Criminal Networks: Who is the Key Player?*†

Xiaodong Liu‡ Eleonora Patacchini§ Yves Zenou¶ Lung-Fei Lee‖

March 8, 2012 (Revised December 18, 2012)

Abstract

We develop a key-player model by allowing for link formation so that when a person is removed from a network the other individuals can form new links while still optimally providing crime effort. We then put our model to the test, using data on adolescent delinquents in the United States, and provide new results regarding the identification of peer effects. This is done by a structural estimation and simulation of our model. Compared to a policy that removes randomly delinquents from the network, a key player policy engenders a crime reduction that can be as large as 35 percent. We discuss how to implement the key-player policy in the real world, primarily within criminal networks, but also within financial, R&D, development, political and tax-evasion networks.

Key words: Crime, Katz-Bonacich centrality, estimation of peer effects, crime policies.
JEL Classification: A14, D85, K42, Z13

*We would like to thank Luigi Pistaferri as well as three anonymous referees for extremely helpful comments. We also thank the participants of the Einaudi Institute of Economics and Finance lunch seminar, the conference on "The Economics of Coordination and Communication: Theory and Empirics" at Ravello in 2012, the Iowa State University departmental seminar, the IIES seminar, and the Italian Association of Labor Economics conference in 2011 for their comments, in particular, Jason Fletcher, David Levine, Matthew Lindquist, Marcus Mobius and Tanya Rosenblat. We are grateful to Tiziano Arduini and Edoardo Rainone for excellent research assistance and Dina Neiman for her editorial services.

†This research uses data from Add Health, a program project designed by J. Richard Udry, Peter S. Bearman, and Kathleen Mullan Harris, and funded by a grant P01-HD31921 from the National Institute of Child Health and Human Development, with cooperative funding from 17 other agencies. Special acknowledgment is due Ronald R. Rindfuss and Barbara Entwisle for assistance in the original design. Persons interested in obtaining data files from Add Health should contact Add Health, Carolina Population Center, 123 W. Franklin Street, Chapel Hill, NC 27516-2524 (addhealth@unc.edu). No direct support was received from grant P01-HD31921 for this analysis.

‡University of Colorado at Boulder, USA. E-mail: xiaodong.liu@colorado.edu.

§La Sapienza University of Rome, EIEF and CEPR. E-mail: eleonora.patacchini@uniroma1.it

¶Corresponding author. Stockholm University, Research Institute of Industrial Economics (IFN) and GAINS. E-mail: yves.zenou@ne.su.se.

‖The Ohio State University, USA. E-mail: lee.1777@osu.edu.

1 Introduction

There are 2.3 million people behind bars at any one time in the United States, and that number continues to grow. It is the highest level of incarceration per capita in the world. Moreover, since the crime explosion of the 1960s, the prison population in the United States has multiplied fivefold, to one prisoner for every hundred adults — a rate unprecedented in American history and unmatched anywhere in the world.[1] However, in spite of a continuously falling crime rate, the prisoner head count continues to rise, and poor people as well as minorities still bear the brunt of both crime and punishment.

One possible way to reduce crime is to detect, apprehend, convict, and punish criminals. This is what has been done in the United States and all of these actions cost money, currently about $200 billion per year nationwide. For example, in California, even if this "brute force" policy has partly worked (the rate of every major crime category is now less than half of what it was 20 years ago), the cost of this policy has been tremendous. Consider, for example, that the cost of the justice system is higher than the cost of education.[2]

In his recent book published in 2009, Mark Kleiman argues that simply locking up more people for lengthier terms is no longer a workable crime-control strategy. But, says Kleiman, there has been a revolution in controlling crime by means other than brute-force incarceration: substituting swiftness and certainty of punishment for randomized severity, concentrating enforcement resources rather than dispersing them, communicating specific threats of punishment to specific offenders, and enforcing probation and parole conditions to make community corrections a genuine alternative to incarceration. As Kleiman shows, "zero tolerance" is nonsense: there are always more offenses than there is punishment capacity.[3]

Is there an alternative to brute force? In this paper, we argue that concentrating efforts by targeting "key criminals", i.e. criminals who once removed generate the highest possible reduction in aggregate crime level in a network, can have large effects on crime because of the snow-ball effects or "social multipliers" at work (see, in particular, Kleiman, 2009; Glaeser et al., 1996; Verdier and Zenou, 2004). That is, as the fraction of individuals participating in a criminal behavior increases, the impact on others is multiplied through social networks. Thus, criminal behaviors can be magnified, and interventions can become more effective. Furthermore, the impact of social networks may be particularly important for adolescents because this developmental period overlaps with the initiation and continuation of many risky, unhealthy, and delinquent behaviors and is a period of maximal response to peer pressure (Thornberry et al., 2003; Warr, 2002).

It is indeed well-established that delinquency is, to some extent, a group phenomenon, and that the sources of crime and delinquency are located in the intimate social networks of individuals (see e.g. Sarnecki, 2001; Warr, 2002; Haynie, 2001; Patacchini and Zenou, 2012). Delinquents often have friends who have themselves committed several offences, and social ties among delinquents are seen

[1] See Cook and Ludwig (2010) and the references therein.

[2] For example, the "Three Strikes" law passed in California in 1994 mandates extremely long prison terms (between 29 years and life) for anyone previously convicted in two serious or violent felonies (including residential burglary) when convicted of a third felony, even for something as minor as petty theft.

[3] In the standard crime literature (Becker, 1968; Garoupa, 1997; Polinsky and Shavell, 2000), punishment is seen as an effective tool for reducing crime.

as a means whereby individuals exert an influence over one another to commit crimes. In fact, not only friends, but also the *structure* of social networks, matters in explaining an individual's own delinquent behavior. This suggests that the underlying structural properties of friendship networks must be taken into account to better understand the impact of peer influence on delinquent behavior and to address adequate and novel delinquency-reducing policies.

The aim of this paper is to investigate the importance of the key player in criminal networks, both from a theoretical and empirical viewpoint and to propose a methodology that helps detect the key player.

For that, we first extend the static model of Ballester et al. (2006) to propose a dynamic network formation model where, at each period in time, delinquents first choose with whom they want to form a link and then how much effort they exert in crime. We have a non-ergodic Markov chain and the network converges to an equilibrium (or an absorbing state) when no criminal has an incentive to create a new link. We then study the key-player policy by adding a first stage where the planner decides which delinquent to remove from the network by anticipating the long-term effect on total crime so that the key player is the delinquent who reduces the (expected) total crime the most. In the *static* key player model developed by Ballester et al. (2006), when the planner removes a delinquent from the network, no other delinquents can form new links. Here, this is not the case anymore. As a result, when the planner removes someone from the network, he/she anticipates the fact that new links may be formed, which may generate more criminal activities.

We then test the model on the data. In order to determine the key-player, we need to estimate the intensity of interactions between criminals in each network. Most of the network models in the literature use the identification approach proposed by Bramoullé et al. (2009), which assumes that the adjacency matrix (or sociomatrix) is *row-normalized* (the so-called *local-average model*). With a row-normalized adjacency matrix, a criminal's effort level depends on the *average* effort level of his/her criminal friends (Patacchini and Zenou, 2012). However, key-player policies only make sense in the context of the *local-aggregate model* developed by Ballester et al. (2006) with a *non-row-normalized* adjacency matrix, where it is the *sum* of friends' efforts that affects own effort. This is because, once we remove a delinquent from a network, there are snow-ball or multiplier effects on other criminals.[4] In this paper, we provide a new identification strategy for the local-aggregate model which is based on the variation of the row sum of the adjacency matrix.[5] We show that the identification condition for the local-aggregate model is weaker than that for the local-average model developed by Bramoullé et al. (2009). We also give some examples of networks for which the local-aggregate model can be identified while the local-average model cannot be.[6]

[4]This idea was also used by Glaeser et al. (1996) in their pioneering study on crime and social interactions. These kinds of effects will not be present in the local average model because, in this model, to affect the behavior of each individual, one needs to change the social norm of all individuals in the network. While it is true that many papers, which typically have no micro-founded model, interpret peer effects as the desire to conform to a social norm and estimate a local average model (e.g., Lin, 2010; Boucher et al, 2012; Bifulco et al., 2011), others, which have a theoretical model, use a local aggregate formulation for peer effects (e.g. Glaeser et al., 1996; Christakis and Fowler, 2007; Calvo-Armengol et al., 2009). The identification conditions for network models with local aggregates have not been studied before.

[5]In a row-normalized matrix the row sum is constant and equal to 1.

[6]Many theoretical models in the network literature use a local aggregate approach. Our results will be useful in identifying these types of models e.g. Bramoulle and Kranton (2007), Belhaj and Deroïan (2011), Zhou and Chen (2012), etc.

Using the AddHealth data of adolescents in the United States, we then estimate and simulate our dynamic network-formation model to determine who is the key player in each of 150 networks. We find that the key player is *not* necessarily the most active criminal in the network. We also find that it is *not* straightforward to determine which delinquent should be removed from a network by only observing his or her criminal activities or position in the network. Compared to other criminals, the key players are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being more socially excluded. They also feel that their parents care less about them, are more likely to come from single-parent families and have more troubles getting along with their teachers.

We finally discuss the policy implications of our results. First, we develop a qualitative approach using simulations to evaluate the cost and benefit of the key-player policy. We show that a random sweep of criminals is less effective than the key-player policy. We show that, for small networks, crime reduction after the implementation of the key-player policy can be as high as 35 percent. Second, our key-player policy can be directly applied to reduce crime in the real world when criminal network data are available (we provide a selective list of such data). In fact, some similar policies aiming at reducing crime have already been implemented in the United States but without the analytical tools of the key-player policy. Finally, we show that we can use our methodology to determine the key player in other types of networks and activities, where network data are easier to obtain. This is particularly true for financial networks, R&D networks, networks in developing countries, political networks and tax-evasion networks.

The rest of the paper unfolds as follows. In the next section, we discuss the related literature and explain our contribution. The theory section (Section 3) is divided into two subsections: the static models (Section 3.1) and the dynamic model (Section 3.2). Our data are described in Section 4. In Section 5, the identification of the econometric network model is discussed while the estimation and empirical results of the impact of peer effects on crime are provided. Section 6 details the simulation of the dynamic network formation model and characterizes the key players. In Section 7, we discuss the policy implications of the key-player policy. Finally, Section 8 concludes.

2 Related literatures

Our paper lies at the intersection of different literatures. We would like to expose them in order to highlight our contribution.

Theories of crime networks There is a growing theoretical literature on the social aspects of crime. Glaeser et al. (1996) were among the first to develop a crime social interaction model in which criminals are located in a circle where some of them mimic what their neighbors do while others decide their criminal activities by themselves. They show that criminal interconnections act as a social multiplier on aggregate crime. Calvó-Armengol and Zenou (2004), Ballester et al. (2010), Patacchini and Zenou (2012) embed criminal activities in a social network model. They study the effect of the structure of the network on crime. They show that the location in the social network of each criminal not only affects his/her direct friends but also friends of friends, etc. Ballester et al. (2010) apply the model developed by Ballester et al. (2006) to the case of delinquent networks and study the policy implications of delinquency network models. They show that a key-player policy can be more efficient than standard punishment policies in reducing crime.

Compared to this literature, we have the following contribution. In all these models of crime with social interactions, the network is fixed and cannot evolve over time. We relax this assumption by considering a dynamic network formation model where criminals not only decide how much effort they put into crime but with whom they want to form links.

Empirical studies of peer effects in crime There is a also a growing body of empirical literature suggesting that peer effects are very strong in criminal decisions. Ludwig et al. (2001) and Kling et al. (2005) study the relocation of families from high- to low-poverty neighborhoods using data from the Moving to Opportunity (MTO) experiment. They find that this policy reduces juvenile arrests for violent offences by 30 to 50 percent, relative to a control group. This also suggests very strong social interactions in crime behaviors. Patacchini and Zenou (2012) find that peer effects in crime are strong, especially for petty crimes. Bayer et al. (2009) consider the influence that juvenile offenders serving time in the same correctional facility have on each other's subsequent criminal behavior. They also find strong evidence of learning effects in criminal activities since exposure to peers with a history of committing a particular crime increases the probability that an individual who has already committed the same type of crime recidivates that crime.

Our contribution here is to estimate a structural dynamic network formation model and to propose a way to identify the key player in criminal networks. This paper is the first to present an empirical implementation of the key player policy, which is grounded on a precise behavioral foundation.

Theories of network formation There is an important literature on dynamic network formation.[7] The first approach is to consider a random network formation (looking at stochastically stable networks) and to study how emerging networks match real world networks (Ehrhardt et al., 2006; Hofbauer and Sandholm, 2007). While sharing some common features with this literature, our model is quite different since *agents do not create links randomly but in a strategic way, i.e. they maximize their utility.* In the economic literature, there are also dynamic network formation models with strategic interactions. Jackson and Watts (2002), Dutta et al. (2005) are prominent papers in this literature. Our dynamic network formation model is different than the ones developed in these papers in the sense that we consider both dynamic models of network formation and agents' optimal actions. This allows us to give a micro-foundation for the network formation process as equilibrium actions transform into equilibrium utility levels. Observe, however, that our dynamic network formation model is only used to determine the key player in the simulation study so we do not need to analytically characterize the steady-state distribution while these papers do.

There are also some papers that, as in our framework, combine both network formation and endogenous actions (see, e.g. Bramoullé et al., 2004, Cabrales et al., 2011, Calvó-Armengol and Zenou, 2004, and Goyal and Joshi, 2003). Most of these models are, however, static and the network formation process is different. König et al. (2012) is the closest to ours since it is both a dynamic network formation model and players choose effort optimally. They, however, impose that individuals have to delete one of their links with some probability, which leads to steady-state networks that

[7] See Goyal (2007), Jackson (2008), Ioannides (2012) and Jackson and Zenou (2013) for overviews on network theory.

have very specific properties (e.g. a diameter of two). In our model, individuals do not delete links but bear a cost of forming links.

