Learning by Employing: A Role for Up-or-Out Contracts*

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May 2007

Abstract

This paper provides a rationale for the use of up-or-out contracts. We consider a learning game in which a firm and a worker commonly observe noisy performance signals about the worker's ability. A worker's choice of effort affects the accuracy of the inference process about ability: the greater the effort, the more informative the output signals are. Output is assumed to be observable but nonverifiable, while effort can be either observable but nonverifiable, or unobservable. In particular, retention decisions based on performance cannot be part of a formal contract. We model contracts as a commitment, on the part of the firm, to a sequence of retention decisions and to a wage schedule conditional on employment. In this setting, contracts act as mechanisms to induce workers to generate nonverifiable information about their ability. We identify conditions under which an up-or-out contract is offered in equilibrium.

Keywords: Up-or-out contracts, Learning, Experimentation.

JEL codes: C73, D82, D83, L14, M12.

1 Introduction

In certain professional service industries, where human capital is a critical input to production, successful employees are awarded virtual permanent tenure after having spent some time working at a lower-paying job position. The use of tenure systems coupled with up-or-out clauses is widespread

*We are indebted to Steven Matthews, Kenneth Burdett, and Jan Eeckhout for their suggestions. We are also grateful to Hector Chade, Edward Schlee, V.V. Chari, and to participants in the 2006 European Meeting of the Econometric Society for their comments.

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in law, medicine, advertising and accounting partnerships, academic departments, and hospitals. Formally, up-or-out contracts are arrangements between a firm and a worker with the following two features. First, the firm commits to retain the worker for a pre-specified period. Second, the worker is considered for promotion only at the end of the probationary period. If promoted, he is granted permanent retention, usually at a higher and better-paying position.[1] Otherwise, he is permanently fired.

In an incomplete information framework, the commitment to a period of probationary employment together with the award of permanent retention upon promotion seems puzzling. Whenever a firm does not observe a worker's ability, and uncertainty about it persists over time, the possibility of firing a worker that performs unsatisfactorily is intuitively beneficial. When deciding whether to employ a worker, the firm has to take into consideration the value of new information about ability as well as the cost of employing a worker whose talent might be inadequate. As long as the firm correctly weighs the value of information against the risk of short-run losses, separation after bad performance may be profitable. With an up-or-out contract, the firm commits to forgo the possibility of firing a worker who is found incompetent.

In this paper, we present an economic rationale for the optimality of up-or-out contracts. We show that by committing to such a contract a firm can induce workers to generate nonverifiable information about their unobserved ability. The commitment to employ workers for a pre-specified number of periods, with an implicit promise of permanent retention only to the ones who perform best, stimulates workers to exert effort and produce output realizations which are on average more informative about their true productivity. This, in turn, allows the firm to make better informed employment decisions.

The model considered consists of a dynamic game between an infinitely-lived firm and a pool of finitely-lived workers. Workers can be of two ability levels, unobserved to both the worker and firm. The firm has the ability to commit to a schedule of retention decisions and wage payments. A worker can only be fired at a period when a retention decision is scheduled, and must be paid at least the corresponding contracted wage. In particular, the firm can commit to employ the worker for more than one period.

When employed, a worker decides whether to exert effort or not. Moreover, he always has the possibility of quitting the firm and collecting a fixed outside option. A worker's output is an imperfect signal of ability, and it is observable but non-verifiable: retention decisions based on performance cannot be part of a formal contract. Effort, on the other hand, can be either observable but not verifiable, or unobservable.

[1] Usually, the employer can dismiss the worker only in extreme cases of misbehavior on the part of the worker, for instance, severe moral misconduct or malpractice.

More specifically, a worker's output is a stochastic function of his ability and effort on the job. A higher ability worker is more likely to produce a higher output, regardless of his effort. Besides, for given worker's ability, effort increases the likelihood of higher output. However, effort is inefficient, that is, the extra output generated when effort is exerted is smaller than its cost.

Given this informational structure, we provide conditions under which a contract with one revision period, a standard up-or-out contract, is offered in equilibrium. In this equilibrium workers only exert effort before the retention decision is made, and are retained only if they produce sufficiently high output. Moreover, under the contract, a wage premium accrues to retained workers. The optimality of up-or-out contracts derives from the firm's willingness to incur the risk of retaining a low ability worker in order to assess more accurately a worker's ability early in his career.

There are a few related papers that provide an explanation for the use of up-or-out contracts. As in this paper, O'Flaherty and Siow [1992] interpret up-or-out contracts as a screening device, but their framework allows the firm to dismiss a promoted worker. This eliminates the trade-off between the improved accuracy of the employment decision and the risk of permanent retention of a low ability worker, which is our focus. Other papers rationalize up-or-out contracts as a mechanism to mitigate a hold-up problem involving a worker's investment in general or firm-specific human capital. Along this line, Kahn and Huberman [1988] motivate up-or-out contracts as a solution to a double static moral-hazard problem. By committing to set a wage higher than a worker's opportunity cost, the firm can induce the worker to invest in firm-specific human capital. The firm's incentive to permanently retain the worker is due to the fact that the worker becomes on average more productive for the firm. Waldman [1990] proves that in an environment in which both a worker's actual and potential employer observe a signal about the worker's productivity, up-or-out contracts provide an incentive to accumulate general human capital. In particular, the firm's retention decision induces competitive bidding for the worker's services by potential employers and this forces the firm to increase post-retention wages.[2] Levin and Tadelis [2004], instead, interpret up-or-out contracts as a commitment device to ensure product quality, when quality is only imperfectly observable in the output market. By dismissing workers who are not most talented, even if they might make a positive contribution to the firm's profit, a firm can commit to ensure a more efficient level of quality if public monitoring is imperfect.[3]

[2] Carmichael [1988] proposes an environment where up-or-out contracts, by guaranteing permanent tenure, provide incumbent workers with the appropriate incentives to select the best junior workers. Within a matching framework, Harris and Weiss [1984] analyze a two-tier job market with finitely-lived workers in which there is uncertainty about ability in the primary market. They show that workers who have reached a certain cumulative output record within a given age remain in the primary market until retirement. Those who do not, move permanently to the secondary market at or before this particular age.

[3] Bar-Isaac [2004] proposes an overlapping generation model of team production and collective reputation. In his

The paper is organized as follows. Section 2 presents the model. Section 3 discusses what we call the spot contracting game, when the firm cannot offer long term contracts. Section 4 analyzes the game in which the firm is allowed to commit to employment and compensation for more than one period. Section 5 finally concludes and discusses directions for future research. An appendix collects proofs that are omitted from the main text.

2 Basic Framework

Here we describe the firm (the principal) and the workers (the agents). We also state some assumptions that are maintained throughout the paper. Time is discrete and indexed by $t \in \mathbb{N}$. The firm is risk-neutral and infinitely lived. Its discount factor is $\delta \in (0,1)$ and it has an outside option that we set to zero.[4] Workers are risk-neutral as well, but, unlike the firm, they live for 3 periods only. We assume that their discount factor is the same as the firm's.[5] They also have an outside option, that we denote by U, where $U \geq 0$. The per-period payoffs of both the firm and the workers are normalized by $(1-\delta)$.

At the beginning of period 1 there is a finite number of workers of ages 1, 2, and 3 available to the firm. Moreover, at the beginning of every subsequent period, a new group of age 1 workers becomes available. In any given period, the firm knows the age of all the living workers. Each worker can be of two types, good/high (H) or bad/low (L). A worker's type is not know to both him and the firm, with ϕ_0 being the probability that a worker of age 1 is of the high type.

The firm can employ at most one worker at a time. If employed, a worker chooses whether to exert a costly effort or not. We denote a worker's choice of effort by $e \in \{\underline{e}, \overline{e}\}$, with $\underline{e}$ denoting no effort and $\overline{e}$ denoting effort. The cost of effort to the worker is $c > 0$. We consider two alternatives in this paper, either effort is observable (by the firm) but not verifiable, or effort is unobservable. It turns out that in both cases the same results are obtained.

When employed, a worker's flow of output is stochastic. The possible values to the firm of this output are $y_1 < y_2 < y_3$. If an employed worker chooses $\underline{e}$, the probability of producing y_3 is zero regardless of his type, while the probability of y_2 is $\alpha \in (0,1)$ if he is of the high type and $0 < \beta < \alpha$ if he is of the low type. If an employed worker of the high type chooses $\overline{e}$, he produces y_3 with probability $1-\gamma$ and y_2 with probability γ, where $\gamma \in (0,1)$. If this worker is, instead, of the low

framework promotion to partnership through an up-or-out mechanism provides a young agent with the incentive to exert effort, by rewarding success with the opportunity to take over the firm.

[4]The value of the firm's outside option plays no role in the analysis that follows as long as it is not too high, in which case the firm never hires any worker.

[5]This is not an innocuous assumption. It simplifies the analysis considerably, and presently we don't know how things change when we allow the firm and the workers to have different discount factors.

type, he produces y_3 with probability $0 < \lambda < 1-\gamma$, y_2 with probability β, and y_1 with probability $1-\lambda-\beta > 0$. The following table summarizes the workers' production technology.

Production Matrix

Ability	**Low**	**Effort**	($\underline{e}$)	**High**	**Effort**	($\overline{e}$)
	y_3	y_2	y_1	y_3	y_2	y_1
H	0	α	$1-\alpha$	$1-\gamma$	γ	0
L	0	β	$1-\beta$	λ	β	$1-\lambda-\beta$

Notice that with the above production technology, if a worker exerts effort and produces y_1, he is immediately revealed as a low type worker. It turns out that this assumption is not crucial for our analysis.[6] For this reason, we don't consider the more general case where the production of y_1 when the worker exerts effort does not reveal the worker's type.

