 + \kappa \frac{\partial \theta}{\partial a}) - 1}{(1 - \delta(1 - \phi))(1 - \alpha)}$ and $\frac{\partial^2 \tilde{\varepsilon}}{\partial a^2} = \frac{\alpha \kappa \frac{\partial^2 \theta}{\partial a^2}}{(1 - \delta(1 - \phi))(1 - \alpha)} > 0$.

Thus, if $f'(\tilde{\varepsilon}) > 0$,

$$\frac{\partial^2 \eta}{\partial a^2} < 0, \quad (28)$$

i.e. the job-finding rate reacts differently to upward and downward shifts of productivity.

If $f'(\tilde{\varepsilon}) > 0$, a positive productivity shock shifts the economy to a thinner part of the idiosyncratic distribution. By contrast, a negative productivity shocks shifts the economy to a thicker part of the idiosyncratic distribution. Thus, negative productivity shocks exert a larger effect on the selection rate and thus the job-finding rate.

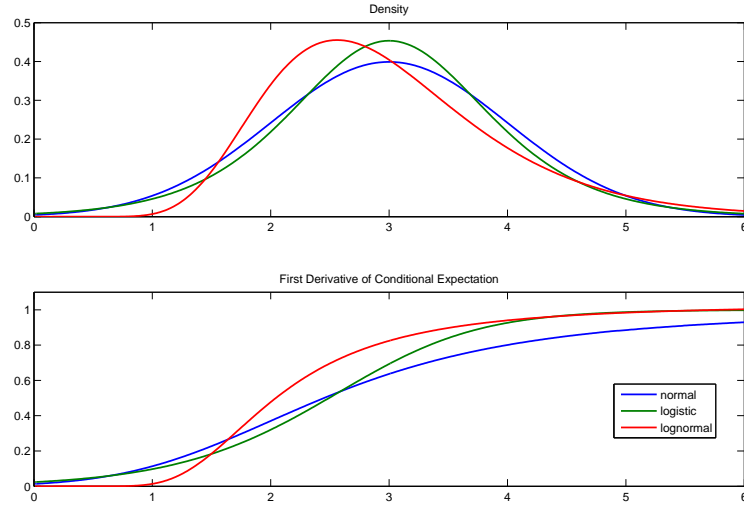
C.2.2 Shifts of the Matching Efficiency

In a pure selection model, the comovement between the job-finding rate and market tightness can be expressed as

$$\frac{\partial \ln(p\eta)}{\partial \ln \theta} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon} \right) = \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}. \quad (29)$$

Figure C.2 illustrates comovement between the job-finding rate and market tightness for different idiosyncratic shock distributions. The upper panel plots the density functions of ε for normal, logistic and lognormal distributions. The lower panel plots equation (29) evaluated at the corresponding cutoff point (on the abscissa). As shown above, this value corresponds to the elasticity of the job-finding rate with respect to market tightness, i.e. the implied weight on vacancies in an estimated matching function. When the cutoff point is at the left hand side of the peak of the density function, the first derivative of the conditional expectation (i.e. the weight on vacancies) is smaller than 0.5, while it is larger than 0.5 on the right hand side.

Figure C.2: Predicted matching coefficients for standard distributions



Notes: Density function and first derivative of conditional expectation for different standard distributions (namely, normal, logistic, and lognormal). For comparability reasons, the variance is normalized to 1 and the mean is set to 3 (the lognormal distribution requires a positive mean).

Figure C.2 illustrates that it is important to take the full nonlinear structure

of the model into account. Under a large negative productivity shock, the cutoff point moves to a thicker part of the distribution (i.e. to the right in the distribution). Or put differently, a larger positive productivity shock leads to a larger weight on vacancies in the backed-out matching function.

In a nutshell: the selection mechanism leads to a time varying backed-out matching function. In good times, the weight on vacancies falls and in bad times the weight on vacancies increases. If one looks at the data through the lens of a time-invariant matching function, it appears as if the matching efficiency drops in bad times. We show in our main part that this effect is present in the nonlinear simulation.