Econometrics of networks The literature on identification and estimation of social network models has progressed significantly recently (see Blume et al., 2011, for a recent survey). In his seminal work, Manski (1993) introduces a linear-in-means social interaction model with endogenous effects, contextual effects, and correlated effects. Manski shows that the linear-in-means specification suffers from the "reflection problem" and the different social interaction effects cannot be separately identified.[8] Bramoullé et al. (2009) generalize Manski's linear-in-means model to a general local-average social network model, whereas the endogenous effect is represented by the average outcome of the peers. They provide some general conditions for the identification of the local-average model (i.e. when the adjacency matrix is row-normalized) using an indirect connection's characteristics as an instrument for the endogenous effect. Here we go further by first providing a micro-foundation for the local-aggregate model and then providing an identification condition for the local-aggregate model (i.e. when the adjacency matrix is not row-normalized). We show that, in this case, the identification condition is weaker than for the local-average model. We provide some examples of networks where the local-aggregate model can be identified while the local-average model cannot be.

The key-player problem The problem of identifying key players in a network is an old one, at least in the sociological literature. Indeed, one of the focuses of this literature is to propose different measures of network centralities and to assert the descriptive and/or prescriptive suitability of each of these measures to different situations (see, in particular, Wasserman and Faust, 1994). Borgatti (2003, 2006) was among the first researchers to really investigate the issue of key players, which is based on explicitly measuring the contribution of a set of actors to the cohesion of a network. The basic strategy is to take any network property, such as density or maximum flow, and derive a centrality measure by deleting nodes and measuring the change in the network property. Borgatti measures the amount of *reduction* in cohesiveness of the network that would occur if some nodes were not present.[9]

Ballester et al. (2006) were the first to define the key-player problem in terms of *behavior* of agents so that total activity is measured as the sum of efforts of all agents at a Nash equilibrium. In the present paper, we extend Ballester et al. (2006) by also allowing for link formation so that, when a person is removed from a network, the other individuals can form new links while still providing optimal crime effort. To the best of our knowledge, this is the first paper that structurally estimates the key player for real-world networks where individuals can choose efforts and link formations. We also provide concrete recommendations on how to implement the key player policy in the real world, primarily from criminal networks but also for financial, R&D, development, political and tax-evasion networks.

[8] Lee (2007) considers a model with multiple networks where an agent is equally influenced by all the other agents in the same network. Lee's social interaction model is identifiable only if there is variation in network size in the sample. The identification, however, can be weak if all of networks are large.

[9] Borgatti gives examples of such problems. In public health, a key-player problem arises whenever the planner needs to select a subset of population members to immunize or quarantine in order to optimally contain an epidemic. In the military or criminal justice context, the key-player problem arises when a planner needs to select a small number of players in a criminal network to neutralize (e.g., by arresting, exposing or discrediting) in order to maximally disrupt the network's ability to mount coordinated action.

3 Theoretical framework

3.1 Static network models: A review of previous results

3.1.1 Model and Nash equilibrium

Borrowing from Ballester et al. (2006, 2010), we develop a network model of peer effects where the network reflects the collection of active bilateral influences.

Network $N = \{1, \ldots, n\}$ is a finite set of agents in a connected network $g \equiv g_N$. We keep track of social connections in a delinquency network g through its adjacency matrix $\boldsymbol{G} = [g_{ij}]$,[10] where $g_{ij} = 1$ if i nominates j ($j \neq i$) as i's friend and $g_{ij} = 0$, otherwise. We set $g_{ii} = 0$. For the ease of the presentation, we focus on directed networks so that $\boldsymbol{G}$ can be asymmetric.[11] For each network g with adjacency matrix $\boldsymbol{G} = [g_{ij}]$, the kth power of $\boldsymbol{G}$ given by $\boldsymbol{G}^k = \boldsymbol{G}^{(k \overset{times}{\ldots})}\boldsymbol{G}$ keeps track of direct and indirect connections in g. More precisely, the (i, j)th cell of $\boldsymbol{G}^k$ gives the number of paths of length k in g from i to j. In particular, $\boldsymbol{G}^0 = \boldsymbol{I}$. Note that, by definition, a path between i and j needs not follow the shortest possible route between those agents. For instance, if $g_{ij} = g_{ji} = 1$, the sequence $i \rightarrow j \rightarrow i \rightarrow j$ constitutes a path of length three in g from i to j.

Definition 1 (Katz, 1953; Bonacich, 1987) *Given a vector $\boldsymbol{u} \in \mathbb{R}^n_+$, and $\phi \geq 0$ a small enough scalar, the vector of Katz-Bonacich centralities of parameter ϕ in network g is defined as:*

$$\boldsymbol{b_u}(g, \phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1}\, \boldsymbol{u} = \sum_{p=0}^{\infty} \phi^p \boldsymbol{G}^p \boldsymbol{u}. \tag{1}$$

Preferences Delinquents in network g decide how much effort to exert. We denote by y_i the delinquency effort level of delinquent i in network g and by $\boldsymbol{y} = (y_1, ..., y_n)'$ the population delinquency profile in network g. Each agent i selects an effort $y_i \geq 0$, and obtains a payoff $u_i(\boldsymbol{y}, g)$ that depends on the effort profile $\boldsymbol{y}$ and on the underlying network g, in the following way:

$$u_i(\boldsymbol{y}, g) = \underbrace{(\pi_i + \xi + \epsilon_i)\, y_i}_{\text{Proceeds}} - \underbrace{\frac{1}{2} y_i^2}_{\text{Cost of crime effort}} - \underbrace{p \cdot f \cdot y_i}_{\text{Cost of being caught}} + \underbrace{\phi \sum_{j=1}^{n} g_{ij} y_i y_j}_{\text{positive peer effects}} \tag{2}$$

where $\phi > 0$. This utility has a standard cost/benefit structure (as in Becker, 1968). The cost of being caught is captured by the probability of being caught $0 < p < 1$ times the fine $f \cdot y_i$, which increases with own effort y_i, as the severity of the punishment increases with one's involvement in crime. Also, individuals have a *direct* cost of committing crime equal to $\frac{1}{2} y_i^2$, which is also increasing in own crime effort y_i. The proceeds from crime are given by $(\pi_i + \xi + \epsilon_i)\, y_i$ and are increasing in own effort y_i. π_i denotes the *exogenous heterogeneity* that captures the *observable* characteristics of individual i (e.g. sex, race, age, parental education) and the observable *average* characteristics of individual i's best friends, i.e. average level of parental education of i's friends, etc. (*contextual*

[10] Matrices and vectors are in bold while scalars are in normal letters.

[11] All our theoretical results also hold under undirected networks since the (a)symmetry of the adjacency matrix $\boldsymbol{G}$ does not play any role in the proof of our theoretical results.

effects). To be more precise, π_i can be written as:

$$\pi_i = \sum_{m=1}^{M} \beta_{1m} x_i^m + \frac{1}{\overline{g}_i} \sum_{m=1}^{M} \sum_{j=1}^{n} \beta_{2m} g_{ij} x_j^m \tag{3}$$

where x_i^m belongs to a set of M variables accounting for observable differences in individual, neighborhood and school characteristics of individual i. β_{1m}, β_{2m} are parameters, and $\overline{g}_i = \sum_{j=1}^{n} g_{ij}$ is the number of friends of individual i. ξ denotes the the unobservable network characteristics, e.g., the prosperous level of the neighborhood/network g (i.e. more prosperous neighborhoods lead to higher proceeds from crime) and ϵ_i is an error term, which captures other uncertainty in the proceeds from crime. Both ξ and ϵ_i are observed by the delinquents (when choosing effort level) but not by the researcher.

To summarize, the utility function can be written as:

$$u_i(\boldsymbol{y}, g) = (\pi_i + \eta + \epsilon_i) y_i - \frac{1}{2} y_i^2 + \phi \sum_{j=1}^{n} g_{ij} y_i y_j, \tag{4}$$

where $\eta = \xi - pf$. Note that the utility (2) is concave in own decisions, and displays decreasing marginal returns in own effort levels.

In this model, referred to as the *local-aggregate model*, it is the sum of friends' efforts that affects own effort. There is an alternative model, the *local-average model* (Patacchini and Zenou, 2012) where it is the deviation from the average effort of own friends that is costly. We adopt the local-aggregate model here because the key-player policy relies on snow-ball and social-multiplier effects in this model. These kinds of effects will not be present in the local-average model because to affect people's decisions you need to change the social norm of all people in the network.

At equilibrium, each agent maximizes his/her utility (2) and the best-reply function, for each $i = 1, ..., n$, is given by:

$$y_i = \phi \sum_{j=1}^{n} g_{ij} y_j + \pi_i + \eta + \epsilon_i, \tag{5}$$

where π_i is defined by (3). Denote by $\mu_1(\boldsymbol{G})$ the spectral radius of $\boldsymbol{G}$ and by $\alpha_i \equiv \pi_i + \eta + \epsilon_i$, with the corresponding non-negative n-dimensional vector $\boldsymbol{\alpha}$. Ballester et al. (2006, 2010) and Calvó-Armengol et al. (2009) have characterized the Nash equilibrium of the game where agents choose their effort level $y_i \geq 0$ simultaneously and shown that, if $\phi\mu_1(\boldsymbol{G}) < 1$, the peer effect game with payoffs (2) has a unique Nash equilibrium in pure strategies given by:

$$\boldsymbol{y}^* \equiv \boldsymbol{y}^*(g) = \boldsymbol{b}_{\boldsymbol{\alpha}}(g, \phi). \tag{6}$$

This means that the Katz-Bonacich centrality is the right network index to account for equilibrium behavior when the utility function is linear-quadratic. Observe that

$$\boldsymbol{b}_{\boldsymbol{\alpha}}(g, \phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1} \boldsymbol{\alpha} = \sum_{p=0}^{\infty} \phi^p \boldsymbol{G}^p \boldsymbol{\alpha} \tag{7}$$

where $\boldsymbol{\alpha} = \boldsymbol{\pi} + \eta \boldsymbol{l}_n + \boldsymbol{\epsilon}$ and $\boldsymbol{l}_n$ is an n-dimensional vector of ones. To simplify notation, we drop the subscript $\boldsymbol{\alpha}$ in $\boldsymbol{b}_{\boldsymbol{\alpha}}(g, \phi)$ whenever there is no ambiguity. Let $b_i(g, \phi)$ be the ith entry of $\boldsymbol{b}(g, \phi)$. From (6), we have, for each individual i, $y_i^* = b_i(g, \phi)$. From (5), it is easy to show that the equilibrium utility is equal to:

$$u_i(\boldsymbol{y}^*, g) = \alpha_i y_i^* - \frac{1}{2} y_i^{*2} + \phi \sum_{j=1}^{n} g_{ij} y_i^* y_j = \frac{1}{2} y_i^{*2} = \frac{1}{2} \left[b_i(g, \phi) \right]^2 . \tag{8}$$

3.1.2 Finding the key player in the static model

In the context of the model above, Ballester et al. (2006, 2010) have studied the "key player" policy, which consists of finding the key player, i.e. the delinquent who, once removed from the network, generates the highest possible reduction in aggregate delinquency level. They assume that, once a person is removed from the network, the links of the remaining players do not change.

In Appendix A.1, following Ballester and Zenou (2012), we show that the intercentrality measure $d_i(g, \phi)$ of delinquent i is defined by (19). This formula highlights the fact that when a delinquent is removed from a network, two effects are at work. The first one is the *contextual effect*, which is due to the change in $\boldsymbol{\alpha}$ after the removal of a delinquent while the network g remains unchanged. The second effect is the *network effect*, which captures the change in the network structure when a delinquent is removed. More generally, the intercentrality measure $d_i(g, \phi)$ of delinquent i accounts both for one's exposure to the rest of the group and for one's contribution to every other exposure. A player i^* is the key player if and only if i^* is a delinquent with the highest intercentrality in g, that is, $d_{i^*}(g, \phi) \geq d_i(g, \phi)$, for all $i = 1, ..., n$ (Ballester et al. 2006, 2010; Ballester and Zenou, 2012). Observe that this result is true for both *undirected* networks (*symmetric* adjacency matrix) and *directed* networks (*asymmetric* adjacency matrix). It is also true for adjacency matrices with weights (i.e. values different than 0 and 1) and self-loops (delinquents have a link with themselves).

3.2 Dynamic network formation models

So far, we have assumed that the network was fixed and that, when a delinquent was removed, no new links were formed. In other words, we have an *invariant assumption* on the reduced network $g^{[-i]}$. This assumption is realistic in the short run but clearly not in the long run.

We would like, now, to develop a dynamic model that extends the key-player approach proposed by Ballester et al. (2006, 2010) and exposed in Section 3.1.2. We first present a dynamic formation model and then analyze the key-player policy within this framework.

3.2.1 The dynamic model

We would like, now, to develop a *dynamic model* where both network formation and effort decisions take place. As we will see below (Section 4), in our dataset, we only observe the network at one point in time. This will correspond to our network at $t = 0$ and to the initial condition of our dynamic network formation process. We will analyze how the network will evolve over time both with and without a key player policy. What will change after $t = 0$ is the structure of the network as well as the crime effort each criminal will provide.

Let us now describe this dynamic model. At each period in time t, we consider a two-stage (*morning-afternoon*) game, with the timing of the game given as follows.

In the *morning* of day t, with *equal probability*, an agent (say, agent i) is chosen and makes a link-formation decision under uncertainty as he/she does not know the realization of a random shock ϵ_i in utility (2). The shock is individual specific and i.i.d. across individuals such that $\mathrm{E}(\epsilon_i) = 0$ and $\mathrm{Var}(\epsilon_i) = \sigma_\epsilon^2 > 0$. The chosen agent can choose to nominate a new friend or keep the status quo but is not allowed to delete an existing link. We assume that the agent i who initiates a friendship relationship with agent j pays the link-formation cost c_i and that he/she is *myopic*, i.e. agent i only maximize expected utility at the end of day t.[12]

At *noon* of day t, a new network is formed, which is denoted by $g_t = g_t^{(i,j)}$ with the adjacency matrix $\boldsymbol{G}_t = \boldsymbol{G}_t^{(i,j)}$. If $j \neq i$, then $\boldsymbol{G}_t^{(i,j)} = \boldsymbol{G}_{t-1} + e_i e_j'$, where e_i is the ith column of the identity matrix. If $j = i$, then $\boldsymbol{G}_t^{(i,j)} = \boldsymbol{G}_{t-1}$.