Let $y(\phi, \underline{e})$ be such that

$$y(\phi, \underline{e}) = \phi[\alpha y_2 + (1-\alpha)y_1] + (1-\phi)[\beta y_2 + (1-\beta)y_1] = \phi(\alpha-\beta)(y_2-y_1) + \underline{y},$$

where $\underline{y} = \beta y_2 + (1-\beta)y_1$. Then $(1-\delta)y(\phi, \underline{e})$ is the expected per-period output of a worker when he exerts no effort and ϕ is the firm's belief that he is of the high type, the firm's belief for short.[7] We assume that $y(\phi_0, \underline{e}) - U > 0$, so that in any period the firm is better off by hiring an untried worker and paying him his outside option than by collecting her outside option. We also assume that effort is inefficient for both a good and a bad worker; that is, if $y(\phi, \overline{e})$ is the expected output realization of a worker when he chooses $\overline{e}$ and the firm's belief is ϕ, then

$$y(1, \overline{e}) - y(1, \underline{e}) = (1-\gamma)y_3 + \gamma y_2 - \alpha y_2 - (1-\alpha)y_1 < c$$

and

$$y(0, \overline{e}) - y(0, \underline{e}) = \lambda y_3 + \beta y_2 + (1-\beta-\lambda)y_1 - \beta y_2 - (1-\beta)y_1 = \lambda(y_3 - y_1) < c.$$

Notice that the first inequality implies that $c > (1-\alpha)(y_2 - y_1)$. The role of this assumption is to emphasize that effort exertion can be desirable not only because it leads to higher output net of the cost of effort exertion (a possibility we are ruling out), but also because it leads to output realizations that are more informative about a worker's ability. This second effect will become clear as we conduct out analysis in Section 4.

[6] We show this in Section 4, when the analyze the full commitment case.

[7] Throughout this paper we normalize lifetime payoffs by $1-\delta$.

3 The Spot Contract Game

In this section we describe and analyze what we call the spot contract game. We are concerned with its weak Perfect Bayesian Equilibria (weak PBE). It turns out that all such equilibria are outcome equivalent; that is, they all imply the same stochastic process over the set of possible firm/worker decisions and output realizations (the outcome space).

3.1 Description

In each period the firm either collects her outside option or offers a worker a wage w in exchange for participation in that period. Participation is verifiable, as well as the wage offer by the firm. The worker then decides whether to participate or not. If he chooses not to participate, he collects his outside option and the firm collects hers. If, on the other hand, the worker chooses to participate, he then makes his effort choice, and output is realized. After this realization, the firm pays the worker his promised wage and chooses whether to pay him a bonus or not. Negative bonus payments are not allowed. Note that when effort is observable, bonus payments contingent on effort exertion and/or output realization are possible, while if effort is unobservable, bonus payments can only be made contingent on output realization. In both cases the firm cannot commit ex-ante to any form of bonus payment.

Workers who are not offered a wage by the firm (and so have to collect their outside option) don't receive any information. In particular, the only way a worker can know if in a given period the firm is engaged in a relationship is if he is the one to receive a wage offer.

3.2 Characterization

Lemma 1. *All weak PBE of this game have workers always choosing $\underline{e}$ upon participation, the firm offering $w = U$ when she wants to induce participation, the workers accepting any wage offer greater than or equal to U, and no bonus payments after any possible history.*

Proof: We proceed by backward induction in the age of an employed worker. First notice that no bonuses are possible for a worker of age 3, as this is his last period of life, and so the firm always reneges on any bonus payment.[8] This is true whether effort is observable or not. Hence, if a worker of age 3 chooses to participate, he chooses no effort, as his future lifetime compensation is independent of his effort choice and output realization in the current period. Consequently, a

[8] The assumption that no workers outside the firm can observe what goes on inside the firm rules out the possibility of trigger strategies that could potentially lead to positive bonus payments.

worker of this age always accepts any wage offer greater than U. This implies that he also accepts any wage offer equal to U, for otherwise no equilibrium would exist.

Consider now a worker of age 2 that is employed by the firm. As before, no bonus payments are possible for this worker. To see why, first notice that this worker always accepts participation in his last period of life in case he is offered $w = U$ by the firm. Therefore, if the firm reneges on a bonus payment, this worker will not punish the firm in the following period by not accepting to participate (if the firm indeed wants him to participate). As such, this worker does not exert effort, since once more his lifetime compensation is independent of his effort choice and output realization in the present period. Consequently, a worker of age 2, like a worker of age 3, always accepts any wage offer greater than or equal to U.

To finish, consider a worker of age 1. The same reasoning as in the previous paragraph shows that no bonus payments are possible for a worker of this age, and so, for the same reason as before, this worker does not exert effort when employed. Consequently, a worker of age 1 always accepts a wage offer w that is not less than U. $\square$

Lemma 2. *The firm never recalls a worker in any weak PBE of the spot contract game.*

Proof: First notice that an untried worker of age k is at least as good to the firm as an untried worker of age $l > k$, with $l, k \in \{1, \ldots, 3\}$. With the younger worker, after $4-l$ periods the firm can choose between this worker and the best alternative available at that point, while with the older worker the firm is forced to choose the latter. Denote by W_1 the first worker that is dismissed by the firm and consider the second time in which the firm wants to discontinue her relationship with a worker. Denote by W_2 the worker the firm is employing at this point in time. From the previous paragraph we know that without any loss, we can assume that the firm has to choose between an age 1 worker and W_1 (assuming he is still alive). At this point in time, the latter is not better to the firm than he was when dismissed. Hence the firm will replace W_2 with an age 1 worker. An induction argument closes the proof.[9] $\square$

Note that the above two results are valid as long as the workers live for a finite number of periods. From Lemma 1 we know that no bonus payments are feasible, an employed worker never exerts effort, and the cheapest way to induce a worker to participate is to offer him $w = U$. Moreover, since $y(\phi_0, \underline{e}) > U$ by assumption, we know that the firm never collects her outside option. Therefore, because of Lemma 2, the problem of the firm consists in choosing, in each period, whether to pay $w = U$ to retain the worker she employed in the previous period, if there is such a worker, or offer

[9] Observe that it is irrelevant for this argument whether the firm fires a worker when she is indifferent between him and the best available alternative or not.

the same wage to an untried worker. If the firm decides for the second alternative, she also needs to choose the age of the untried worker. A consequence of this is that in any weak PBE of the spot contract game, the firm's lifetime expected payoff must be the same in any subgame where the firm hires a new worker.

Lemma 3. *The firm never hires untried workers of ages 2 or 3 in a weak PBE of this game.*

Proof: Suppose that it is optimal for the firm to hire an untried worker of age 3 at some point in time. Let V_0 denote the firm's lifetime payoff in any subgame where she hires a new worker. In particular, V_0 is the firm's lifetime payoff in the spot contract game. The previous paragraph implies that

$$V_0 = y(\phi_0, \underline{e}) - U.$$

Consider then the following strategy. The firm hires an untried worker of age 2 in the first period and retains him only if he produces y_2. Whether this worker is retained or not, afterwards the firm only hires an untried worker of age 3. Let $\phi^y(\phi_0, \underline{e})$ denote the firm's updated belief about an untried worker when he exerts no effort and produces $y \in \{y_1, y_2\}$ and let $p_1 = [\phi_0\alpha + (1-\phi_0)\beta]$ be the ex-ante probability that this worker produces y_2. The firm's lifetime payoff from this strategy is

$$(1-\delta)[y(\phi_0, \underline{e}) - U] + \delta p_1 \{(1-\delta)[y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U] + \delta V_0\} + \delta(1-p_1)V_0,$$

which is greater than V_0 since $\phi^{y_2}(\phi_0, \underline{e}) > \phi_0$ and $y(\phi, \underline{e})$ is strictly increasing in ϕ. Therefore it constitutes a profitable deviation, and so the firm never hires an untried worker of age 3.

Suppose now that it is optimal for the firm to hire an untried worker of age 2 at some period t. Since $\phi^{y_1}(\phi_0, \underline{e}) < \phi_0$, this worker is not retained from t to $t+1$ if he produces y_1, as an untried worker of age 3 in period $t+1$ is strictly better for the firm and does not alter continuation payoffs. We then have two possibilities. Either he is retained if he produces y_2 in period t or not. Since the second alternative corresponds to the case considered above we can, without loss, assume that the worker is retained from t to $t+1$ if he produces y_2. Use now V_1 to denote firm's lifetime payoff in any subgame where she hires a new worker. Then

$$V_1 = (1-\delta)[y(\phi_0, \underline{e}) - U] + \delta p_1\{(1-\delta)[y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U] + \delta V_1\} + \delta(1-p_1)V_1,$$

which implies that

$$V_1 = \frac{y(\phi_0) - U + \delta p_1[y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U]}{1 + \delta p_1} < y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U. \tag{1}$$

Consider then the following strategy for the firm. The firm hires an untried worker of age 1 in the first period and retains him for the rest of his life if he produces y_2. Once this worker leaves the firm, the firm only hires untried workers of age 2 and only retain such workers if they produce y_2 in their first period of employment. The payoff to the firm from such strategy is

$$V_1' = (1-\delta)[y(\phi_0, \underline{e}) - U] + \delta p_1\{(1-\delta^2)[y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U] + \delta^2 V_1\} + \delta(1-p_1)V_1,$$

which is greater than V_1 from (1). Hence the firm never hires an untried worker of age 2 as well. $\square$

We are now ready to establish the main result of this section, concerning the firm's retention decisions in any weak PBE of the game under consideration.

Theorem 1. *Suppose that $\alpha(1-\alpha) < \beta(1-\beta)$. Then in all weak PBE of the spot contract game the following two facts are true:* (i) *A worker is retained at the end of his first period of employment only if he produces y_2;* (ii) *A worker is retained at the end of his second period of employment only if he produces y_2 in that period.*

Proof: (i) Suppose that a newly hired worker produces y_1. Denote by t the period at which this happens. Since

$$\phi^{y_2}((\phi^{y_1}(\phi_0), \underline{e}), \underline{e}) = \frac{\alpha(1-\alpha)\phi_0}{\alpha(1-\alpha)\phi_0 + \beta(1-\beta)(1-\phi_0)} < \phi_0,$$

if this worker is retained by the firm, then, he is not going to be retained from $t+1$ to $t+2$ regardless of what he produces in $t+1$. An untried worker of age 3 in $t+2$ is better than him in flow terms and leads to the same continuation payoffs from $t+3$ on. Therefore this worker should not be retained from t to $t+1$, for otherwise the firm can do better by hiring an untried worker of age 3 in $t+1$ and reverting to the original course of play in $t+2$. We then have two alternatives, either the firm retains a newly hired worker that produces y_2 or not. From the reasoning in the proof of the last lemma we know that the second alternative is not possible, and so a newly hired worker should always be retained after producing y_2.

(ii) Since $\phi^{y_1}((\phi^{y_2}(\phi_0), \underline{e}), \underline{e}) = \phi^{y_2}((\phi^{y_1}(\phi_0), \underline{e}), \underline{e}) < \phi_0$, any worker that produces y_1 in his second period of employment is worse in his third and last period of life than an untried worker of age 3, and as such should not be retained. Once more from the proof of the previous lemma we know that any worker who produces y_2 in his second period of employment should be retained. $\square$

Let V^* be the lifetime payoff obtained by the firm in any weak PBE of the spot contract game. Simple algebra shows that if $\alpha(1-\alpha) < \beta(1-\beta)$, then

$$V^* = \frac{y(\phi_0, \underline{e}) - U + \delta p_1 \{y(\phi_1, \underline{e}) - U + \delta p_2[y(\phi_2, \underline{e}) - U]\}}{1 + \delta p_1(1 + \delta p_2)}, \tag{2}$$

where $\phi_1 = \phi^{y_2}(\phi_0, \underline{e})$, $p_2 = \phi_1\alpha + (1-\phi_1)\beta$, and $\phi_2 = \phi^{y_2}(\phi_1, \underline{e})$. To finish, simple algebra shows that V^* is strictly greater than $y(\phi_0, \underline{e}) - U$.

4 The Full Commitment Case

In this section we consider what happens when the firm can commit to what we call long-term contracts. A long-term contract is a list $C = \{w_i, T_i\}_{i=1}^k$, where $k \leq 3$, $w_i \geq 0$ and $\sum_{i=1}^k T_i = A$, the age of the worker to whom C is offered. The wages w_1 to w_k are what we call the committed wages, and we refer to $j \in \{1, \ldots, k\}$ as the j^{th} probationary period. If the list $\{w_i, T_i\}_{i=j}^k$, with $j \geq 2$, is non-empty, we refer to it as a continuation contract.

When the firm offers a contract $\{w_i, T_i\}_{i=1}^k$ to a worker and he accepts this contract, she is committed to the following: (i) For T_1 periods the firm pays the worker, upon participation, a wage of no less than w_1; (ii) If at the end of the first T_1 periods the continuation contract $\{w_i, T_i\}_{i=2}^k$ is non-empty, the firm must decide whether to retain the worker or not. If she retains the worker, she pays him, upon participation, a wage of at least w_2 for T_2 periods; (iii) If at the end of the first $T_1 + T_2$ periods the continuation contract $\{w_3, T_3\}$ is non-empty, then once more the firm has to decide whether to retain the worker or not. If she decides for retention, she pays him, upon participation, a wage of at least w_3 for the rest of the worker's life.

A **standard up-or-out contract** is a list $\{w_1, T_1 = 1, w_2, T_2 = 2\}$. By definition, such a contract can only be offered to (necessarily) untried workers of age 1. Any worker that accepts such a contract is said to be granted **tenure** if he is retained at the end of the first probationary period. For simplicity, we refer to the first probationary period of a standard up-or-out contract as the probationary period only.

Notice that in the class of contracts described above, rules governing the retention decision or decisions of the firm are not part of the contract. Otherwise, this would be equivalent to allowing for output contingent contracts, and in this case we know that the firm is able to induce effort unless the worker she employs is of age 3. Notice as well that we impose that any long-term contract the firm offers to a worker must specify the terms of their relationship for the worker's entire lifetime. In what follows we assume that no committed wage can be smaller than the worker's outside option; that is, no long-term contract $\{w_i, T_i\}_{i=1}^k$ is possible with $w_i < U$. We refer to this assumption as limited liability. At the end of this section we consider what happens when we drop this restriction.

4.1 The Game

We now describe in the detail the game between the firm and the workers when the firm can offer the long-term contracts described above. We refer to this game as the full commitment game. There are three types of periods t:

1. $t > 1$, the firm employed a worker in $t - 1$, and no retention decisions have to be made in this period. Hence the firm must employ in t the same worker employed in $t - 1$. In this case, like in the spot contract game, the firm offers a wage w to the worker. Since t is, for some $j \in \{2, 3\}$, the j^{th} probationary period of the worker the firm is currently employing, it must be that $w \geq w_j$. After this the worker chooses whether to participate or collect his outside option (forcing the firm to collect her outside option as well). In other words, employment is at will. If the worker participates, he then chooses whether to exert effort or not. Output is realized and the firm pays a non-negative bonus to the worker.

2. Either $t = 1$ or t is such that in $t - 1$ the firm did not employ a worker. In such periods the firm decides whether to offer a contract C to a worker or collect her outside option. In the first case, the worker that is offered C decides whether to accept it or not. If he rejects the contract, both the worker and the firm collect their respective outside options. If he accepts, then the timing of moves is as in the previous item.

3. The period t is one where a retention decision has to be made. If the firm decides to retain the worker she employed in the previous period, the timing of the moves is as in **1**. If, on the other hand, the firm decides not to retain this worker, the timing of moves is as in **2**.

Similarly to the Spot Contract Game, workers who are not offered a contract by the firm don't receive any information.

4.2 Characterization

We start by defining a (time-invariant) anonymous strategy profile. A strategy profile for the full commitment game is said to be **anonymous** if it satisfies the following three conditions: (i) The workers play symmetric strategies; (ii) Whenever the firm has the chance to offer a contract, she either always offers the same contract $C = \{w_i, T_i\}_{i=1}^k$ or she always collects her outside option; (iii) The contingencies that lead to a worker's retention at the end of his i^{th} probationary period, $i \in \{1, \ldots, k\}$, are the same. In particular, in a anonymous strategy profile, the retention decisions for any worker can only depend on his previous output realizations and, in case effort is observable, on his previous effort choices.

Since $y(\phi_0, \underline{e}) - U > 0$ by assumption, the anonymous strategy profile where the firm collects her outside option in every period cannot be an equilibrium. So we can assume, without any loss, that in an anonymous strategy profile the firm always offers the same contract whenever possible. Notice also that the restriction that $\sum_i T_i = A$, the age of the worker to whom C is offered, implies that in any such profile only workers of a certain age are offered contracts. Moreover, since there are only untried workers in the first period, only they are offered contracts. The final observation we make about anonymous strategy profiles is that in such profiles the firm's lifetime payoff at any subgame that begins when the firm has to offer a contract, including the game itself, is the same. This fact plays a central role in what follows.

In the remainder of this subsection we establish the main result of the paper, namely, that under certain assumptions all weak PBE in anonymous strategies of the full commitment game are such that: (a) The firm offers a standard up-or-out contract whenever she has the chance; (b) Any worker that accepts this contract exerts effort in the probationary period; (c) A worker is granted tenure only if he produces y_3 in the probationary period; (d) Once granted tenure, the worker exerts no effort. The approach we employ to establish this result is, through a long sequence of lemmas, to rule out as weak PBE all anonymous strategy profiles for which conditions (a) to (d) above are not satisfied. Given that all statements that follow are for weak PBE equilibria, from now on we omit the weak PBE qualification when talking about equilibria of the game under consideration.

STEP 0: We first establish two auxiliary results that will be used several times in what follows. The first one is an analogue of Lemma 1 for the full commitment game. The proof is identical.

Lemma 4. *The following holds in all equilibria of the game under consideration: (i) No bonus payments are made after any history; If a wage is paid in a given period, it is equal to the wage prescribed by the prevailing contract for that period.*

Lemma 5. *Let V^{**} be the the firm's lifetime payoff in an equilibrium (not necessarily in anonymous strategies) of the full commitment game. Then V^{**} is no less than V^{*} given by (2).*

Proof: Notice first that any worker of age 1 always accepts $C = \{w_1 = U, T_1 = 1, w_2 = U, T_2 = 1, w_3 = U, T_3 = 1\}$. Moreover, if the firm offers C, no worker that accepts it ever exerts effort when employed, since his continuation payoff is independent of his effort choice. Consequently, if the firm always offers C when possible and employs the retention decision described in Theorem 1, she obtains a lifetime payoff of V^{*}. Hence the desired result holds, otherwise a profitable deviation for the firm is possible. □

STEP 1: The next step consists in showing that the only possible contracts in an anonymous equilibrium are the ones with $T_1 = 1$; that is, the ones that stipulate a first probationary period that is one period long. Since in any anonymous strategy profile only untried workers are offered a contract, from now on all workers are assumed to be untried. We begin with the $T_1 = 3$ case.