In the *afternoon* of day t, the random shock ϵ_i in utility (2) is realized and its value becomes complete information for all agents. As in Section 3.1.1, all agents in the (new) network simultaneously choose their effort level to maximize their utility at time t. In particular, agent i will choose an effort level $y_{i,t} \equiv y_{i,t}(g_t^{(i,j)})$ to maximize the utility

$$u_{i,t}(\boldsymbol{y}_t, g_t^{(i,j)}) = (\pi_i + \eta + \epsilon_i)y_{i,t} - \frac{1}{2}y_{i,t}^2 + \phi \textstyle\sum_{k=1}^n g_{ik,t}y_{i,t}y_{k,t} - c_i(\bar{g}_{i,t} - \bar{g}_{i,t-1}) \tag{9}$$

where $\bar{g}_{i,t} = \sum_{k=1}^n g_{ik,t}$ and c_i is the marginal cost of a link.

In Appendix A.2, we solve this model by backward induction and determine a lower bound $\underline{c}_{i,t}$, defined by (25), such that if $c_i > \underline{c}_{i,t}$, then agent i will have no incentive to create a link at period t. We also analyze, in detail, the network formation process $(g_t)_{t=0}^{\infty}$, where g_t is the random variable realized at time $t \geq 0$. We have a *discrete time Markov chain* for the network formation process $(g_t)_{t=0}^{\infty}$ with $g_t = (N, L_t)$ comprising the set of delinquents $N = \{1, ..., n\}$ together with a set of links L_t at time t between them. The key question is how individuals choose among their potential linking partners. At every t, an agent i, selected uniformly at random from the set N, enjoys an opportunity of updating his/her current links. If an agent i receives such an opportunity, then he/she initiates a link to agent j which *increases his/her (expected) equilibrium payoff the most.* Agent j is said to be the best response of agent i given the network g_t. If the utility of not creating a link is higher than that of creating a link with individual i's best response, then a link is not created and the network is unchanged.

Definition 7 in Appendix A.2 says that, starting from a network at $t = 0$ (denoted by G_0), it converges to an equilibrium network g_T at time $t = T$ when each of the n delinquents in the

[12] The assumption of selecting one individual at a time is standard in the dynamic network formation literature (see e.g. Ehrhardt et al., 2006 or König et al., 2012) because, otherwise, it would be impossible to characterize the equilibria given the standard coordination problems prevalent in strategic network formation (see e.g. Jackson, 2008, chap. 6). The assumption of myopic behavior is also standard in this literature because of the numerous possible networks that can be formed in the future (for n players, there are $2^{n(n-1)/2}$ possible networks that can be formed). Observe, however, that, in our model, where agents form links to the most central agents, then assuming that agents will form links based on their discounted life time expected utility (forward-looking agents), instead of current utility, will not change the main results since they will still create links with the agent who has the highest Katz-Bonacich centrality in the network. This is because this agent has a higher probability than any other agent in the network to have the highest Katz-Bonacich centrality in the future (at any period in time). In other words, there exists a farsighted equilibrium which has the same properties as ours. In the simulation exercises (Section 6.1), we look at both myopic and forward-looking behavior and find the same results for the key-player policy.

network g_T has no incentive to create a new link at time $t = T$, that is, for $i \neq j$, $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})] > \max_j \mathrm{E}[u_{i,T}(\boldsymbol{y}_t^*, g_t^{(i,j)})]$ for all $i = 1, ..., n$. In terms of Markov chain, this means that the equilibrium network g_T is an *absorbing state*. From now on, when we use the word "equilibrium" or "equilibrium network" in the dynamic network formation model, we refer to the equilibrium concept defined in Definition 7.

Observe that the discrete time Markov chain is not ergodic (since it is not irreductible) and thus the absorbing state a network will converge to will strongly depend on the initial network G_0. In the empirical implementation of this model, the initial condition G_0 will be determined by the network we observe in the data.

3.2.2 Finding the key player in the dynamic model

We will now implement a key-player policy in the dynamic network formation model. At $t = 0$, before the dynamic network formation game described above starts, the planner will choose the key player i^* in the following way.[13] The planner will compare the (expected) total crime that will emerge in equilibrium (Definition 7) when he/she removes different delinquents from the network. As in Section 3.1.2, the key player i^* will be the delinquent whose removal leads to the lowest total expected crime in the equilibrium network.

Let us now calculate the total expected crime of the equilibrium network after removing a delinquent at $t = 0$. Let the adjacency matrix for the equilibrium network $g_T^{[-i]}$ without delinquent i be denoted by $\boldsymbol{G}_T^{[-i]}$. In that case, the expected equilibrium effort outcome is equal to:

$$\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})] = (\boldsymbol{I} - \phi \boldsymbol{G}_T^{[-i]})^{-1}(\boldsymbol{\pi}^{[-i]} + \eta \boldsymbol{l}_{n-1}).$$

Then the total expected crime for the equilibrium network is: $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})]$.

The planner's objective is to find the delinquent whose removal leads to the lowest total expected crime in the equilibrium network. Formally, the planner's problem is:

$$\min_i \left\{ \sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})] \mid i = 1, ..., n \right\}. \tag{10}$$

In the following sections, we present the empirical implementation of the key player policy using networks of juvenile delinquents in US schools as a case study. For that, we will first describe the data, then expose the econometric methodologies and finally comment on the empirical results.

4 Data

Our analysis is made possible by the use of a unique database on friendship networks from the National Longitudinal Survey of Adolescent Health (AddHealth). The AddHealth database has been designed to study the impact of the social environment (i.e. friends, family, neighborhood and

[13] We assume that, after the removal of the key player, a random person in the network initiates the search for new links. One could argue that, following the arrest of a crime leader, it is his/her closest associates that initiate the search for new links. Doing so will in fact not change any of our results in the simulation exercises (see Section 6.1). Indeed, even if we choose a person at random, we reach an equilibrium only when nobody in the network wants to form a link, including the closest associate of a crime leader. This is mainly because the networks are relatively small and we start from an equilibrium situation (nobody in the network wants to form a link) and move to another equilibrium situation (where after removing the key player nobody in the network wants to form a link).

school) on adolescents' behavior in the United States by collecting data on students in grades 7-12 from a nationally representative sample of roughly 130 private and public schools in years 1994-95. Every pupil attending the sampled schools on the interview day is asked to complete a questionnaire (*in-school survey*) containing questions on respondents' demographic and behavioral characteristics, education, family background and friendships. This sample contains information on roughly 90,000 students. A subset of adolescents selected from the rosters of the sampled schools, about 20,000 individuals, is then asked to complete a longer questionnaire containing more sensitive individual and household information (*in-home survey* and parental data).

For the purposes of our analysis, the most interesting aspect of the AddHealth data is the information on friendships. Indeed, the friendship information is based upon actual friend nominations. Pupils were asked to identify their best friends from a school roster (up to five males and five females).[14] Friend nominations, however, are not always reciprocal. If i nominates j as a friend, we set $g_{ij} = 1$, which implies that i is influenced by j according to the best-reply function (5). In other words, i can be influenced by j even though j does not consider i as his/her best friend.[15] For each school, we thus obtain all the networks of (best) friends. By matching the identification numbers of the friendship nominations to respondents' identification numbers, one can obtain information on the characteristics of nominated friends.

The in-home questionnaire contains an extensive set of questions on juvenile delinquency, that are used to construct our dependent variable. Specifically, the AddHealth dataset contains information on 15 delinquency items.[16] The survey asks students how often they participate in each of these activities during the past year.[17] Each response is coded using an ordinal scale ranging from 0 (i.e. never participate) to 1 (i.e. participate 1 or 2 times), 2 (participate 3 or 4 times) up to 3 (i.e. participate 5 or more times). To derive quantitative information on crime activity using qualitative answers to a battery of related questions, we calculate an index of delinquency involvement for each respondent.[18] The delinquency index ranges between 1.51 and 11.04, with mean equal to 2.35 and standard deviation equal to 1.09. Non-criminal individuals are defined as those who report never participating in any delinquent activity. This data allows us to get valid information on 6,993 (criminal and non-criminal) students distributed over 1,596 networks, with network size ranging between 2 and 1,050 individuals.[19] This sample is used to investigate the decision to commit crime

[14] The limit in the number of nominations is not binding (even by gender). Less than 1% of the students in our sample show a list of ten best friends.

[15] As highlighted by Wasserman and Faust (1994), centrality indices for directional relationships generally focus on choices made.

[16] Namely, painting graffiti or signs on someone else's property or in a public place; deliberately damaging property that didn't belong to you; lying to your parents or guardians about where you had been or whom you were with; taking something from a store without paying for it; getting into a serious physical fight; hurting someone badly enough to need bandages or care from a doctor or nurse; runing away from home; driving a car without its owner's permission; stealing something worth more than \$50; going into a house or building to steal something; use or threaten to use a weapon to get something from someone; selling marijuana or other drugs; stealing something worth less than \$50; taking part in a fight where a group of your friends was against another group; acting loud, rowdy, or unruly in a public place.

[17] Respondents listened to pre-recorded questions through earphones and then entered their answers directly on laptop computers. This administration of the survey for sensitive topics minimizes the potential for interview and parental influence, while maintaining data security.

[18] This is a standard factor analysis, where the factor loadings of the different variables are used to derive the total score.

[19] The large reduction in sample size with respect to the original AddHealth sample is mainly due to missing values in

(see Section 5.4.1). The analysis of the key player policy in crime focuses on networks of criminals only. Because the strength of peer effects may vary with network size (see Calvó-Armengol et al., 2009), the large variation in network size forces us to exclude networks at the extremes of the network size distribution. Indeed, our theoretical model assumes homogenous peer effects in crime ϕ, and we thus need to estimate only one peer-effect parameter.[20] Our selected sample consists of 1,297 criminals distributed over 150 networks, with network size ranging between 4 and 77 individuals.[21] The mean and standard deviation of network size are roughly 9 and 12 pupils.[22]

Table A.1 in Appendix B provides a description of the control variables used in our study and Table A.2 collects the summary statistics distinguishing between criminals and non criminals.[23] As expected, Table A.2 shows that delinquent students are less likely to be a female, religious, attached to the school and to come from less educated families than non-delinquent students. They are also more likely to reside in urban areas. Table A.2 also shows that the final sample we use does not lose representativeness.

5 Peer effects and network centrality

5.1 Econometric model

To determine the key player, first we need to estimate the endogenous peer effect parameter ϕ in the best-reply function (5). Let $\bar{r}$ be the total number of networks in the sample, n_r be the number of individuals in the rth network g_r, and $n = \sum_{r=1}^{\bar{r}} n_r$ be the total number of sample observations. Let $\boldsymbol{x}_{i,r} = (x_{i,r}^m, \cdots, x_{i,r}^M)'$. The econometric model corresponding to the best-reply function (5) of agent i in network g_r can be written as

$$y_{i,r} = \phi \sum_{j=1}^{n_r} g_{ij,r} y_{j,r} + \boldsymbol{x}_{i,r}'\boldsymbol{\beta}_1 + \frac{1}{\bar{g}_{i,r}} \sum_{j=1}^{n_r} g_{ij,r}\boldsymbol{x}_{j,r}'\boldsymbol{\beta}_2 + \eta_r + \epsilon_{i,r}, \tag{11}$$

where $\epsilon_{i,r}$'s are i.i.d. innovations with zero mean and variance σ^2 for all i and r. In matrix form, model (11) can be written as

$$\boldsymbol{y}_r = \phi \boldsymbol{G}_r \boldsymbol{y}_r + \boldsymbol{X}_r \boldsymbol{\beta}_1 + \boldsymbol{G}_r^* \boldsymbol{X}_r \boldsymbol{\beta}_2 + \eta_r \boldsymbol{l}_{n_r} + \boldsymbol{\epsilon}_r.$$

For a sample with $\bar{r}$ networks, stack up the data by defining $\boldsymbol{y} = (\boldsymbol{y}_1', \cdots, \boldsymbol{y}_{\bar{r}}')'$, $\boldsymbol{X} = (\boldsymbol{X}_1', \cdots, \boldsymbol{X}_{\bar{r}}')'$, $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1', \cdots, \boldsymbol{\epsilon}_{\bar{r}}')'$, $\boldsymbol{G} = \mathrm{diag}\{\boldsymbol{G}_r\}_{r=1}^{\bar{r}}$, $\boldsymbol{G}^* = \mathrm{diag}\{\boldsymbol{G}_r^*\}_{r=1}^{\bar{r}}$, $\boldsymbol{L} = \mathrm{diag}\{\boldsymbol{l}_{n_r}\}_{r=1}^{\bar{r}}$ and $\boldsymbol{\eta} = (\eta_1, \cdots, \eta_{\bar{r}})'$, where $\mathrm{diag}\{\boldsymbol{A}_k\}$ is a "generalized" block diagonal matrix in which the diagonal blocks are $m_k \times n_k$ matrices $\boldsymbol{A}_k$'s. For the entire sample, the model is

$$\boldsymbol{y} = \phi \boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta} + \boldsymbol{\epsilon}. \tag{12}$$

variables and to the network construction procedure. Indeed, roughly 20% of the students do not nominate any friends and another 20% cannot be correctly linked (for example because the identification code is missing or misreported).

[20] From an empirical point of view, the estimation of heterogenous peer effects requires the exclusion of network fixed effects, which are an important aspect of our identification strategy, allowing us to control for unobserved factors.