Lemma 6. *There is no anonymous equilibrium of the full commitment game where contracts with $T_1 = 3$ are offered.*

Proof: Suppose σ is an anonymous strategy profile where contracts with $T_1 = 3$ are offered. From Lemma 4 any worker that accepts this type of contract exerts no effort and receives w_1. Therefore, the firm's lifetime payoff from σ is at most $V_0 = y(\phi_0, \underline{e}) - U$, since $w_1 \geq U$, and so cannot be an equilibrium by Lemma 5. $\square$

We now consider the $T_1 = 2$ case. We first establish two preliminary results. The first one is an immediate corollary to the proof of the last lemma. It implies that we only need to deal with the case where contracts with $T_1 = 2$ are offered to age 1 workers.

Corollary 1. *There is no anonymous equilibrium of the full commitment game where contracts with $T_1 = 2$ are offered to age 2 workers.*

Lemma 7. *The firm's retention decision in any anonymous equilibrium where a contract with $T_1 = 2$ is offered to age 1 workers is a cutoff rule. In other words, there exists a cutoff belief ϕ_R such that the firm retains the worker at the end of the first probationary period if, and only if, her belief is not less that ϕ_R. Moreover, $\phi_R > \phi_0$.*

Proof: Let σ be an anonymous strategy profile where a contract C with $T_1 = 2$ is offered to age 1 workers. Denote by V' the firm's lifetime payoff at any subgame that begins when she offers C and by ϕ her belief about a worker to whom C is offered at the end of his fist probationary period. By Lemma 4, this worker exerts no effort if retained. Hence he is retained if, and only if,

$$(1-\delta)[y(\phi, \underline{e}) - w_2] + \delta V' \geq V',$$

where w_2 is the post-retention wage, and so the firm's retention decision is indeed a cutoff rule. Moreover, by Lemma 5, V' must be no less than V^*, so that

$$y(\phi_R, \underline{e}) > y(\phi_0, \underline{e}) + w_2 - U.$$

Since $w_2 \geq U$ by limited liability, we can then conclude that ϕ_R must be bigger than ϕ_0.[10] $\square$

We now introduce some notation. Suppose that a worker hired by the firm for two periods in a row chooses $e_i \in \{\underline{e}, \overline{e}\}$ and produces $y_{j_i} \in \{y_1, y_2, y_3\}$ in his i^{th} period of employment, $i = 1, 2$. If ϕ is the firm's initial belief about this worker, we denote her updated belief at the end of these two periods of employment by $\phi^{y_{j_1}, y_{j_2}}(\phi, e_1, e_2)$.

Lemma 8. *There exists $\underline{\gamma}$ such that if $\gamma \leq \underline{\gamma}$, then in any anonymous equilibrium where a contract with $T_1 = 2$ is offered and effort is exerted with positive probability in both periods of the first probationary period, retention only happens if y_3 is produced twice. Moreover, effort is exerted in the second period of the first probationary period only after y_3 is produced in the first period.*

[10] Notice that the result just obtained holds even without limited liability, since no worker that accepts C ever stays in the firm in his last period of life if $w_2 < U$. Consequently, the case where $w_2 < U$ reduces to the one covered by Corollary 1.

Proof: Let $\underline{\gamma}$ be the smallest solution to the equation $\gamma(1-\gamma) = \beta\lambda$. If $\gamma \leq \underline{\gamma}$, then $\underline{\gamma}(1-\underline{\gamma}) < \beta\lambda$, and so

$$\phi^{y_3,y_2}(\phi_0, \overline{e}, \overline{e}) = \frac{\gamma(1-\gamma)\phi_0}{\gamma(1-\gamma)\phi_0 + \beta\lambda(1-\phi_0)} \leq \phi_0.$$

Hence, as a consequence of Lemma 7, retention at the end of the first probationary period does not happen if y_3 is produced in the first period of employment and y_2 is produced in the second. Because $\phi^{y_2,y_3}(\phi_0, \overline{e}, \overline{e}) = \phi^{y_3,y_2}(\phi_0, \overline{e}, \overline{e})$, a worker that produces y_2 in the first period of the first probationary period has no incentives to exert effort in the second period, since he is not retained even if y_3 is produced. To finish, notice that retention must happen if y_3 is produced twice, otherwise there are no incentives for effort exertion, in which case a profitable deviation for the firm is possible by Theorem 1. □

Lemma 9. *Suppose that $\gamma \leq \underline{\gamma}$. In this case there is no anonymous equilibrium of the full commitment game that involves:* (i) *A contract C with $T_1 = 2$ being offered;* (ii) *The worker that accepts C exerting, with positive probability, effort in both periods of the first probationary period.*

Proof: Consider an anonymous strategy profile such that (i) and (ii) in the above statement are satisfied. From Lemma 8, we know that any worker who accepts C exerts effort in his second period of employment only if he produces y_3 in the first one, and he is retained only if he produces y_3 in this period as well. Because of limited liability, C must have $w_1 = U$. Hence a worker that produces y_1 or y_2 in his first period of employment stays in the firm for one more period (without exerting effort). Consider then the following deviation by the firm:

A. Offer $\{w'_1 = U, T_1 = 1, w'_2 = U, T_2 = 1, w'_3 = w_2, T_3 = 1)$ in period 1. Denote the worker to whom C is offered by W;

B. Retain W from period 1 to period 2 only if he produces y_3. Otherwise, offer $\{w''_1 = U, T_1 = 1, w''_2 = U, T_2 = 1\}$ to an untried worker of age 2 and never retain this worker from period 2 to period 3;

C. If W is retained from period 1 to period 2, retain him from period 2 to period 3 only if he produces y_3 in period 2;

D. Behave, from period 4 on if no dismissal occurs in period 3 and from period 3 on if it does occur, in the same way as specified by the original strategy.

It is straightforward to see that this deviation is profitable for the firm, and so the desired result indeed holds. □

Lemma 10. *There is no anonymous equilibrium of the full commitment game that involves:* (i) *A contract C with $T_1 = 2$ being offered;* (ii) *The worker that accepts C exerts effort in the first period and no effort in the second period of his first probationary period.*

As in the previous lemma, the fact that a worker who produces y_1 in his first period of employment remains in the firm for one more period allows the firm a profitable deviation. The details can be found in the Appendix.

Lemma 11. *Suppose that $\alpha(1-\alpha) < \beta(1-\beta)$. There is no anonymous equilibrium of the full commitment game that involves:* (i) *A contract C with $T_1 = 2$ being offered;* (ii) *The worker that accepts C exerting no effort in the first period of his first probationary period.*

The proof of this result can be found in the Appendix. Once more it relies on the fact that if (i) is satisfied, then a worker who produces y_1 in his first period of employment remains in the firm for one more period.

STEP 2: The previous step established that only the anonymous strategy profiles where the firm offers contracts with $T_1 = 1$ can be an equilibrium of the full-commitment game. In what follows we show that all anonymous strategy profiles where contracts with $T_1 = 1$ are offered to workers of age greater than 1 cannot be an equilibrium. The first result is obvious.

Lemma 12. *There is no anonymous equilibrium of the full commitment game where a contract with $T_1 = 1$ is offered to a worker of age 3.*

Before we can establish the second result, we need some results about anonymous strategy profiles where a standard up-or-out contract is offered.

Lemma 13. *There is no anonymous equilibrium of the full commitment game where:* (i) *A standard up-or-out contract is offered;* (ii) *The worker that accepts it exerts no effort in the probationary period.*

Proof: This follows immediately from limited liability together with Theorem 1 and Lemma 4. □

Consequently, standard up-or-out contracts are only possible in an anonymous equilibrium if any worker who accepts them exerts effort in his first period of employment. This, however, can only happen if the difference between w_2, the tenure wage, and U is big enough to compensate the worker for his effort during the probationary period. Since the firm's retention decision is an equilibrium decision as well, we must check that the tenure wages that induce effort are not so high that the firm finds it too expensive to grant tenure.

Consider then an anonymous strategy profile where the firm offers a standard up-or-out contract and any worker who accepts it exerts effort during the probationary period. As in the proof of Lemma 7, it is possible to show that the firm's tenure decision must be a cutoff rule, with the cutoff belief not lower than ϕ_0. Since $\phi^y(\phi_0, \overline{e}) < \phi_0$ if $y \in \{y_1, y_2\}$, we can then conclude that a worker is granted tenure only if he produces y_3 during the probationary period. We refer to such a strategy profile as a standard up-or-out profile. The next two results show that under certain parameter restrictions there are standard up-or-out profiles that are feasible; i.e., the workers' effort choice in the probationary period and the firm's retention decision are both incentive compatible.

Lemma 14. *Suppose that effort is unobservable and $\alpha > (3+\beta)/4$. There exist $(1-\alpha)(y_2-y_1) < \overline{c}$, $\underline{\gamma}' \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $\underline{\lambda} \in (0,1)$, $\underline{\delta} \in (0,1)$, and $\underline{y_3} > y_2$ such that if $\gamma < \underline{\gamma}'$, $\lambda < \underline{\lambda}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta > \underline{\delta}$, and $y_3 \in (y_2, \underline{y_3})$, then there are standard up-or-out profiles that are feasible when $c \in ((1-\alpha)(y_2-y_1), \overline{c})$.*

Proof: Let σ be a standard up-or-out profile. Since effort is unobservable, a worker that deviates during the probationary period and exerts no effort is never retained. Hence the IC constraint for the worker's effort decision during the probationary period is

$$\delta(1+\delta)p_T[w_2 - U] \geq c \quad \Leftrightarrow \quad w_2 - U \geq \frac{c}{\delta(1+\delta)p_T}, \tag{3}$$

where $p_T = [\phi_0(1-\gamma) + (1-\phi_0)\lambda]$ is the probability of tenure under σ. The term on the left of the first inequality is the worker's net gain from effort exertion.