[21] Our results, however, do not depend crucially on these network size thresholds. They remain qualitatively unchanged when moving the network size window slightly.

[22] On average, delinquents declare having 1.33 delinquent friends with a standard deviation of 1.29.

[23] Information at the school level, such as school quality and teacher/pupil ratio, is unnecessary given our fixed effects estimation strategy.

In model (12), ϕ represents *the endogenous effect*, where an agent's choice/outcome may depend on those of his/her friends about the same activity; and $\boldsymbol{\beta}_2$ represents *the contextual effect*, where an agent's choice/outcome may depend on the exogenous characteristics of his/her friends. It is well known that endogenous and contextual effects cannot be separately identified in a linear-in-means model due to *the reflection problem*, first formulated by Manski (1993). We will discuss the identification of endogenous and contextual effects in the following subsection.

The vector of network fixed effects given by $\boldsymbol{\eta}$ captures *the correlated effect* where agents in the same network may behave similarly as they have similar unobserved individual characteristics or they face a similar (institutional) environment. The network fixed effect serves as a remedy for the selection bias that originates from the possible sorting of individuals with similar unobserved characteristics into a network. The underlying assumption is that such unobserved characteristics are common to the individuals within each network. This is extremely reasonable in our case study where the networks are extremely small (see Section 4).[24]

From the best-reply function (5), the network fixed effect also captures the deterrence effect on crime (pf in the theoretical model). Indeed, because in our sample networks are within schools, the use of network fixed effects also accounts for differences in the strictness of anti-crime regulations across schools (e.g. differences in the expected punishment for a student who is caught possessing illegal drugs, stealing school property, verbally abusing a teacher). As a result, instead of directly estimating deterrence effects (i.e. including in the model specification observable measures of deterrence, such as local police expenditures or the arrest rate in the local area), we focus our attention on the estimation of peer effects in crime, accounting for network fixed effects.[25]

In the econometric model, we allow network fixed effects $\boldsymbol{\eta}$ to depend on $\boldsymbol{G}$, $\boldsymbol{G}^*$ and $\boldsymbol{X}$ by treating $\boldsymbol{\eta}$ as a vector of unknown parameters (as in a fixed effect panel data model). When the number of groups $\bar{r}$ is large, we may have the incidental parameter problem. To avoid the incidental parameter problem, we transform (12) using the deviation from group mean projector $\boldsymbol{J} = \text{diag}\{\boldsymbol{J}_r\}_{r=1}^{\bar{r}}$, where $\boldsymbol{J}_r = \boldsymbol{I}_{n_r} - \frac{1}{n_r}\boldsymbol{l}_{n_r}\boldsymbol{l}'_{n_r}$. This transformation is analogous to the *within* transformation for the fixed effect panel data model. As $\boldsymbol{JL} = 0$, the transformed model is

$$\boldsymbol{Jy} = \phi\boldsymbol{JGy} + \boldsymbol{JX}\boldsymbol{\beta}_1 + \boldsymbol{JG}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{J}\boldsymbol{\epsilon}. \tag{13}$$

The proposed estimators and identification conditions are based on the transformed model (13).

5.2 Identification of the peer effects

In this subsection, we will first review the identification results when the endogenous effect takes the form of a local-average effect. Then, we will present new identification results when the endogenous effect takes the form of a local-aggregate effect.

[24] In fact, the use of an over-identifying restrictions test (Lee, 2007b) supports the validity of this assumption (see section 5.3).

[25] The identification of deterrence effects is a difficult empirical exercise because of the well-known potential simultaneity and reverse causality issues (Levitt, 1997).

5.2.1 The local-average endogenous effect

Bramoullé et al. (2009) and Lee et al. (2010) have considered the identification of the following network model

$$\boldsymbol{y} = \phi \boldsymbol{G}^* \boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta} + \boldsymbol{\epsilon}. \tag{14}$$

In (14), the term $\boldsymbol{G}^*\boldsymbol{y}$ represents the average effort level of an individual's friends. Therefore, we refer to this specification as the local-average model.

To estimate the transformed local-average model

$$\boldsymbol{J}\boldsymbol{y} = \phi \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y} + \boldsymbol{J}\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{J}\boldsymbol{\epsilon}$$

using linear IV estimators, the deterministic part of the right-hand-side (RHS) variables, $[\mathrm{E}(\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y}), \boldsymbol{J}\boldsymbol{X}, \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}]$, needs to have full column rank for large enough sample size. When $(\boldsymbol{I} - \phi\boldsymbol{G}^*)$ is nonsingular, from the reduced-form equation of the local-average model, we have

$$\mathrm{E}(\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y}) = \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_1 + (\boldsymbol{J}\boldsymbol{G}^{*2}\boldsymbol{X} + \phi\boldsymbol{J}\boldsymbol{G}^{*3}\boldsymbol{X} + \cdots)(\phi\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2). \tag{15}$$

If $\phi\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 = 0$, then $\mathrm{E}(\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y}) = \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_1$. Thus, the model cannot be identified as $[\mathrm{E}(\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y}), \boldsymbol{J}\boldsymbol{X}, \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}]$ does not have full column rank. On the other hand, if $\phi\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 \neq 0$, then $\boldsymbol{J}\boldsymbol{G}^{*2}\boldsymbol{X}$, $\boldsymbol{J}\boldsymbol{G}^{*3}\boldsymbol{X}$, etc. can be used as instruments for the endogenous effect variable $\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{y}$. The term $\boldsymbol{G}^{*2}\boldsymbol{X}$ represents the characteristics of "second-order" indirect friends. If individuals i, j are friends and j, k are friends, it does not necessarily imply that i, k are also friends. Thus, *the intransitivity in social connections* provides an exclusion restriction to identify endogenous and contextual effects.[26] Based on this important observation, Bramoullé et al. (2009) have argued that the local-average model is identified if $\boldsymbol{I}, \boldsymbol{G}^*, \boldsymbol{G}^{*2}, \boldsymbol{G}^{*3}$ are linearly independent.

5.2.2 The local-aggregate endogenous effect

In model (12), the term $\boldsymbol{G}\boldsymbol{y}$ represents the aggregate effort level of an individual's friends. Therefore, we refer to this specification as the local-aggregate model. When $(\boldsymbol{I} - \phi\boldsymbol{G})$ is nonsingular, from the reduced-form equation of (12), we have

$$\mathrm{E}(\boldsymbol{J}\boldsymbol{G}\boldsymbol{y}) = \boldsymbol{J}(\boldsymbol{G}\boldsymbol{X} + \phi\boldsymbol{G}^2\boldsymbol{X} + \cdots)\boldsymbol{\beta}_1 + \boldsymbol{J}(\boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X} + \phi\boldsymbol{G}^2\boldsymbol{G}^*\boldsymbol{X} + \cdots)\boldsymbol{\beta}_2 + \boldsymbol{J}(\boldsymbol{G}\boldsymbol{L} + \phi\boldsymbol{G}^2\boldsymbol{L} + \cdots)\boldsymbol{\eta}. \tag{16}$$

So even if $\boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = 0$, the deterministic part of the RHS variables in (13), $[\mathrm{E}(\boldsymbol{J}\boldsymbol{G}\boldsymbol{y}), \boldsymbol{J}\boldsymbol{X}, \boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}]$, can still have full column rank as $\mathrm{E}(\boldsymbol{J}\boldsymbol{G}\boldsymbol{y}) = \boldsymbol{J}(\boldsymbol{G}\boldsymbol{L} + \phi\boldsymbol{G}^2\boldsymbol{L} + \cdots)\boldsymbol{\eta}$. According to Definition 1 (Section 3), the matrix $\boldsymbol{G}\boldsymbol{L} + \phi\boldsymbol{G}^2\boldsymbol{L} + \cdots$ collects the Katz-Bonacich centrality for all individuals. The leading order term of the Katz-Bonacich centrality matrix is $\boldsymbol{G}\boldsymbol{L} = \mathrm{diag}\{\boldsymbol{G}_r\boldsymbol{l}_{n_r}\}_{r=1}^{\bar{r}}$, where $\boldsymbol{G}_r\boldsymbol{l}_{n_r}$ is a vector of *row-sums* of $\boldsymbol{G}_r$. When considering social network interactions, $\boldsymbol{G}_r\boldsymbol{l}_{n_r}$ has a natural interpretation, with its ith element being the number of friends student i nominates. Therefore, for the local-aggregate model, besides the characteristics of friends, the number of friends

[26] In a linear-in-means model, individuals are affected by all individuals belonging to their group and by nobody outside the group. Hence, simultaneity in behavior of individuals in the same group introduces a perfect collinearity between the endogenous effect and the contextual effect (i.e. the so-called *reflection problem*, Manski, 1993)

can be used as an additional instrument for the endogenous effect variable.

Let $\boldsymbol{Z} = [\boldsymbol{Gy}, \boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X}]$. For identification of (12) via (13) through linear IV estimators, $\mathrm{E}(\boldsymbol{JZ})$ needs to have full column rank for large enough sample size. The following proposition gives sufficient conditions for the rank condition. Henceforth, let c (possibly with subscripts) denote a constant scalar that may take different values for different uses.

Proposition 2 *When $\boldsymbol{G}_r$ has non-constant row sums for some network r, $\mathrm{E}(\boldsymbol{JZ})$ has full column rank if:* (i) *$\boldsymbol{I}_{n_r}, \boldsymbol{G}_r, \boldsymbol{G}_r^*, \boldsymbol{G}_r\boldsymbol{G}_r^*$ are linearly independent and $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \eta_r$ are not all zeros; or* (ii) *$\boldsymbol{G}_r\boldsymbol{G}_r^* = c_1\boldsymbol{I}_{n_r} + c_2\boldsymbol{G}_r + c_3\boldsymbol{G}_r^*$ and $\boldsymbol{\Lambda}_1$ given by (29) has full rank.*

When $\boldsymbol{G}_r$ has constant row sums such that $\bar{g}_{i,r} = \bar{g}_r$ for all r, $\mathrm{E}(\boldsymbol{JZ})$ has full column rank if: (iii) *$\boldsymbol{I}, \boldsymbol{G}, \boldsymbol{G}^*, \boldsymbol{GG}^*, \boldsymbol{G}^{*2}, \boldsymbol{GG}^{*2}$ are linearly independent and $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ are not both zeros;* (iv) *$\boldsymbol{I}, \boldsymbol{G}, \boldsymbol{G}^*, \boldsymbol{GG}^*, \boldsymbol{G}^{*2}$ are linearly independent, $\boldsymbol{GG}^{*2} = c_1\boldsymbol{I} + c_2\boldsymbol{G} + c_3\boldsymbol{G}^* + c_4\boldsymbol{GG}^* + c_5\boldsymbol{G}^{*2}$, and $\boldsymbol{\Lambda}_2$ given by (30) has full rank; or* (v) *$\bar{g}_r = \bar{g}$ for all r, $\boldsymbol{I}, \boldsymbol{G}^*, \boldsymbol{G}^{*2}, \boldsymbol{G}^{*3}$ are linearly independent, and $\phi_1\bar{g}\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 \neq 0$.*

The proof of Proposition 2 is given in Appendix C. From Proposition 2, we can see that, for a given network, sometimes identification of the local-aggregate model is easier to achieve than the local-average model. Figure 1 gives some examples where identification is possible for the local-aggregate model but fails for the local-average model. First, consider a dataset where each network is represented by graph (a) of Figure 1 (a star-shaped network). The corresponding adjacency matrix $\boldsymbol{G}$ is a block-diagonal matrix with diagonal blocks $\boldsymbol{G}_r$ representing graph (a). For the row-normalized adjacency matrix $\boldsymbol{G}^*$, it is easy to see that $\boldsymbol{G}^{*3} = \boldsymbol{G}^*$. Therefore, it follows from Proposition 5 of Bramoullé et al. (2009) that the local-average model (14) is not identified. On the other hand, as $\boldsymbol{G}_r$ corresponding to the graph (a) has non-constant row sums and $\boldsymbol{I}_{n_r}, \boldsymbol{G}_r, \boldsymbol{G}_r^*, \boldsymbol{G}_r\boldsymbol{G}_r^*$ are linearly independent, it follows from our Proposition 2(i) that the local-aggregate model can be identified for this network.

[Insert Figure 1 here]

Graphs (b) and (c) in Figure 1 provide other examples where the local-average model cannot be identified while the local-aggregate model can be. Consider a dataset that consists of two types of networks. The first $\bar{r}_1$ networks are represented by graph (b) of Figure 1 (a regular or circle network). The rest $\bar{r}_2$ networks are represented by graph (c) of Figure 1 (a bi-partite network). Suppose $\bar{r}_1 > 0$, $\bar{r}_2 > 0$ and $\bar{r}_1 + \bar{r}_2 = \bar{r}$. The corresponding adjacency matrix $\boldsymbol{G}$ is a block-diagonal matrix with the first $\bar{r}_1$ diagonal blocks $\boldsymbol{G}_{1r}$ representing graph (b) and the rest $\bar{r}_2$ diagonal blocks $\boldsymbol{G}_{2r}$ representing graph (c). For the row-normalized adjacency matrix $\boldsymbol{G}^*$, it is easy to see that $\boldsymbol{G}^{*3} = \boldsymbol{G}^*$. Therefore, it follows from Proposition 5 of Bramoullé et al. (2009) that the local-average model (14) is not identified. On the other hand, as $\boldsymbol{G}_{1r}$ and $\boldsymbol{G}_{2r}$ have different row sums, $\boldsymbol{I}, \boldsymbol{G}, \boldsymbol{G}^*, \boldsymbol{GG}^*, \boldsymbol{G}^{*2}$ are linearly independent and $\boldsymbol{GG}^{*2} = \boldsymbol{G}$. Therefore, the local-aggregate model can be identified by our Proposition 2(iv).