Let V be the firm's lifetime payoff from σ. Straightforward algebra shows that

$$V = \frac{y(\phi_0, \overline{e}) - w_1 + \delta(1+\delta)p_T[y(\hat{\phi}, \underline{e}) - w_2]}{1 + \delta(1+\delta)p_T},$$

where $\hat{\phi} = \phi^{y_3}(\phi_0, \overline{e})$. Since a worker accepts a standard up-or-out contract only if

$$w_1 + \delta(1+\delta)p_T w_2 + \delta(1+\delta)(1-p_T)U \geq U + \delta(1+\delta)U - c, \tag{4}$$

an upper bound for V is

$$\overline{V} = \frac{y(\phi_0, \overline{e}) - U - c + \delta(1+\delta)p_T[y(\hat{\phi}, \underline{e}) - U]}{1 + \delta(1+\delta)p_T} = y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + \delta(1+\delta)p_T}.[11]$$

Therefore, the firm grants tenure to a worker after he produces y_3 in the first period only if

$$(1-\delta^2)[y(\hat{\phi}, \underline{e}) - w_2] + \delta^2\overline{V} \geq \overline{V} \quad \Leftrightarrow \quad y(\hat{\phi}, \underline{e}) - w_2 \geq y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + \delta(1+\delta)p_T}.$$

[11] Observe that under limited liability (4) is automatically satisfied once (3) is satisfied.

Rearranging terms in the last inequality, we can then conclude that the firm's retention decision is IC compatible if

$$w_2 - U \leq \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + \delta(1+\delta)p_T}. \tag{5}$$

We can then conclude that if

$$\frac{c}{\delta(1+\delta)p_T} < \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + \delta(1+\delta)p_T} \tag{6}$$

there exists a tenure wage w_2 such that both (3) and (5) are satisfied. Observe that the above equation is obviously satisfied if we take c sufficiently small. However, effort exertion is assumed inefficient for both the good and the bad workers, and so we cannot simply take c as small as we want. We must also have

$$(1-\gamma)y_3 + \gamma y_2 - \alpha y_2 - (1-\alpha)y_1 = (1-\gamma)(y_3 - y_2) + (1-\alpha)(y_2 - y_1) < c. \tag{7}$$

and

$$\lambda(y_3 - y_1) < c. \tag{8}$$

The last inequality is obviously satisfied is we take λ small enough, since c is bounded away from zero. So we only worry about (6) and (7). Taking the limit of both inequalities as $\delta \to 1$, $\lambda \to 0$, $\gamma \to 0$, $y_3 \to y_2$, and $\phi_0 \to 1/2$, we have that c must be such that

$$c < \frac{y(1, \underline{e}) - y(1/2, \overline{e}) + c}{2} \Rightarrow c < y(1, \underline{e}) - y(1/2, \overline{e}) = \frac{2\alpha - 1 - \beta}{2}(y_2 - y_1).$$

and $(1-\alpha)(y_2 - y_1) < c$. Therefore, as long as

$$(1-\alpha) < \frac{2\alpha - 1 - \beta}{2} \Leftrightarrow \alpha > \frac{(3+\beta)}{4}$$

the desired result holds. □

Notice, in particular, that we need $2\alpha - 1 - \beta \geq 0$ in order for the above result to hold. From now on we assume that this is always the case (and we know that this is true if $\alpha(3+\beta)/4$). The proof of the next result is relegated to the Appendix, given that its similarity to the proof of the previous lemma.

Lemma 15. *Suppose now that effort is observable, that* $(\alpha - \beta)^2 \leq 2(1-\alpha)(\alpha + \beta)$, *and that* $\alpha > (3+\beta)/4$. *There exist* $(1-\alpha)(y_2 - y_1) < \overline{c}, \underline{\gamma}' \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $\underline{\lambda} \in (0,1)$, $\underline{\delta} \in (0,1)$, *and* $\underline{y_3} > y_2$ *such that if* $\gamma < \underline{\gamma}'$, $\lambda < \underline{\lambda}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta > \underline{\delta}$, *and* $y_3 \in (y_2, \underline{y_3})$, *then there are standard up-or-out profiles that are feasible when* $c \in ((1-\alpha)(y_2 - y_1), \overline{c})$.

Lemma 16. *There is no anonymous equilibrium where a contract with $T_1 = 1$ is offered to an age 2 worker.*

Proof: Consider an anonymous strategy profile where a contract with $T_1 = 1$ is offered to an age 2 worker. By Lemma 4, we know that a worker never exerts effort when he is of age 3. Hence we have two cases to analyze. Either effort is exerted in the first period of employment or not. The second case is not possible from Lemma 3, so we can assume, without loss, that hired workers exert effort when they are of age 2. As above, a worker should only be granted tenure if he produces y_3 in his first period of employment. Therefore, the firm's payoff in such a strategy profile is

$$V' = \frac{y(\phi_0, \overline{e}) - w_1 + \delta p_T[y(\hat{\phi}, \underline{e}) - w_2]}{1 + \delta p_T},$$

where $\hat{\phi}$ and p_T are the same as in the proof of Lemma 14, w_1 is the wage in the first probationary period, and w_2 is the wage in the second. Now observe that we must have

$$w_1 + \delta p_T w_2 + \delta(1 - p_T)U \geq (1 + \delta)U - c$$

in order for any untried worker of age 2 to accept the contract under consideration. Therefore, an upper bound for V' is

$$\overline{V}' = y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + \delta p_T} < y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + (1 + \delta)\delta p_T},$$

where the right-hand side of the above inequality is the payoff to the firm in a standard up-or-out profile where the probationary wage is U and the tenure wage is chosen so that (3) binds. Hence there is a profitable deviation for the firm. □

STEP 3: The next sequence of results identifies conditions under which we can ignore contracts with $T_1 = T_2 = 1$ in an anonymous equilibrium. From the previous lemma we know that it is enough to restrict attention to anonymous strategy profiles where such contracts are offered to workers of age 1. It is possible to show that the retention decisions at the end of the first and second probationary periods must be cutoff decision rules, with the corresponding cutoff beliefs not smaller than ϕ_0. The proof is identical to the proof of Lemma 7. The proof of the next result can be found in the Appendix.

Lemma 17. *There exists $\underline{\gamma}'' \in (0, 1)$ such that if $\gamma < \underline{\gamma}''$, then there is no anonymous equilibrium where:* (i) *A contract $C = \{w_1, T_1 = 1, w_2, T_2 = 1, w_3, T_3 = 1\}$ is offered;* (ii) *Any worker who accepts it exerts effort in the first probationary period;* (iii) *There is an output realization in the first probationary period that leads to retention; (iv) With positive probability any worker who is retained from the first to the second probationary period exerts effort in his second period of employment.*

The above result is very intuitive. If γ is small, effort is very informative about ability. In particular, if a worker exerts effort in his first period of employment, he is only retained if he produces y_3, after which the firm is very optimistic about him. In other words, when γ is small and the worker exerts effort in his first period of employment, there is very little to learn about him in case he is retained. Therefore, since effort is inefficient by assumption, requiring this worker to exert effort in his second period of employment is suboptimal.

Lemma 18. *Suppose that $\alpha < 2\beta$. There exist $\underline{\gamma}' \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $\underline{\lambda} \in (0,1)$, $\underline{\delta} \in (0,1)$, and $\underline{y_3} > y_2$ such that if $\gamma < \underline{\gamma}'$, $\lambda < \underline{\lambda}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta > \underline{\delta}$, and $y_3 \in (y_2, \underline{y_3})$, then there is no anonymous equilibrium of the full commitment game such that:* (i) *A contract with $T_1 = T_2 = 1$ is offered to an age 1 worker;* (ii) *Workers never exert effort in their first period of employment;* (iii) *If employed when of age 2, workers exert effort.*

Proof: Consider an anonymous strategy profile where conditions (i) to (iii) in the above statement are satisfied. Denote by w_3 the wage for the last probationary period. First notice that since $\phi^{y_1}(\phi_0, \underline{e}) < \phi_0$, a worker is only retained from the first to the second probationary period if he produces y_2.[12] Since

$$\phi^{y_2,y_2}(\phi_0, \underline{e}, \overline{e}) = \frac{\alpha\gamma\phi_0}{\alpha\gamma\phi_0 + \beta^2(1-\phi_0)},$$

$\phi^{y_2,y_2}(\phi_0, \underline{e}, \overline{e}) < \phi_0$ for γ sufficiently small. Hence, a worker who is retained from the first to the second probationary period is retained once more only if he produces y_3 in the latter period. Consequently, the on-the-equilibrium-path IC constraint for effort exertion for workers that are employed when of age 2 is

$$\delta\left[(1-\gamma)\phi^{y_2}(\phi_0, \underline{e}) + \lambda(1-\phi^{y_2}(\phi_0, \underline{e}))\right](w_3 - U) \geq c. \tag{9}$$

Let V' denote the firm's payoff in the strategy profile under consideration. Since we want to find conditions under which this strategy profile is not an equilibrium of the full commitment game, we can restrict attention to the case where V' is greater than or equal to the payoff V the firm obtains in a standard up-or-out profile when (3) binds. In this case, the firm's retention decision from the second to the third probationary period is not IC compatible if

$$y(\phi^{y_2,y_3}(\phi_0, \underline{e}, \overline{e}), \underline{e}) - w_3 < V.$$

Since V is given by

$$V = y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + (1+\delta)\delta p_T},$$

[12] Retention must occur after y_2 in the first period of employment, otherwise the firm's payoff from the strategy profile under consideration is $(1-\delta)^{-1}[y(\phi_0) - U]$, in which case a profitable deviation is possible for her.

we can rewrite the last inequality as

$$w_3 - U > y(\phi^{y_2,y_3}(\phi_0, \underline{e}, \overline{e}), \overline{e}) - y(\hat{\phi}, \underline{e}) + \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + (1+\delta)\delta p_T}.$$

We can then conclude that if

$$\frac{c}{\delta\left[(1-\gamma)\phi^{y_2}(\phi_0, \underline{e}) + \lambda(1 - \phi^{y_2}(\phi_0, \underline{e}))\right]} > y(\phi^{y_2,y_3}(\phi_0, \underline{e}, \overline{e}), \overline{e}) - y(\hat{\phi}, \underline{e}) + \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + (1+\delta)\delta p_T}, \quad (10)$$

then it is too expensive for the firm to induce a worker to exert effort if he is retained from the first to the second probationary period: The wage w_3 required is so high that the firm cannot commit to retain the worker if he produces the desired output. Taking the limits $\gamma, \lambda \to 0$, $\delta \to 1$, $\phi_0 \to 1/2$, and $y_3 \to y_2$, equation (10) becomes

$$\frac{c}{\phi^{y_2}(1/2, \underline{e})} > \frac{y(1, \underline{e}) - y(1/2, \overline{e}) + c}{2} \quad \Leftrightarrow \quad c > \frac{\alpha(2\alpha - 1 - \beta)(y_2 - y_1)}{\alpha + 2\beta}.$$

Now remember, from Lemma 14, that we must have

$$c < y(1, \underline{e}) - y(1/2, \overline{e}) = \frac{2\alpha - 1 - \beta}{2}(y_2 - y_1).$$

Consequently, the desired result holds. □

The intuition for this result is straightforward. If α and β are far apart, producing y_2 in the first probationary period leads to a high updated belief, since a worker's output is quite informative about his ability even if he exerts no effort. In this case, the IC constraint (9) is easy to satisfy, making it cheap for the firm to retain a worker from the second to the third probationary period. Therefore, one should expect the above lemma to hold if a worker's output is revealing *only* when he exerts effort.

Lemma 19. *Suppose once again that $\alpha < 2\beta$ and consider an anonymous strategy profile such that:* (i) *A contract with $T_1 = T_2 = 1$ is offered to age 1 workers;* (ii) *Workers only exert effort in their first period of employment. There exist $\underline{\gamma}' \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $\underline{\lambda} \in (0,1)$, $\underline{\delta} \in (0,1)$, and $\underline{y_3} > y_2$ such that if $\gamma < \underline{\gamma}'$, $\lambda < \underline{\lambda}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta > \underline{\delta}$, and $y_3 \in (y_2, \underline{y_3})$, then the only feasible strategy profiles of the above type are outcome equivalent to a standard up-or-out profile. In other words, any worker that is retained from the first to the second probationary period is not fired afterwards.*

Proof: Consider an anonymous strategy profile where (i) and (ii) in the above statement are satisfied. We have two cases to rule out:

1. Suppose the firm never retains a worker from the second to the third probationary period. If effort is unobservable, the IC constraint for effort exertion in the first period is

$$\delta[(1-\gamma)\phi_0 + \lambda(1-\phi_0)](w_2 - U) \geq c. \quad (11)$$

Since $\phi_0 < \phi^{y_2}(\phi_0, \underline{e})$, the above IC constraint is more stringent then the IC constraint for effort exertion (9) of the previous lemma. Hence, if $\lambda, \gamma, \delta, \phi_0$, and y_3 satisfy the conditions in the above statement, the firm's retention decision from the first to the second probationary period is not IC compatible.

Suppose now that effort is observable. We have two alternatives for the off-the-equilibrium-path behavior of the firm. Either she retains a worker that deviates in the first probationary period and produces y_2 or not.[13] The second alternative leads to an IC constraint for effort exertion in the first probationary period identical to (11). So we can, without any loss, consider only the first alternative. In this case the IC constraint for effort exertion is

$$\delta[(1-\gamma-\alpha)\phi_0 + (\lambda-\beta)(1-\phi_0)](w_2 - U) \geq c,$$

which is more stringent than (11).

2. Suppose now that a worker is retained from the second to the third probationary period only if the produces y_2 in the second probationary period. Moreover, suppose that effort is unobservable. The case where effort is observable is dealt with in a similar way. In this case the IC constraint for effort exertion in the first probationary period is

$$\delta p_T[w_2 - U + \delta\overline{p_T}(w_3 - U)] \geq c, \tag{12}$$

where, as before, $p_T = (1-\gamma)\phi_0 + \lambda(1-\phi_0)$ and $\hat{\phi} = \phi^{y_3}(\phi_0, \overline{e})$, and $\overline{p_T} = \hat{\phi}\alpha + (1-\hat{\phi})\beta$. Let V' be the firm's payoff in the strategy profile under consideration. The IC constraints for the firm's retention decisions are

$$y(\phi^{y_2}(\hat{\phi}, \underline{e}), \underline{e}) - w_3 \geq V' \tag{13}$$

and

$$(1-\delta)[y(\hat{\phi}, \underline{e}) - w_2] + \delta\overline{p_T}\left\{(1-\delta)[y(\phi^{y_2}(\hat{\phi}, \underline{e}), \underline{e}) - w_3] + \delta V'\right\} + \delta(1-\overline{p_T})V' \geq V',$$

which can be rewritten as

$$y(\hat{\phi}, \underline{e}) - w_2 + \delta\overline{p_T}\left\{y(\phi^{y_2}(\hat{\phi}, \underline{e}), \underline{e}) - w_3\right\} \geq [1 + \delta\overline{p_T}]V'. \tag{14}$$

Suppose then, by contradiction, that there exists a pair $(\overline{w}_2, \overline{w}_3)$ satisfying (12) to (14). Now let (w_2, w_3) be such that $w_3 = U$ and $w_2 = \overline{w}_2 + \delta\overline{p_T}(\overline{w}_3 - U)$. Then

$$w_2 - U + \delta\overline{p_T}(w_3 - U) = \overline{w}_2 + \delta\overline{p_T}(\overline{w}_3 - U),$$

[13] If the worker deviates and produces y_1, the firm's updated belief about this worker is smaller than ϕ_0, and so he should not be retained for reasons already discussed.

and so, since $\overline{w}_3 \geq U$ by limited liability, (w_2, w_3) also satisfies the IC constraints (12) to (14). In particular, it must be that

$$\delta[\phi_0(1-\gamma) + (1-\phi_0)\lambda](w_2 - U) \geq c.$$

By the previous case, however, this is not possible if $\lambda, \delta, \gamma, \phi_0$, and y_3 satisfy the conditions in the above statement. $\square$

STEP 4: We are now ready to state and prove the central result of this paper.

Theorem 2. *Suppose that α and β are such that*

$$\beta \in \left(\frac{3}{7}, \frac{3}{7} + \eta\right) \text{ and } \alpha \in \left(\frac{3+\beta}{4}, \min\left\{\frac{6}{7} + \frac{\eta}{4}, 2\beta\right\}\right).$$

Then there exist $\overline{\eta} \in (0,1)$, $\underline{\gamma} \in (0,1)$, $\underline{\lambda} \in (0,1)$, $\overline{\delta} \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $(1-\alpha)(y_2 - y_1) < \overline{c}$, and $\underline{y_3} > y_2$ such that if $\eta \in (0, \overline{\eta})$, $\gamma < \underline{\gamma}$, $\lambda < \underline{\lambda}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta \in (\overline{\delta}, 1)$, $y_3 \in (y_2, \underline{y_3})$, and $c \in ((1-\alpha)(y_2 - y_1), \overline{c})$, then all anonymous equilibria of the full commitment are outcome equivalent to the standard up-or-out profile where

$$w_1 = U \quad \textit{and} \quad w_2 = U + \frac{c}{\delta(1+\delta)[\phi_0(1-\gamma) + (1-\phi_0)\lambda]}. \tag{15}$$

Proof: First notice

$$2\beta > \frac{3+\beta}{4} \quad \Rightarrow \quad \beta > \frac{3}{7},$$

and so it is necessary to have $\beta > 3/7$ in order for the interval where α must lie to be well-defined. Moreover, $(3 + 3/7 + \eta)/4 = 6/7 + \eta/4$, and so this interval is non-empty. Suppose, from now on, that $\eta < 1/14$. Then

$$\beta > \frac{3}{7} \;\Rightarrow\; \frac{3+\beta}{4} > \beta \;\Rightarrow \alpha \in (\beta, 2\beta) \;\Rightarrow\; (\alpha - \beta)^2 < \beta^2 < \frac{1}{4},$$

and that

$$1 - \alpha > \frac{1}{8} \quad \text{and} \quad \alpha + \beta \geq \frac{3 + 5\beta}{4} > \frac{9}{7} \Rightarrow 2(1-\alpha)(\alpha+\beta) > \frac{1}{4} \cdot \frac{9}{7}.$$

Hence α and β are such that

$$\alpha > \frac{3+\beta}{4}\,, \quad (\alpha-\beta)^2 \leq 2(1-\alpha)(\alpha+\beta)\,, \quad \text{and} \quad \alpha < 2\beta. \tag{16}$$

Consider now the anonymous strategy profile σ where: (i) The firm always offers the contract $C = \{w_1', T_1 = 1, w_2', T_2 = 1, w_3', T_3 = 1\}$ to an age 1 worker; (ii) The worker never exerts effort when employed; (iii) The firm adopts the retention decisions of Theorem 1, i.e., she retains a worker as long as he keeps producing y_2. The fact that α and β satisfy the three conditions in (16) implies that there exist $\overline{\delta}, \underline{\lambda}, \underline{\gamma} \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $0 < (1-\alpha)(y_2 - y_1) < \overline{c}$, and $\underline{y_3} > y_2$ such that if $\gamma < \underline{\gamma}$, $\lambda < \underline{\lambda}$, $\delta > \overline{\delta}$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $c \in ((1-\alpha)(y_2 - y_1), \overline{c})$, and $y_3 \in (y_2, \underline{y_3})$, then the only anonymous strategy profiles that are feasible are either a standard up-or-out profile or a strategy profile that is outcome equivalent to σ.[14] It is obvious that in order for σ to be an equilibrium (and any other strategy profile that is outcome equivalent to it), we must have $w_1' = w_2' = w_3' = U$. In the same way, we must have w_1 and w_2 in a standard up-or-out profile given by (15).