5.3 Estimation methods and results

We estimate the local-aggregate network model (12) using the 2SLS and GMM estimators proposed in Liu and Lee (2010).[27] As discussed above, for the local-aggregate model, besides the "conventional" instruments $\boldsymbol{G}^{*2}\boldsymbol{X}$ based on characteristics of friends' friends (Bramoullé et al., 2009), the number of friends given by $\boldsymbol{GL}$ can be used as additional instruments to improve estimation efficiency. The GMM estimator uses additional quadratic moment conditions and can be helpful when the conventional instruments are weak. However, as $\boldsymbol{GL}$ has $\bar{r}$ columns, there would be a large number of instruments if the number of networks $\bar{r}$ is large. As a result, the 2SLS and GMM estimators with the instruments $\boldsymbol{GL}$ may have the "many-instrument" bias if $\bar{r}$ is large relative to the sample size. Liu and Lee (2010) have suggested a bias-correction procedure based on the estimated leading-order "many-instrument" bias. The details of the 2SLS and GMM estimators considered in this empirical study are given in Appendix D.

The validity of our identification strategy and the moment conditions employed by the 2SLS and GMM estimators rests on the exogeneity of the adjacency matrix $\boldsymbol{G}$. We test the exogeneity of $\boldsymbol{G}$ using the over-identifying restrictions (OIR) test (Lee, 1992). If the OIR test cannot reject the null hypothesis that the moment conditions are correctly specified, then it provides evidence that $\boldsymbol{G}$ is uncorrelated with the error term, when $\boldsymbol{X}$, contextual effects and network fixed effects are controlled.

[Insert Table 1 here]

Table 1 collects the estimation results of the local-aggregate network model (12). The first stage partial F-statistics (see Stock et al., 2002 and Stock and Yogo, 2005) reveal that our instruments are quite informative. Hence, the 2SLS and GMM deliver similar results. The GMM estimators are only more precise in the estimation of ϕ. The p-value of the OIR test is larger than conventional significance levels, which provides evidence on *the exogeneity of network structure* (conditional on $\boldsymbol{X}, \boldsymbol{G}^{*}\boldsymbol{X}$ and network fixed effects). All our estimates of ϕ are within the parameter space given by $\phi < 1/\mu_1(\boldsymbol{G})$. Such a condition is needed to guarantee the existence of the equilibrium in our model (equation (6) in Section 3). According to the bias-corrected many-IV GMM estimator, in a group of two friends, a standard deviation increase in the level of delinquent activity of the friend translates into a roughly 5 percent increase of a standard deviation in the individual level of activity. If we consider an average group of 4 best friends (linked to each other in a network), a standard deviation increase in the level of delinquent activity of each of the peers translates into a roughly 17 percent increase of a standard deviation in the individual level of activity. This is a non-negligible effect, especially given our long list of controls.

5.4 Robustness of the estimation results

We would like to propose different robustness checks for the estimation of ϕ in the local-aggregate network model (12).

[27] The GMM estimator improves estimation efficiency of the 2SLS estimator by using quadratic moment conditions based on the correlation structure of the error term for estimation. See Appendix D for more details.

5.4.1 Endogenous participation in criminal activities

First, we introduce the endogenous participation in criminal activities into the model (12). We consider a type-II Tobit model. We specify *the participation equation* as

$$\boldsymbol{y}_{1\bar{n}}^{*} = \boldsymbol{X}_{1\bar{n}}\boldsymbol{\gamma} + \boldsymbol{\epsilon}_{1\bar{n}}, \tag{17}$$

where $\bar{n}$ is the total number of individuals in the population, $\boldsymbol{y}_{1\bar{n}}^{*} = (y_{1,1}^{*}, \cdots, y_{\bar{n},1}^{*})'$, $\boldsymbol{X}_{1\bar{n}} = (\boldsymbol{x}_{1,1}, \cdots, \boldsymbol{x}_{\bar{n},1})'$, and $\boldsymbol{\epsilon}_{1\bar{n}} = (\epsilon_{1,1}, \cdots, \epsilon_{\bar{n},1})'$. Agent i will become a criminal if and only if $y_{i,1}^{*} > 0$. For the individuals who decide to be criminals, *the outcome equation* given by (12) determines the crime effort levels.

Without loss of generality, we assume the first n (of the $\bar{n}$) individuals will choose to become criminals. By assuming joint normality of a criminal's error terms across the participation and outcome equations, the *selection-bias-corrected* outcome equation is given by

$$\boldsymbol{y} = \phi\boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^{*}\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta} + \sigma_{12}\boldsymbol{\lambda} + \boldsymbol{\epsilon}^{*}, \tag{18}$$

where $\boldsymbol{\lambda} = [\lambda(\boldsymbol{x}_{1,1}'\boldsymbol{\gamma}), ..., \lambda(\boldsymbol{x}_{n,1}'\boldsymbol{\gamma})]'$ and $\lambda(\boldsymbol{x}_{i,1}'\boldsymbol{\gamma})$ is the inverse Mills ratio (IMR) for criminal i. Equation (18) can be estimated following Heckman's two-step approach, which is detailed in Appendix E.1.

Observe that, if $\boldsymbol{X}_{1\bar{n}}$ in (17) contains the same set of exogenous variables as $\boldsymbol{X}$ in (18), then the identification of the IMR term in (18) solely relies on the nonlinear functional form of $\lambda(\cdot)$. As $\lambda(\cdot)$ is approximately linear in a wide range of its domain, this identification can be weak. Hence, we need to find variables in $\boldsymbol{X}_{1\bar{n}}$ that are excluded from $\boldsymbol{X}$. Our exclusion restrictions are inspired by the entry model of Ballester et al. (2010) according to which there is a cost that individuals have to pay when entering criminal activity (sunk cost) and individual characteristics are the key determinants of the crime participation. The cost can also be interpreted as the outside option of non-criminals. We then use two types of exclusion restrictions.

The first instrument captures the idea that there is a moral cost of committing crime. Children coming from families with stronger moral and family values may pay an higher "entry cost". However, once a crime is committed for the first time (hence parents and relatives are already disappointed), there is no additional cost of this sort when committing more crimes.[28] To measure this moral cost, we use the information on religion practice, which is the answer to the following question: "In the past 12 months, how often did you attend religious services", coded as 1= never, 2= less than once a month, 3= once a month or more, but less than once a week, 4= once a week or more, coded as 5 if the previous is skipped because of response "none" to the question: "What is your religion?" (see Appendix B). Table 2 (upper panel, column IV1: moral cost) reports the first stage of the Heckman selection model when we use this instrument (Religion Practice). One can see that it is a strong predictor of criminal activity (with an expected negative association).

The second instrument aims to capture the idea that there are outside options for crime in terms

[28] Another interpretation is that kids from families with strong moral values can have a higher cost of anticipating being found out. In that case, only kids with low "disappointing their parents" price will become criminals. Then, once a kid has decided to become a criminal, this anticipation cost becomes insignificant.

of activities and money. A student receiving a generous weekly allowance from his/her parents and/or having money from other jobs and/or being busy in other activities is less likely, in the first place, to consider crime activity as an appealing option. We have thus constructed an indicator capturing the relevance of the outside options for each student. We use a similar factor analysis that we employed when constructing an index of criminal activity to derive a composite score using the answers to the following questions: (i) During the past week, how many times did you do work around the house, such as cleaning, cooking, laundry, yardwork, or caring for a pet? (ii) During the past week, how many times did you do hobbies, such as collecting baseball cards, playing a musical instrument, reading, or doing arts and crafts? (iii) During the past week, how many times did you go rollerblading, roller-skating, skate-boarding, or bicycling? (iv) How much money do you earn in a typical non-summer week from all your jobs combined? (v) How much is your allowance each week? If you don't receive your allowance weekly, how much would it be each week? (see Appendix B). Table 2 (upper panel, column IV2: moral cost & outside options) reports the first stage of the Heckman selection model when we add this instrument (Outside Options). It appears that this outside option indicator seems to be a strong predictor of criminal activity and the relationship is of the predicted (negative) sign.

[Insert Table 2 here]

In Table 2, we also report the estimation results of the selection-bias-corrected outcome equation (18) without exclusion restrictions (column 1), with the moral cost exclusion restriction (column 2), and with both the moral cost and outside option exclusion restrictions (column 3). We find that the results are qualitatively the same as the results reported in Table 1, which provides evidence that the estimation results are robust to endogenous crime participation.[29,30]

5.4.2 Sampling on criminals

Second, we take into account a possible sampling issue because questions on delinquency activities are only asked to the students in the in-home sample.[31] Thus, if a criminal has a friend who is not in the in-home sample, we do not know if this friend is a criminal or not. As a result, when constructing criminal networks this node is excluded from the network and we face a possible misspecification of the network topology (if this friend is criminal).

As a robustness check, we estimate our model using the subsample of students whose nominated friends are all in the in-home sample. For those individuals the information on the network connection is complete. The estimates are reported in column (1) of Table 3 and remain largely unchanged

[29] The significant estimate of σ_{12} suggests that the IMR term is an omitted variable in the error term of the outcome equation without selection-bias correction. The OIR test for the finite-IV 2SLS estimator of the outcome equation without selection-bias correction suggests that the instruments are uncorrelated with the error term (including the omitted IMR). This evidence is consistent with the robustness of estimation results with respect to endogenous crime participation.

[30] The inclusion of "religion practice" in the outcome equation returns an insignificant estimated coefficient. The estimation results which are obtained using only the outside option indicator as exclusion restrction remain roughly unchanged.

[31] This is a different sampling issue from the one studied by Chandrasekhar and Lewis (2012) and Liu (2013) where only a subsample of the population is surveyed to nominate their friends. For the AddHealth data, all the students in a school are surveyed to nominate their friends but a student's crime effort cannot be observed if he/she is not in the in-home subsample.

with respect to those reported in the first column of Table 1.[32]

[Insert Table 3 here]

5.4.3 The role of non-criminal friends

Finally, we consider the role of non-criminal friends. In our theoretical model, a criminal's effort level depends on the aggregate effort level of his/her friends. As the crime effort level of a non-criminal friend is zero, a non-criminal friend has no direct influence on a criminal's effort level. However, non-criminal friends may affect the crime effort decision through the following two channels: (i) a non-criminal friend may bridge two criminals and pass on the endogenous peer effect; (ii) a non-criminal friend's characteristics may affect a criminal's crime effort decision through the contextual effect.

Regarding (i), we estimate our model using a subsample of criminals whose friends are all criminals. For these criminals (who only nominate criminal friends), there is clearly no uncaptured effect of non-criminal friends. The results of this robustness check are contained in column (2) of Table 3. Our estimate of peer effects is roughly unchanged with respect to those reported in the first column of Table 1, which suggests that the "bridge" effect of non-criminal friends is negligible.[33]

Regarding (ii), we repeat our regression analysis using the entire network of social contacts, i.e. both criminals and non criminals, thus considering the average characteristics of non-criminal friends as additional regressors in model (12). Observe that, in this exercise, because we also consider non-criminal friends in the network, our IVs (which are based on the characteristics of indirect friends) might change (because the indirect friends might be different). The estimation results are contained in the last column of Table 3 and are, again, very similar. Our robustness checks thus show that the influence of non-criminal friends does not seem to matter much in the estimation of peer effects.

6 Who is the key player?

Let us now calculate empirically who is the key player in each of our real-world networks. For this part, we will mainly use the dynamic network formation model developed in Section 3.2 and determine the key player as we did in Section 3.2.2, by solving the program given by (10). We will *structurally simulate* this model. To study the convergence of the dynamic network formation model after the removal of the key player, we assume that the criminal networks observed in the AddHealth data in 1994/1995 are *equilibrium* networks and thus, to guarantee that this is true, we need to set

[32] The sample size is reduced by about 50% by using this subsample. This robustness check can be justified only if this subsample of (criminal) students is a random subsample of the in-home (criminal) sample. Panel (a) of Table A3 in Appendix E.2 shows the degree distributions of the criminals in the in-home sample and of those nominating friends only inside the in-home sample. The two distributions are remarkably similar and a formal comparison of the two distributions using non parametric tests does not reject the null hypothesis that they are two random samples drawn from the same population.

[33] The sample size is reduced by about 60% by using this subsample. We compare the degree distributions of the in-home criminals and the subsample of criminals whose friends are all criminals. Panel (b) of Table A3 in Appendix E.2 shows that the two distributions are remarkably similar and a formal comparison of the two distributions using non-parametric tests does not reject the null hypothesis that the two samples are two random drawings from the same population. Observe that our data are on teenagers and thus report mainly on petty crimes, which include for example lying to parents. It is thus reasonable to think that, in our context, a criminal with all criminal friends is not so dissimilar from a criminal with both criminal and non-criminal friends.

the cost c_i of forming links high enough so that no one in the observed network has incentive to form new links. In Appendix A, we explain in detail how we do this.

6.1 Simulation results

As described in Section 4, we have 1,297 criminals distributed over 150 networks. For the dynamic structural model, we only retain networks that satisfy the eigenvalue condition $\phi\mu_1(\boldsymbol{G}_t) < 1$, as $\boldsymbol{G}_t$ evolves.[34] Fortunately, only 5 networks fail to satisfy this condition and thus we end up with 1,038 criminals distributed over 145 networks. Figure 2 displays the distribution of these 145 networks by their size. The distribution is clearly very skewed to the left as we focus on small networks. The average network is size 7. The networks contain a minimum of 4 and a maximum of 64 delinquents.