Now observe that

$$\alpha(1-\alpha) < 2\beta\left(1 - \frac{3+\beta}{4}\right) = \frac{\beta(1-\beta)}{2} < \beta(1-\beta).$$

Hence, as a consequence of Theorem 1 in Section 3, the firm's payoff from σ, or any other strategy profile outcome equivalent to it, is V^* given by (2). Straightforward algebra shows that

$$V^* = \underline{y} - U + \frac{1 + \delta\alpha + \delta^2\alpha^2}{1 + \delta\phi_0[\alpha + \beta + \delta(\alpha^2 + \beta^2)]}\phi_0(\alpha - \beta)(y_2 - y_1),$$

where $\underline{y} = \beta y_2 + (1-\beta)y_1$. Therefore

$$V^* = \underline{y} - U + (\alpha - \beta)(y_2 - y_1)\underbrace{\left\{\frac{1 + \alpha + \alpha^2}{2 + \alpha + \beta + \alpha^2 + \beta^2}\right\}}_{\mu}$$

in the limiting case ($\lambda = \gamma = 0$, $\delta = 1$, $\phi_0 = 1/2$ and $y_3 = y_2$). The payoff to the firm from the standard up-or-out profile under consideration is

$$V = \frac{y(\phi_0, \overline{e}) - U - c + \delta(1+\delta)p_T[y(\hat{\phi}, \underline{e}) - U]}{1 + \delta(1+\delta)p_T},$$

which in the limiting case becomes

$$V = \underline{y} - U + \frac{3}{4}(\alpha - \beta)(y_2 - y_1) + \frac{1}{4}(1-\alpha)(y_2 - y_1) - \frac{c}{2}.$$

Hence, still in the limiting case,

$$V - V^* = (\alpha - \beta)(y_2 - y_1)\left(\frac{3}{4} - \mu\right) + \frac{1}{4}(1-\alpha)(y_2 - y_1) - \frac{c}{2}.$$

Since $c > (1-\alpha)(y_2 - y_1)$, as effort is inefficient, $V - V^* > 0$ if, and only if,

$$(\alpha - \beta)\left(\frac{3}{4} - \mu\right) > \frac{1-\alpha}{4}. \tag{17}$$

Suppose $\beta = \frac{3}{7}$ and $\alpha = (3+\beta)/4 = \frac{6}{7}$. Then $\mu = \frac{97}{216} < \frac{1}{2}$, and so (17) holds if

$$\frac{\alpha - \beta}{4} > \frac{1-\alpha}{4} \quad \Leftrightarrow \quad \alpha > \frac{1+\beta}{2} \quad \Leftarrow \quad \alpha = \frac{3+\beta}{4}.$$

We can then conclude that $V > V^*$ for η sufficiently small and so the desired result holds. □

[14] These strategy profiles differ from σ in how the workers behave off the equilibrium path.

4.3 Limited Liability

(*To be added*).

5 Conclusion

This paper rationalizes the use of up-or-out contracts in an environment where a worker's ability is unobserved to both a firm and the worker. Information about ability is acquired by observing the worker's performance over time. Differently from standard experimentation problems, information about the worker's ability is affected by his choice of effort. In particular, the likelihood of high output increases in the effort a worker exerts. We show that when prior information about ability is diffuse and the informativeness of output signals is sufficiently large when the worker exerts effort, a firm benefits by offering an up-or-out contract.

An issue of interest in this framework is the extent to which commitment on post-retention compensation can be relaxed. In this case, if employment outcomes are to some extent observable to outside labor market participants, a worker's incentive to effort exertion can be sustained through the wage bidding triggered by a firm's retention decision. Modeling outside labor market competition in the context of the up-or-out contract game, as well as extending the results to the case in which the worker is T-period lived, is the object of current research.

References

[1] Bar-Isaac, H. (2002): "Something to Prove: Reputational Incentives in Teams and Hiring to Introduce Uncertainty", in Proceedings of the 2002 North American Summer Meetings of the Econometric Society: Game Theory, edited by David K. Levine, William Zame, Larry Samuelson, and John Van Huyck.

[2] Charmichael, H.L. (1988): "Incentives in Academics: Why is There Tenure?", *Journal of Political Economy* 96(3), 453-472.

[3] Harris, M., and Y. Weiss (1984): "Job Matching with Finite Horizon and Risk Aversion", *Journal of Political Economy* 92(4), 758-779.

[4] Kahn, C., and G. Huberman (1988): "Two-Sided Uncertainty and 'Up-or-Out' Contracts", *Journal of Labor Economics* 6(4), 423-444.

[5] Levin, J., and S. Tadelis (2002): "A Theory of Partnerships", Stanford University, mimeo.

[6] O'Flaherty, B., and A. Siow (1992): "On the Job Screening, Up or Out Rules, and Firm Growth", *Canadian Journal of Economics* 25(2), 346-368.

[7] Waldman, M. (1990): "Up-or-Out Contracts: A Signaling Perspective", *Journal of Labor Economics* 8(2), 230-250.

Appendix

Lemma 10. *There is no anonymous equilibrium of the full commitment game that involves:* (i) *A contract C with $T_1 = 2$ being offered;* (ii) *The worker that accepts C exerts effort in the first period and no effort in the second period of his first probationary period.*

Proof: Consider a strategy profile such that (i) and (ii) in the above statement are satisfied. Since $\phi^{y_1}(\phi_0, \underline{e}) = 0$, a worker who exerts effort in his first period of employment and produces y_1 is never retained. The following deviation is then profitable for the firm, as it allows her to replace a worker who produces y_1 in his first period of employment with better worker:

A. Offer $\{w_1' = U, T_1 = 1, w_2' = U, T_2 = 1, w_3' = w_2, T_3 = 1\}$, where w_2 is the post-retention wage in the contract C, in period 1;

B. Retain the worker from period 1 to 2 only if he produces y_2 or y_3. Otherwise, offer $\{w_1'' = U, T_1 = 1, w_2'' = U, T_2 = 1\}$ in period 2 to an untried worker of age 2 and never retain this worker from period 2 to 3;

C. Return to the original strategy in period 2 if a retention happens in that period and in period 3 if a retention does not happen in period 2.

□

Lemma 11. *Suppose that $\alpha(1-\alpha) < \beta(1-\beta)$. There is no anonymous equilibrium of the full commitment game that involves:* (i) *A contract C with $T_1 = 2$ being offered;* (ii) *The worker that accepts C exerting no effort in the first period of his first probationary period.*

Proof: Let σ be an anonymous strategy profile for which conditions (i) and (ii) in the above statement are satisfied. From Lemma 4, a worker of age 3 never exerts effort when employed. Hence there are two cases to consider. Either effort is not exerted when the worker is of age 2 or effort is exerted with positive probability when the worker is of this age. In the first case, because of limited liability, it must be that $C = \{w_1 = U, T_1 = 2, w_2 = U, T_2 = 1\}$ if σ is to be an equilibrium. The proof of Theorem 1 then shows that a profitable deviation is possible for the firm. Consequently, we only need to deal with the second case.

We first establish that there are two sub-cases to be considered. Either the workers only exert effort when they are of age 2 if they produce y_2 in their first period of employment, or they always exert effort when they are of age 2. Since the workers are supposed to exert effort in the second period of their first probationary period, we need $w_2 > U$ and there must be some output realizations that lead to retention and others that lead to dismissal. Otherwise there are no incentives to exert effort in that period. Moreover, given our assumptions about γ, β, and λ ($\lambda < 1-\gamma$ and $\beta + \lambda < 1$), the higher is the output realization under effort, the higher is the firm's updated belief about the worker. Hence, if producing $y \in \{y_1, y_2, y_3\}$ when of age 2 leads to a worker's retention, any output realization higher than y leads to retention as well. This follows from the previous lemma. Consequently, there exists $\underline{y}(y') > y_1$, that depends on the first period's output realization y', such that a worker is retained if, and only if, he produces at least $\underline{y}(y')$ in his second period of employment.

Now observe that $\Pr(y_2|H, \underline{e}) > \Pr(y_2|L, \underline{e})$. Hence, the firm's belief ϕ about a worker is higher at the beginning of this worker's second period of employment if he produces y_2 in his first period of employment. Therefore, once more from Lemma 7, $\underline{y}(y_1) \geq \underline{y}(y_2)$, and so $\Pr\{y \geq y(y_2)|\overline{e}\} \geq \Pr\{y \geq y(y_1)|\overline{e}\}$. We can then conclude that if a worker has an incentive to choose $\overline{e}$ after he produces y_1 in his first period of employment, then he has an incentive to choose $\overline{e}$ after y_2 as well. In other words, the two sub-cases listed above cover all the possibilities.