[Insert Figure 2 here]

In Figure 3, we investigate the three following questions concerning the key player found in the dynamic network-formation model: Is he/she the most active delinquent in the network (left panel)? Does he/she have the highest betweenness centrality (middle panel)?[35] Is he/she also the key player in the static network model, which is determined by the intercentrality measure (19)[36] (right panel)?[37]

Figure 3 indicates that it is *not* straightforward to determine which delinquent should be removed from a network by only observing his or his/her criminal activities or position in the network.[38] Indeed, Figure 3, left panel, shows that, in many cases, the key player is *not* the most active delinquent (i.e. the delinquent with the highest Katz-Bonacich centrality) in the network. For example, for networks of size 4, 66 percent have key players who are not the most active deliquents while, for networks with six delinquents, this number is 76%. If we now consider Figure 3, middle panel, we find that an even higher number of key players do not have the highest betweenness centrality in the network. Indeed, for networks of size 4, 55 percent of key players do not have the highest betweenness centrality in the network while, for networks of a bigger size, this number can be over 80 percent (for networks of size 9 and 10, this number is 100 percent). As a result, if a planner does not use a key-player policy but his/her intuition or "a rule of thumb" like targeting the most active criminal or the one who bridges different clusters in a network (i.e. highest betweenness centrality), then he/she will, in many cases, target the "wrong" person, i.e. *not* the delinquent who

[34] In fact, this condition needs to be satisfied for all networks during the dynamic link-formation process. Since the eigenvalue of a network (and thus the largest eigenvalue condition) changes over time, we use a sufficient condition: $\phi\sqrt{\sum_{i,j} g_{ij} + n - 1} < 1$, which uses an upper bound on the largest eigenvalue (see Corrolary 1 page 1409 in Ballester et al., 2006). The main advantage of this new condition is that one does not need to calculate the largest eigenvalue of the network at each period in time.

[35] The betweenness centrality is another centrality measure that counts the number of shortest paths that goes through an agent (see Wasserman and Faust, 1994).

[36] Remember that this formula gives the key player when other delinquents cannot create new links after the removal of the key player; this is referred to as the *invariant case*.

[37] Observe that, in Figure 3, we display the simulation results for networks of size between 4 and 10 because they are extremely few networks of size greater than 10 delinquents.

[38] In Figure 3, we consider only networks of size between 4 and 10; there are 130 networks of this size. We only consider networks of size between 4 and 10 to save space and because, for networks larger than 10, there are, usually, only one or two networks for each network size. For example, there is only one network with 12 individuals, one with 13 individuals and one with 15 individuals.

will reduce total crime in the network the most.

Finally, if we look at the right hand panel of Figure 3, we see that in most cases (90 percent for networks of size 4 and 100 percent for networks of sizes 8 and 10), the key players are the same in the static and the dynamic models, meaning that the invariant assumption is relatively good in our context. This is because our networks are relatively small and we set the link-formation cost relatively high to ensure that the observed networks in the data are in equilibrium. As a result, the convergence to the new equilibrium network is very fast in the simulation exercises with an average of 0.45 days and a maximum of 5 days and the equilibrium networks in the dynamic model are usually quite similar to the initial networks right after a delinquent is removed.

Observe that, with different data, for example adult criminal networks, there may be important differences between a static and a dynamic model, especially because the structure of criminal groups has changed from hierarchical, centralized organizations to highly adaptable networks that are decentralized and flat. There are still leaders but leaders of networks, not leaders of hierarchies. So when someone is removed from such a network, he/she is quickly replaced by new leaders and that is why a dynamic network formation is needed. Furthermore, in many datasets, information on link formation is usually available at many points in time (see Section 7.2.1) and a dynamic model will then be indispensable.

[Insert Figure 3 here]

Figure 4a shows how much change in the network density[39] occurs after the removal of the key player where, on the left panel, we consider the static (or invariant) network case while, on the right panel, the dynamic network-formation model is displayed. We see that there is no clear pattern. The density could either increase or decrease after the removal of the key player. We also see that there is little difference between the static and the dynamic model for the same reasons given above. In Figure 4b, we examine the density change from the static network to the dynamic network (after the removal of the key player). In that case, the network density increases in the dynamic network formation process as compared to the static model.

[Insert Figures 4a and 4b here]

6.2 Characteristics of the key player

Once we have identified the key player for each network, we can draw his or her "profile" by comparing the characteristics of these key players with those of the other criminals in the network. Table 4 displays the results only for the variables where the difference in means between these two samples is statistically significant.

Compared to other criminals, key players are less likely to be a female, are less religious, belong to families whose parents are less educated and have the perception of being socially more excluded. They also feel that their parents care less about them, are less likely to come from families where both parents are married and have more trouble getting along with teachers. An interesting feature

[39]Network *density* is simply the fraction of links present in a network over all possible ones. It ranges from 0 to 1 as networks get denser.

is that key players are more intelligent (i.e. higher mathematics scores) than the average criminal and are more likely to have friends who are older (i.e. in higher grades), more religious and whose parents are more educated. Also, even though key players themselves do not have better self-esteem, are not more physically developed nor are they more likely to be urbanites than other criminals.[40,41]

[Insert Table 4 here]

7 Policy implications

Let us address the fundamental policy issues of the key player. First, we further develop the qualitative approach (simulations) of Section 6 to evaluate the benefits versus costs of a key-player policy. Second, we provide evidence that there exist data on criminal networks and discuss the extent to which our analysis can be used in practice to address general policies against crime. Finally, we show that the methodology developed in this paper can be used to address policies in other activities such as financial networks, R&D networks, social networks in developing economies, political networks and tax-evasion networks.

7.1 Qualitative approach of the key-player policy using the AddHealth data

The first key issue is *when* a key-player policy should be implemented and when it should not. We perform a simulation exercise comparing the key player policy with another "potential" public policy from which relevant lessons could be drawn.

For each of the 145 networks, we calculate the reduction in total crime,[42] following the removal of the key player. We report, in Figure 5, the relationship between crime reduction and network size where, in the left panel, we consider the static (or invariant) network case while, in the right panel, the dynamic network-formation model is displayed. First, not surprisingly, there are nearly no difference between the static and the dynamic model. Second, one can see that there are very large variations in crime reduction between different networks. Indeed, for some networks, total crime is reduced by less than 5 percent while, for other networks, the reduction in crime can be as high as 35 percent. Moreover, the crime reduction is much more important in small networks than in large networks. This is because we remove only one key player, and in large networks the effect is clearly lower than in small networks.

[Insert Figure 5 here]

Figure 6 plots the average crime reduction for all the 145 networks when a key-player (solid curve) and a random-target policy (dash curve) is implemented. To plot this figure, we have put together networks of the same size and calculated the average crime reduction for this network size under the two policies. For example, for all networks of size 4 (horizontal axis), the average crime

[40] Summary statistics of the other characteristics of the key players, as well as that of their friends are not reported for brevity. They remain available upon request.

[41] Our analysis of the key player has also been performed for petty crimes and more serious crimes separately. The results are available upon request.

[42] This reduction is calculated as: [Expected total crime of the converged network (before the removal of the key player) - Expected total crime of the converged network (after the removal of the key player)] / Expected total crime of the converged network (before the removal of the key player).

reduction is 29.94 percent on average when the key-player policy is implemented while it is 23.86 percent when a random-target policy is implemented. The results displayed in Figure 6 confirms the fact that a key-player policy is important since the difference in crime reduction between these two policies can be extremely large, especially for big networks where *implementing a random-target policy can backfire by increasing rather than decreasing crime*. For example, for the network of 34 individuals, randomly removing someone from this network *increases* total crime by 9.82 percent because the remaining individuals form new links that generates more crime.[43] We believe that we obtain these results because targeting key players generate large *multiplier and amplifying effects* as opposed to a random-target policy. Indeed, targeting the key player, not only affects his/her criminal activities, but also the criminal activities of his/her direct friends, which, in turn, affects that of their direct friends, etc. In a random-target policy, these chain effects are similar but the target is on the "wrong" person. Our simulations show that a key-player policy should be implemented when the crime reduction difference between a key-player and a random-target policy is relatively high. In the real-world, there are some administrative costs of collecting detailed data on the social networks linking individuals and on their criminal activity. However, in Section 7.2.1, we provide a list of data on criminal networks suggesting that the administrative costs of gathering information on network data are not that high.

[Insert Figure 6 here]

7.2 How the key-player policy can be useful in fighting crime

7.2.1 Data about criminal networks

In order to apply the key-player policy one needs to collect detailed data on the social networks linking individuals and on their criminal activity. Despite the importance of the key-player problem for law enforcement agencies, to the best of our knowledge, there is no widespread research on centrality analysis in large scale criminal networks to show if and how this kind of analysis can help in crime reduction and prevention or other efforts aiming at destabilizing the criminal network structure. The lack of research in this field can be explained with the perception that the sensitivity of crime data does not permit the release of public real-world crime data sets. However, in the real-world, there is a lot of information and data about criminal networks that could easily be used to implement our key-player policy. Let us give a (selective) list of them:[44]

(i) *Juvenile deliquency in schools (survey data):* There are data similar to the AddHealth data for students in schools. For example, Weerman (2011) uses data from the Netherlands Institute for the Study of Crime and Law Enforcement (NSCR) "School Study", a study that focused on social networks and the role of peers in delinquency with two waves, conducted in the spring of 2002 and

[43] In Tables 14a and 14b in Liu et al. (2012), we provide several examples where total crime *increases* after the removal of a delinquent if the latter is not the key player.

[44] There is a recent literature which explores theoretical and empirical methods of inferring networks from node attributes and from functions of these attributes that are computed for connected nodes. In particular, Polanski and McVicar (2012) discuss the conditions, under which an underlying connection structure can be (probabilistically) recovered, and propose a Bayesian recovery algorithm. See also Costebander and Valente (2003) who show that centrality measures based on connectivity (rather than betweenness) are robust to mispecifications in sociometric data, and thus open the door to estimations of centrality measures with incomplete samples of network data.

2003. Students were provided with a numbered list of all students in the same grade in their school and were asked to fill in the numbers of those fellow students they spend time with at school ("with which of these students do you associate regularly?"), with a maximum of ten possible nominations. Students' delinquent behavior was measured using self-reports on a variety of offenses. The final sample consisted of 1,156 students in ten schools that participated in both waves.

(ii) *Adult crime (police data)*: the police has in fact quite a lot of information on criminal networks. For example, in his book, Sarnecki (2001) was able to construct the network of all criminals in Sweden for several years. The way a link is defined is as follows. Each time two (or more) persons are suspected of a crime (*co-offenders*), the police in Sweden register this information. A link in a network is then created between individuals i and j, i.e. $g_{ij} = g_{ji} = 1$, whenever individuals i and j are suspected of a crime together. Once we have this information, we can match each individual's social security number with his/her characteristics (education, age, ethnicity, gender, etc.). In that case, we can literally apply all the tools developed in this paper and identify the key players in criminal networks.[45]

This type of information can be obtained from the police in many countries. Indeed, it is important to identify criminal networks in data resources readily available to investigators, such as police arrest data and court data. For example, Tayebi et al. (2011) use a data set made available by the "E" Division of the Royal Canadian Mounted Police (RCMP), as a result of a research memorandum of understanding between ICURS[46], the RCMP and the Ministry of Public Safety and the Solicitor General. The data contains five years (2001-2006) of real-world crime data (arrest-data) and was made available for research purposes. This data was retrieved from the RCMP's Police Information Retrieval System (PIRS), a large database system keeping information for the regions of the Province of British Columbia, which are policed by the RCMP. PIRS contains information about all reported crime events (4.4 million) and all persons associated with a crime (9 million), from complainant to charged.[47] As for the Swedish data described above, Tayebi et al. (2011) defines a *co-offending network*, that is a network of offenders who have committed crimes together.

In the United States, there are also similar data. For example, Coplink (Hauck et al., 2002) was one of the first large scale research projects in crime data mining, and an excellent work in criminal network analysis. It is remarkable in its practicality, being integrated with and used in the workflow of the Tucson Police Department. Colink has information about the perpetrators' habits and close associations in crime to capture the *connections* between people, places, events, and vehicles, based on past crimes. Xu and Chen (2005) built on this when they created CrimeNet Explorer, a framework for criminal network knowledge discovery incorporating hierarchical clustering, Social Network Analysis (SNA) methods, and multidimensional scaling.[48]

(iii) *Gang networks*: McGloin (2004, 2005) use data from the Newark portion of the North Jersey

[45]If a link is created, one can also check what the probability that these two persons are condemned is and also that this link re-appears over time (i.e. the two individuals are also suspected of another crime later on in their life).

[46]The Institute for Canadian Urban Research Studies (ICURS) is a university research centre at Simon Fraser University.

[47]In addition, PIRS also contains information about vehicles used in crimes (1.4 million), and businesses which were involved in crimes (1.1 million).

[48]Xu and Chen (2005) expand the research in Xu and Chen (2003) and design a fully fledged system capable of incorporating outside data, such as phone records and report narratives, in order to establish stronger ties between individual offenders.

Gang Task Force, a regional problem analysis project that sought to define the local gang landscape in Northern New Jersey. These data came from the experiential knowledge of representatives of various criminal justice agencies, including the Newark Police Department, Essex County Sheriff's Office, Essex County Department of Parole, and Juvenile Justice Commission of New Jersey. In particular, groups of law enforcement officials from this variety of criminal justice agencies engaged in collective semi-structured interviews — 32 over the course of one year — that solicited information on the gang landscape. In particular, they provided information on known gang members, as well as the quantity and type of their respective associates. The classification of the profession of gang members in the questionnaire relied on New Jersey code, which defines a gang as three or more people who are, in fact, associated, that is people who have a common group name; identifying sign, tattoos, or other indices of association; and who have committed criminal offenses while engaged in gang-related activity.

Another example of available gang network data is the recent paper by Mastrobuoni and Patacchini (2012). They use a data set provided by the Federal Bureau of Narcotics on criminal profiles of 800 US Mafia members active in the 1950s and 1960s and on their connections within the Cosa Nostra network.[49] Patacchini and Mastrobuoni shed light on the extent to which family relationships, community roots and ties, legal and illegal activities predict the criminal ranking of the "men of honor," suggesting the main characteristics that can be used to detect criminal leaders.

7.2.2 How to implement a key-player policy

Once we have criminal network data, there are different ways of implementing a key-player policy. Indeed, once we have identified a key player in a network, one cannot put him/her in prison if he/she hasn't committed any crime. However, different policies can be implemented to reduce crime using a key-player approach.