We begin with the second sub-case. Given limited liability, it must be that $C = \{w_1 = U, T_1 = 2, w_2 > U, T_2 = 1\}$. Consider then the following deviation for the firm:

A. Offer $C =' \{w_1' = U, T_1 = 1, w_2' = U, T_2 = 2, w_3' = w_2, T_3 = 1\}$ in the first period. Denote the worker to whom C is offered by W;

B. Retain W from period 1 to period 2 only if he produces y_2 in the first period. Otherwise, offer the contract given by $\{w_1'' = U, T_1 = 1, w_2'' = w_2, T_2 = 1\}$ to an untried worker of age 2. Denote this worker by W';

C. Suppose W is retained from period 1 to 2. In this case, adopt the same retention decision at the end of period 2 as the one used with the original contract;

D. If W is replaced with W' at the beginning of the second period, retain W' from period 2 to 3 only if he produces $y \geq \underline{y}(y_1)$;

E. Behave, from period 4 on if no dismissal occurs in period 3 and from period 3 on if it does occur, in the same way as prescribed by σ.

We argue that this deviation improves the firm's payoff. To see why, first observe that if y_2 realizes in the first period, then the payoff to the firm is the same as in the original strategy profile. If, on the other hand, y_1 realizes in the first period, the firm replaces an age 2 worker with belief $\phi^{y_1}(\phi_0, \underline{e}) < \phi_0$, W, with an age 2 worker with belief ϕ_0, W'. Since in the original strategy profile, W exerts effort after y_1, W' also exerts effort in period 2, as he is more optimistic about himself, is retained in period 3 under exactly the same circumstances as W is retained, and receives the same wage upon retention. Therefore, if y_1 is realized in the first period, the firm's lifetime payoff from period 2 on is strictly bigger with the deviation, as her per-period payoffs are strictly increasing in her beliefs.

The other alternative is dealt with in a similar way. As above, the firm should replace the worker she hires in period 1 if he produces y_1. The only difference is that in this case the firm should offer the "flat" contract $\{w_1'' = U, T_1 = 1, w_2'' = U, T_2 = 1\}$ to an untried worker of age 2 at the beginning of period 2. □

Lemma 15. *Suppose now that effort is observable, that $(\alpha - \beta)^2 \leq 2(1-\alpha)(\alpha+\beta)$, and that $\alpha > (3+\beta)/4$. There exist $0 < (1-\alpha)(y_2 - y_1) < \overline{c}$, $\underline{\gamma}' \in (0,1)$, $0 < \underline{\phi_0} < \overline{\phi_0} < 1$, $\underline{\lambda} \in (0,1)$, $\underline{\delta} \in (0,1)$, and $\underline{y_3} > y_2$ such that if $\gamma < \underline{\gamma}'$, $\phi_0 \in (\underline{\phi_0}, \overline{\phi_0})$, $\delta > \underline{\delta}$, and $y_3 \in (y_2, \underline{y_3})$, then there are standard up-or-out profiles that are feasible when $c \in ((1-\alpha)(y_2 - y_1), \overline{c})$.*

Proof: We know that a worker that exerts effort during the probationary period is only granted tenure if he produces y_3. The firm, however, now observes the worker's choice of effort, and so if he deviates an produces y_2, the firm's belief increases to $\phi^{y_2}(\phi_0, \underline{e})$. Hence, there are two possible alternatives, either the worker is granted tenure if he deviates and produces y_2 or not, and they lead to different IC constraints for the worker's choice of effort. Both alternatives, however, lead to the same payoff V to the firm, where V is the same as in the previous lemma. Therefore, we can, without loss of generality, restrict attention to the case where w_2 is such that

$$y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - w_2 < V, \tag{18}$$

so that a worker that deviates and produces y_2 is not retained. This ensures that the IC constraint for the worker's choice of effort is the same as in the previous lemma. Moreover, we restrict attention to the case where $w_1 = U$ and w_2 is such that (3) binds, so that $V = \overline{V}$.[15]

Consequently, if (6), (7), (8), and (18) are satisfied, we know that there are feasible standard up-or-out profiles when effort is observable. From the previous lemma and the assumption that $\alpha > (6+2\beta)/8$, we know that (6), (7), and (8) are satisfied for a certain range of costs if we take δ close to 1, γ and λ close to zero, ϕ_0 close to 1/2, and y_3 close to y_2. Let $\underline{V}$ be such that

$$\underline{V} = y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - U - \frac{y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - y(\phi_0, \underline{e}) + c}{1 + \delta(1+\delta)p_T}.$$

[15] After all, we know that if a standard up-or-out profile is to be an equilibrium, these two conditions must hold, otherwise there is a profitable deviation for the firm.

It is straightforward to see that $\underline{V} < V$, and so (18) is satisfied if

$$y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - w_2 \leq \underline{V} \quad \Leftrightarrow \quad w_2 - U \geq \frac{y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - y(\phi_0, \underline{e}) + c}{1 + \delta(1+\delta)p_T}$$

is satisfied. Since from equation (3) we must have $w_2 - U \geq [\delta(1+\delta)p_T]^{-1}c$, we are done if

$$\frac{c}{\delta(1+\delta)p_T} \geq \frac{y(\phi^{y_2}(\phi_0, \underline{e}), \underline{e}) - y(\phi_0, \underline{e}) + c}{1 + \delta(1+\delta)p_T}.$$

The above equation can be rewritten as

$$c \geq \frac{\delta(1+\delta)[\phi_0(1-\gamma) + (1-\phi_0)\lambda]\phi_0(1-\phi_0)(\alpha-\beta)^2(y_2 - y_1)}{\phi_0\alpha + (1-\phi_0)\beta}.$$

Taking the appropriate limits it becomes

$$c \geq \frac{(\alpha-\beta)^2}{2(\alpha+\beta)}(y_2 - y_1),$$

which is satisfied – since by assumption $c > (1-\alpha)(y_1 - y_2)$ – if $(\alpha-\beta)^2 \leq 2(1-\alpha)(\alpha+\beta)$. □

Lemma 17. *There exists $\underline{\gamma}'' \in (0,1)$ such that if $\gamma < \underline{\gamma}''$, then there is no anonymous equilibrium where:* (i) *A contract $C = \{w_1, T_1 = 1, w_2, T_2 = 1, w_3, T_3 = 1\}$ is offered;* (ii) *Any worker who accepts it exerts effort in the first probationary period;* (iii) *There is an output realization in the first probationary period that leads to retention; (iv) With positive probability any worker who is retained from the first to the second probationary period exerts effort in his second period of employment.*

Proof: Consider an anonymous strategy profile where (i) to (iv) in the above statement are satisfied. Since $\phi^y(\phi_0, \overline{e}) < \phi_0$ if $y \in \{y_1, y_2\}$, it follows that a worker is retained from the first to the second probationary period only if he produces y_3. As before, let $\underline{\gamma}$ be the smallest solution to $\gamma(1-\gamma) = \beta\lambda$ and suppose $\gamma < \underline{\gamma}$. Recall that $\phi^{y_3,y}(\phi_0, \overline{e}, \overline{e}) < \phi_0$ for $y \in \{y_1, y_2\}$ when $\gamma < \underline{\gamma}$. Consequently, any worker who is retained from the first to the second probationary period is retained one more time only if he produces y_3 in the second probationary period.

Let $p'_T = \hat{\phi}(1-\gamma) + (1-\hat{\phi})\lambda$, where $\hat{\phi} = \phi^{y_3}(\phi_0, \overline{e})$. From the previous paragraph we know that

$$w_1 + \delta p_T \left[w_2 + \delta p'_T w_3 + \delta(1-p'_T)U\right] + \delta(1+\delta)(1-p_T)U \geq (1+\delta+\delta^2)U - (1+\delta)c,$$

must be satisfied if C is to be accepted. Therefore, an upper bound for the firm's payoff in this strategy profile is

$$\overline{V}' = (1-\delta) \cdot \frac{y(\phi_0, \underline{e}) - U - c + \delta p_T \left\{y(\hat{\phi}, \overline{e}) - U - c + \delta p'_T[y(\hat{\hat{\phi}}, \underline{e}) - U]\right\}}{1 - \delta p_T \left[\delta^2 p'_T + \delta(1-p'_T)\right] - \delta(1-p_T)},$$

where $\hat{\hat{\phi}} = \phi^{y_3,y_3}(\phi_0, \overline{e}, \overline{e})$. Straightforward algebra shows that

$$\overline{V}' < \overline{V}'' = \frac{y(\phi_0, \underline{e}) - U - c + \delta p_T[y(\hat{\phi}, \overline{e}) - U - c] + \delta^2 p_T[y(\hat{\hat{\phi}}, \underline{e}) - U]}{1 + \delta(1+\delta)p_T},$$

where $\overline{V}''$ is obtained by setting $p'_T = 1$ in $\overline{V}'$.

Now let $\underline{\gamma}'$ be such that

$$\delta[y(\hat{\hat{\phi}}, \underline{e}) - y(\hat{\phi}, \underline{e})] = c + y(\hat{\phi}, \underline{e}) - y(\hat{\phi}, \overline{e}).$$

Since $c > (1-\alpha)(y_2 - y_1)$ in order for effort to be inefficient, the above equation has a positive solution. Moreover, $\gamma \in (0, \underline{\gamma}')$ implies that the left-hand side of the above equation is smaller than its right-hand side, in which case

$$\overline{V}'' < y(\hat{\phi}, \underline{e}) - U - \frac{y(\hat{\phi}, \underline{e}) - y(\phi_0, \overline{e}) + c}{1 + (1+\delta)\delta p_T}.$$

We can then conclude that if $\gamma < \underline{\gamma}''$, where $\underline{\gamma}'' = \min\{\underline{\gamma}, \underline{\gamma}'\}$, the desired result holds. □