(i) First, the police can offer to the key player(s) incentives to leave the gang or the criminal network. For example, the police can offer them a job or a conditional transfer (by asking them to move to another city) or monitor them more. These types of policies have been implemented in the US. For example, in Canada, some gang members of criminal networks were persuaded to abandon gang life in return for needed employment training, educational training, and skills training (Tremblay et al., 1996).

(ii) Second, the police can target key players in a meaningful way. A very similar type of policy has actually be implemented in the US. Indeed, a recent innovation in policing that capitalizes on the growing evidence of the effectiveness of police deterrence strategies in the "focused deterrence" framework, which is often referred to as "pulling-levers policing" (Kennedy, 1998, 2008). This strategy was pioneered in Boston (*Boston Gun Project*) with the intent of understanding the purported nexus of rising youth violence and use of firearms. As part of its problem analysis, representatives of various criminal justice agencies defined and characterized problematic local gangs (Braga et al., 2001; Kennedy et al., 1996, 1997, 2001). This process included elaborating on the relationships among the street gangs, which Kennedy et al. (1996, 1997, 2001) translated into sociograms illustrating connections within the gang landscape. This seemingly simple information was invaluable for

[49] The information is also contained in a book by the United States Treasury Department (2007).

the problem analysis and construction of *Operation Ceasefire*. This latter policy combines a strong law enforcement response with a "pulling levers" deterrence effort aimed at chronic gang offenders. The key to the success is to use a "lever pulling" approach, which is a crime deterrence strategy that attempts to prevent violent behavior by using a *targeted individual or group's vulnerability* to law enforcement as a means of gaining their compliance. Operation Ceasefire was first launched in Boston and youth homicide fell by two-thirds after the Ceasefire strategy was put in place in 1996 (Kennedy, 1998). It was then implemented in Los Angeles in 2000: police beefed up patrols in the area, attempting to locate gang members who had *outstanding arrest warrants* or *had violated probation or parole regulations*. Gang members who had *violated public housing rules*, *failed to pay child support*, or were similarly vulnerable were also subjected to stringent enforcement (Tita et al., 2003). The strategy and tactics resulted from months of meticulous planning and coordination by 19 public and private agencies throughout the city and county of Los Angeles, all of whom came together to work with researchers in defining the problem and designing a response. The results from the law enforcement components were surprisingly good. In the area of Hollenbeck where gang activities were most active and enforcement was most intensive, both gang crime and violent crime fell. Operation Ceasefire in Los Angeles demonstrated the potential for using data-driven research to identify problems and design interventions, obtain the commitment of disparate criminal justice agencies to work together on a discrete problem, and secure the support of an array of partners in the community.

In early 2004, High Point Police Department (HPPD) officials in North Carolina decided to implement a policy similar to Operation Ceasefire. The logic model of the High Point intervention (High Point, NC) centered on three specific phases: the *identification phase*, the *notification phase*, and *resource delivery*. Following recommendations outlined by Eck et al. (2005), researchers identified the high-density violent crime areas that were influenced by coterminous drug markets (*identification phase*). Within each targeted area, narcotics investigators collaborated with probation and parole officers, reviewed prior drug incident reports, and conducted detailed surveillance, including the use of video-recorded purchases, for at least 1 month (and sometimes up to 3 months) where they identified *key offenders* within each site. The project team also worked to construct resolute cases to have the capacity to prosecute established and chronic drug dealers. The *notification phase* centered on the "call-in" sessions. The High Point team focused its intensive efforts in unique geographic contexts at distinct points in time, which included a call-in session in the West End Neighborhood in May 2004, in the Daniel Brooks Neighborhood in April 2005, in the Southside Neighborhood in June 2006, and in the East Central area in August 2007. Thus, *identified offenders* are either arrested based on their violent criminal histories (felony convictions) or called-in to notification sessions (nonviolent, nonfelony convictions). Regarding *resource delivery*, notified offenders completed a "needs assessment" the night of the call-in session, and they were subsequently assigned to local social service providers and community outreach officials for extensive follow-up. With this data, we believe that using the key-player methodology developed in this paper could lead to a more effective policy and a larger reduction in crime.

(iii) Finally, a key-player policy can also help for related issues. For example, there is a lot of debate in the US on how to allocate under-age adolescents who have committed an offence into juvenile facilities (detention centers). We know that there is a lot of learning in crime in prisons (Bayer et al., 2009). If we can rank these adolescents by their key-player centralities, then our model predict that we should put together the delinquents with high key-player centralities while grouping together young delinquents with low key-player centralities.

7.3 Using our methodology to find key players for other types of networks and activities

Our paper can also be seen as a methodological one. Our methodology can be applied to other contexts where network data are much easier to obtain. Let us provide some examples.

Financial networks

There is an abundance of information available on financial networks where links are usually bank loans. For example, Cohen-Cole et al. (2011) use transaction level data on interbank lending from an electronic interbank market, the e-MID SPA (or e-MID), which was the reference marketplace for liquidity trading in the Euro area from January 2002 to December 2009. Boss et al., 2004 analyze the network of Austrian banks in the year 2008 where links in the network represent exposures between Austrian-domiciled banks on a non-consolidated basis (i.e. no exposures to foreign subsidiaries are included). They have 770 banks with 2,454 links between them and an average 20.54 links. Basically, any central bank could provide this type of information. Using this type of data and the methodology developed in this paper, we could study the key player policy by reinterpreting the model where efforts would be the amount of loans between banks (Cohen-Cole et al. 2011). In that case, the key player policy would be: Which bank should we bail out in order to reduce systemic risk or maximize total activity? This is an extremely important issue because the recent financial crisis has shown that the problem is not necessary "too big to fail" but "too interconnected to fail". Moreover, on a daily basis, central banks do monitor "risky" banks to prevent them from taking too many risks that could be harmful to the system. The problem is to define which banks are too risky. We believe that our framework, adapted to the financial networks, could help in designing effective key-player policies in the financial markets.

R&D networks

There is also a lot of information on R&D networks. For example, García-Canal et al. (2008) use alliance data stemming from the Thomson SDC Platinum data base. Three types of alliances are reported in the SDC database: (1) alliances that imply the transmission of an existing technology from one partner to another or to the alliance; (2) alliances that imply the cross-transfer of existing technologies between two or more partners or between these and the alliance, and (3) alliances that include the undertaking of R&D activities.

Another dataset that has been used is the one on interfirm collaborations from the NBER Patent Data File (Hall et al., 2001). These data provide information about all researchers who were involved in creating the innovation along with information on the patenting company, its geographic location, and the types of technology involved. The names of the inventors are recorded along with the name of the corporate assignee claiming each patent. Hanaki et al. (2010) match the lists of inventors' names across different assignee companies to see if they are connected via common inventors. If the same inventors work on a particular research project across two innovating companies, they ascribe the project to an R&D partnership and identify those companies as collaborators through the inventors. Longitudinal data on an evolving R&D network are then created by collecting annual snapshots of instantaneous networks.

Our model is easily adapted to R&D networks where efforts correspond to quantity produced and a link (i.e. an alliance) decreases the cost of producing goods. We can easily adapt our methodology to study the key-player policy by identifying the key firms that are the most critical for industry productivity. We can also answer the question: Which firm should we subsidize in order to generate maximum total activity?

Networks in developing economies

There are many network data for developing countries (see e.g. Fafchamps and Lund, 2003, who conducted a survey in four villages in the Cordillera mountains of northern Philippines between July 1994 and March 1995; Krishnan and Sciubba, 2009, who use the second round of the Ethiopian Rural Household Survey, conducted in 1994). There is also a recent paper by Banerjee et al. (2012) which study a problem related to the key-player issue. Their data come from a survey on 75 rural villages in Karnataka, India, that the authors conducted to obtain information on network structure and various demographics. The data are available online. They look at the diffusion of a microfinance program in these villages and show that, if the bank in charge of this program had targeted individuals in the village with the highest eigenvector centrality (a measure related to the Katz-Bonacich centrality), the diffusion of the microfinance program (i.e. take-up rates) would have been much higher. There is, however, no economic model that can justify the choice of the eigenvector centrality. We believe that targeting the key player(s) as we define them in this paper would have had an even bigger effect on the take-up rates. More generally, in developing countries, one could apply the key player policy to the issue of adoption of a new technology since there is strong evidence of social learning (Conley and Udry, 2010) and take-up rates in microfinance programs.

Political networks

Another application of a key player policy could be the political world. There is evidence that personal connections amongst politicians have a significant impact on the voting behavior of U.S. politicians (Cohen and Malloy, 2010). There is also evidence on lobbying to persuade public opinion when members of the public influence each other's opinion (Lever, 2010). When people are deciding how to vote or which product to buy, they discuss their decision with people in their social environment. Competitions to persuade public opinion are the essence of political campaigns, but also occur in marketing between rival firms or in lobbying by interests groups on opposite sides of a legislation. Matching data on campaign contributions by lobby groups with data on co-sponsorship networks in the US House of Representatives, Lever (2010) finds that changes in both network in influence and pivot probabilities are significant predictors of changes in campaign contributions. Our key-player policy suggests that resources should be spent on *key voters* who have an influential position in the social network.

Tax evasion

There is strong evidence that there are strong social interaction effects in tax evasion (Fortin et al., 2007). Galbiati and Zanella (2012) estimate social externalities of tax evasion in a model where congestion of the auditing resources of local tax authorities generates a social multiplier.

Identification is based on a contrast of the variance of tax evasion at different levels of aggregation. They use a unique data set that contains audits of about 80,000 small businesses and professionals in Italy and also provide an exact measure of reference groups that easily allows us to apply our model. They find a social multiplier of about 3, which means that the equilibrium response to a shock that induces an exogenous variation in mean concealed income is about 3 times the initial average response. This is a short-run effect that persists to the extent that auditing resources are not adjusted to internalize the congestion externality. In this context, a relevant question would be: Which person(s) should we target to reduce total tax evasion in a country?

8 Concluding remarks

This paper presents a methodology for determining the key-player whose exclusion from his/her network would result in the greatest impact on the outcome of interest (adolescent crime rates in the case of our AddHealth data). We provide a structural estimation of the model, and a simulation describing how such a key-player is identified, and furthermore, that this "greatest impact" key-player cannot be accurately identified by the other measures prominent in the literature. This methodology provides a cost effective instrument for a wide variety of policy interventions. Implementation of a key-player based policy intervention in networks would result in drastic cost reductions by taking advantage of multiplier effects.

The key-player methodology has great scope for practical implementation, since it takes advantage of multiplier effects in naturally occurring networks. Given the right data (which in several instances already exists) it can be implemented in any context. Consider the recent Obama-Romney US presidential election, which recorded the most extensive campaign spending in US history. Implementation of our key-player methodology would shift attention away from "swing states" and rather target "swing voters", who would have the greatest possible impact on voter decision within their social networks. Such an approach could easily have tempered the USD 2.3 billion total campaign cost.[50] The cost of gathering the necessary data could not be expected to be more than a fraction of this. Alternatively consider vaccination in developing countries, targeting individuals who would reduce the spread of infectious diseases the most would be highly cost effective, freeing up resources to make more vaccination projects viable, and reducing the overall infection rate. Even where data is not yet freely available, implementing the key-player policy could be cost-effective overall, if the cost of data gathering is recouped by the cost saving in the actual policy intervention that stems from targeting key-players rather than all individuals.

References

[1] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), "Who's who in networks. Wanted: the key player," *Econometrica* 74, 1403-1417.

[2] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2010), "Delinquent networks," *Journal of the European Economic Association* 8, 34-61.

[50] See http://www.opensecrets.org/pres12/index.php#out.

[3] Ballester, C. and Y. Zenou (2012), "Key player policies when contextual effects matter," Unpublished manuscript, Stockholm University.

[4] Banerjee, A., Chandrasekhar, A.G., Duflo, E. and M.O. Jackson (2012), "The diffusion of microfinance," NBER Working Paper No. 17743.

[5] Bayer, P., Hjalmarsson, R. and D. Pozen (2009), "Building criminal capital behind bars: Peer effects in juvenile corrections," *Quarterly Journal of Economics* 124, 105-147.

[6] Becker, G. (1968) "Crime and punishment: An economic approach," *Journal of Political Economy* 76, 169-217.

[7] Bekker, P. (1994), "Alternative approximations to the distributions of instrumental variable estimators," *Econometrica* 62, 657-681.

[8] Bekker, P. and J. van der Ploeg (2005), "Instrumental variable estimation based on grouped data," *Statistica Neerlandica* 59, 239-267.

[9] Belhaj, M. and F. Deroïan (2012), "Risk taking under heterogenous revenue sharing," *Journal of Development Economics* 98, 192-202.

[10] Bifulco, R., Fletcher, J.M. and S.L. Ross (2011), "The effect of classmate characteristics on post-secondary outcomes: Evidence from the AddHealth," *American Economic Journal: Economic Policy* 3, 25-53.

[11] Blume, L.E., Brock, W.A., Durlauf, S.N. and Y.M. Ioannides (2011), "Identification of social interactions," In: J. Benhabib, A. Bisin, and M.O. Jackson (Eds.), *Handbook of Social Economics*, Amsterdam: Elsevier Science.

[12] Bonacich, P. (1987), "Power and centrality: A family of measures," *American Journal of Sociology* 92, 1170-1182.

[13] Borgatti, S.P. (2003), "The key player problem," In: R. Breiger, K. Carley and P. Pattison (Eds.), *Dynamic Social Network Modeling and Analysis: Workshop Summary and Papers*, New York: National Academy of Sciences Press, pp. 241-252.

[14] Borgatti, S.P. (2006), "Identifying sets of key players in a network," *Computational, Mathematical and Organizational Theory* 12, 21-34.

[15] Boss, M., Elsinger, H., Summer, M. and S. Thurner (2004), "Network topology of the interbank market," *Quantitative Finance* 4, 677-684.

[16] Boucher, V., Bramoullé, Y., Djebbari, H. and B. Fortin (2012), "Do peers affect student achievement? Evidence from Canada using group size variation," *Journal of Applied Econometrics*, forthcoming.

[17] Braga, A.A., Kennedy, D.M., Waring, E.J. and A.M. Piehl (2001), "Problem oriented policing, deterrence, and youth violence: An evaluation of Boston's Operation Ceasefire," *Journal of Research in Crime and Delinquency* 38, 195-225.

[18] Bramoullé, Y., and R. Kranton (2007), "Public goods in networks," *Journal of Economic Theory* 135, 478-494.

[19] Bramoullé, Y., Lopez, D., Goyal, S. and F. Vega-Redondo (2004), "Social interaction in anti-coordination games," *International Journal of Game Theory* 33, 1-19.

[20] Bramoullé, Y., Djebbari, H. and B. Fortin (2009), "Identification of peer effects through social networks," *Journal of Econometrics* 150, 41-55.

[21] Cabrales, A., Calvó-Armengol, A. and Y. Zenou (2011), "Social interactions and spillovers," *Games and Economic Behavior* 72, 339-360.

[22] Calvó-Armengol, A., Patacchini, E. and Y. Zenou (2009), "Peer effects and social networks in education," *Review of Economic Studies* 76, 1239-1267.

[23] Calvó-Armengol, A. and Y. Zenou (2004), "Social networks and crime decisions: The role of social structure in facilitating delinquent behavior," *International Economic Review* 45, 935-954.

[24] Chao, J, C., Hausman, J., Newey, W.K., Swanson, N.R. and T.Woutersen (2010), "Testing overidentifying restrictions with many instruments and heteroskedasticity," Unpublished manuscript, University of Maryland.

[25] Chandrasekhar, A. and R. Lewis (2012), "Econometrics of sampled networks," Unpublished manuscript, MIT.

[26] Christakis N. and J. Fowler (2007), "The spread of obesity in a large social network over 32 years," *New England Journal of Medecine* 357, 370-379.

[27] Cohen-Cole, E., Patacchini, E. and Y. Zenou (2011), "Systemic risk and network formation in the interbank market," CEPR Discussion Paper No. 8332.

[28] Cohen, L. and C. Malloy (2010), "Friends in high places," NBER Working Paper No. 16437.

[29] Conley, T.J. and C.R. Udry (2010), "Learning about a new technology: Pineapple in Ghana," *American Economic Review* 100, 35-69.

[30] Cook, P.J. and J. Ludwig (2010), "Economical crime control," NBER Working Paper No. 16513.

[31] Costebander, E. and T.W. Valente (2003), "The stability of centrality measures when networks are sampled," *Social Networks* 25, 283-307.

[32] Dutta, B., Ghosal, S. and D. Ray (2005), "Farsighted network formation," *Journal of Economic Theory* 122, 143-164.

[33] Eck, J.E., Chainey, S., Cameron, J.G., Leitner, M. and R.E.Wilson (2005), *Mapping Crime: Understanding Hot Spots*, Washington, DC: Office of Justice Programs, U.S. Department of Justice.

[34] Ehrhardt, G., Marsili, M. and F. Vega-Redondo (2006), “Diffusion and growth in an evolving network,” *International Journal of Game Theory* 34, 383-397.

[35] Fafchamps, M. and Lund, S. (2003), “Risk sharing networks in rural Philippines,” *Journal of Development Economics* 71, 261-287.

[36] Fortin, B., Lacroix, G. and M.-C. Villeval (2007), “Tax evasion and social interactions,” *Journal of Public Economics* 91, 2089-2112.

[37] Galbiati, R. and G. Zanella (2012), “The tax evasion social multiplier,” *Journal of Public Economics* 96, 485-494.

[38] García-Canal, E., Valdés-Llaneza, A. and P. Sánchez-Lorda (2008), “Technological flows and choice of joint ventures in technology alliances,” *Research Policy* 37, 97-114.

[39] Garoupa, N. (1997), “The theory of optimal law enforcement,” *Journal of Economic Surveys* 11, 267-295.

[40] Glaeser, E.L., Sacerdote, B. and J. Scheinkman (1996), “Crime and social interactions,” *Quarterly Journal of Economics* 111, 508-548.

[41] Goyal, S. (2007), *Connections: An Introduction to the Economics of Networks*, Princeton: Princeton University Press.

[42] Goyal, S. and S. Joshi (2003), “Networks of collaboration in oligopoly,” *Games and Economic Behavior* 43, 57-85.

[43] Hall, B.H., Jaffe, A.B. and M. Trajtenberg (2001), “The NBER patent citations data file: Lessons, insights and methodological tools,” NBER Working Paper No. 8498.

[44] Hanaki, N., Nakajimac, R. and Y. Ogurad (2010), “The dynamics of R&D network in the IT industry,” *Research Policy* 39, 386-399.

[45] Hansen, C., Hausman, J. and W. Newey (2008), “Estimation with many instrumental variables,” *Journal of Business and Economic Statistics* 26, 398-422.

[46] Hauck, R.V., Atabakhsh, H., Ongvasith, P., Gupta, H. and H. Chen (2002), “Using Coplink to analyze criminal-justice data,” *IEEE Computer* 35, 3: 3037.

[47] Haynie, D.L. (2001), “Delinquent peers revisited: Does network structure matter?” *American Journal of Sociology* 106, 1013-1057.

[48] Hofbauer, J. and W. Sandholm (2007), “Evolution in games with randomly disturbed payoffs,” *Journal of Economic Theory* 132, 47-69.

[49] Ioannides, Y.M. (2012), *From Neighborhoods to Nations: The Economics of Social Interactions*, Princeton: Princeton University Press.

[50] Jackson, M.O. (2008), *Social and Economic Networks*, Princeton: Princeton University Press.

[51] Jackson, M.O. and A. Watts (2002), "The evolution of social and economic networks," *Journal of Economic Theory* 106, 265-295.

[52] Jackson, M.O. and Y. Zenou (2013), "Games on networks," In: P. Young and S. Zamir (Eds.), *Handbook of Game Theory, Vol. 4*, Amsterdam: Elsevier, forthcoming.

[53] Katz, L. (1953), "A new status index derived from sociometric analysis," *Psychometrika* 18, 39-43.

[54] Kennedy, D.M. (1998), "Pulling levers: Getting deterrence right," *National Institute of Justice Journal* 236, 2-8.

[55] Kennedy, D.M. (2008), *Deterrence and Crime Prevention: Reconsidering the Prospect of Sanction*, New York: Routledge.

[56] Kennedy, D.M., Braga, A.A. and A.M. Piehl (1996), *Youth Gun Violence in Boston: Gun Markets, Serious Youth Offenders, and a Use Reduction Strategy*, Boston, MA: John F. Kennedy School of Government, Harvard University Press.

[57] Kennedy, D.M., Braga, A.A. and A.M. Piehl (1997), "The (un)known universe: Mapping gangs and gang violence in Boston," In: D. Weisburd and T. McEwen (Eds.), *Crime Mapping and Crime Prevention*, Monsey, N.Y: Criminal Justice Press.

[58] Kennedy, D.M., Braga, A.A., Piehl, A.M. and E.J. Waring (2001), *Reducing Gun Violence: The Boston Gun Project's Operation Ceasefire*, Washington, D.C: U.S. National Institute of Justice.

[59] Kleiman, M.A. (2009), *When Brute Force Fails. How to Have Less Crime and Less Punishment*, Princeton: Princeton University Press.

[60] Kling, J.R., Ludwig, J. and L.F. Katz (2005), "Neighborhood Effects on Crime for Female and Male Youth: Evidence from a Randomized Housing Voucher Experiment," *Quarterly Journal of Economics* 120, 87-130.

[61] König, M., Tessone, C. and Y. Zenou (2012), "Nestedness in networks," Unpusblished manuscript, Stockholm University.

[62] Krishnan, P. and E. Sciubba (2009), "Links and architecture in village networks," *Economic Journal* 119, 917-949.

[63] Lee, L.F. (2002), "Consistency and efficiency of least squares estimation for mixed regressive, spatial autoregressive models," *Econometric Theory* 18, 252-277.

[64] Lee, L.F. (2003), "Best spatial two-stage least squares estimators for a spatial autoregressive model with autoregressive disturbances," *Econometric Reviews* 22, 307-335.

[65] Lee, L.F. (2007a), "Identification and estimation of econometric models with group interactions, contextual factors and fixed effects," *Journal of Econometrics* 140, 333-374.

[66] Lee, L.F. (2007b), "GMM and 2SLS estimation of mixed regressive, spatial autoregressive models," *Journal of Econometrics* 137, 489-514.

[67] Lee, L.F., Liu, X. and X. Lin (2010), "Specification and estimation of social interaction models with network structures," *Econometrics Journal* 13, 145-176.

[68] Lee, L.F. and R.P. Trost (1978), "Dependent variable models with an application to housing demand," *Journal of Econometrics* 8, 357-383.

[69] Leitner, Y. (2005), "Financial networks: Contagion, commitment, and private sector bailouts," *Journal of Finance* 6, 2925-2953.

[70] Lever, C. (2010), "Strategic spending in voting competitions with social networks," Banco de México Working Paper No. 2010-16.

[71] Levitt, S.D. (1997), "Using electoral cycles in police hiring to estimate the effect of police on crime," *American Economic Review* 87, 270-90.

[72] Lin, X. (2010), "Identifying peer effects in student academic achievement by a spatial autoregressive model with group unobservables," *Journal of Labor Economics* 28, 825-860.

[73] Liu, X (2013), "Estimation of a local-aggregate network model with sampled networks," *Economics Letters* 118, 243-246.

[74] Liu, X. and L.F. Lee (2010), "GMM estimation of social interaction models with centrality," *Journal of Econometrics* 159, 99-115.

[75] Liu, X., Patacchini, E., Zenou, Y. and L.-F. Lee (2012), "Criminal networks: Who is the key player?" CEPR Discussion Paper No. 8772.

[76] Ludwig, J., Duncan, G.J. and Hirschfield, P. (2001), "Urban poverty and juvenile crime: Evidence from a randomized housing-mobility experiment," *Quarterly Journal of Economics* 116, 655-679.

[77] Manski, C.F. (1993), "Identification of endogenous effects: The reflection problem," *Review of Economic Studies* 60, 531-542.

[78] McGloin, J.M. (2004), *Associations among Criminal Gang Members as a Defining Factor of Organization and as a Predictor of Criminal Behavior: The Gang Landscape of Newark, New Jersey*, Ann Arbor, MI: University of Michigan Press.

[79] McGloin, J.M. (2005), "Policy and intervention considerations of a network analysis of street gangs," *Criminology and Public Policy* 4, 607-636.

[80] Mastrobuoni, G. and E. Patacchini (2012), "Organized crime networks: An application of network analysis techniques to the American mafia," *Review of Network Economics* 11(3), Article 10.

[81] Patacchini, E. and Y. Zenou (2012), "Juvenile delinquency and conformism," *Journal of Law, Economic, and Organization* 28, 1-31.

[82] Polinsky, M.A. and S. Shavell (2000), "The economic theory of public enforcement of law," *Journal of Economic Literature* 38, 45-76.

[83] Polanski, A. and D. McVicar (2012), "Recovering social networks from individual attributes," *Journal of Mathematical Sociology* 35, 287-311.

[84] Sarnecki, J. (2001), *Delinquent Networks: Youth Co-Offending in Stockholm*, Cambridge: Cambridge University Press.

[85] Stock, J.H, Wright J.H. and M. Yogo (2002), "A survey of weak instruments and weak identification in generalized method of moments," *Journal of Business and Economic Statistics* 2, 518-529.

[86] Stock, J.H and M. Yogo (2005), "Testing for weak instruments in linear IV regression," In: D.W.K. Andrews and J.H. Stock (Eds.), *Identification and Inference for Econometric Models: Essays in Honor of Thomas Rothenberg*, Cambridge: Cambridge University Press, pp. 80-108.

[87] Sutherland, E.H. (1947), *Principles of Criminology*, fourth edition, Chicago: J.B. Lippincott.

[88] Tayebi, M.A., Bakker, L., Glässer, U. and V. Dabbaghian (2011), "Organized crime structures in co-offending networks," *IEEE Ninth International Conference on Autonomic and Secure Computing* (DASC), pp. 846-853.

[89] Tita, G.K. Riley, J., Ridgeway, G., Grammich, C., Abrahamse, A. and P.W. Greenwood (2003), *Reducing Gun Violence: Results from an Intervention in East Los Angeles*, Santa Monica, CA: RAND Corporation.

[90] Thornberry, T.P., Krohn, M.D., Lizotte, A.J., Smith, C.A. and K. Tobin (2003), *Gangs and Delinquency in Developmental Perspective*, Cambridge: Cambridge University Press.

[91] Tremblay, R.E., Masse, L., Pagani, L. and F. Vitaro (1996), "From childhood physical aggression to adolescent maladjustment: The Montreal Prevention Experiment," In R.D. Peters and R.J. McMahon (Eds.), *Preventing Childhood Disorders, Substance Abuse, and Delinquency*, Thousand Oaks, CA: Sage Publications.

[92] United States Treasury Department (2007), *MAFIA: The Government's Secret File on Organized Crime*, Bureau of Narcotics, New York: Harper Collins Publishers.

[93] Verdier, T. and Y. Zenou (2004), "Racial beliefs, location and the causes of crime," *International Economic Review* 45, 727-756.

[94] Warr, M. (2002), *Companions in Crime: The Social Aspects of Criminal Conduct*, Cambridge: Cambridge University Press.

[95] Wasserman, S. and K. Faust (1994), *Social Network Analysis. Methods and Applications*, Cambridge: Cambridge University Press.

[96] Weerman, F.M. (2011), "Delinquent peers in context: A longitudinal network analysis of selection and influence effects," *Criminology* 49, 253-286.

[97] Xu, J.J. and H. Chen (2005), "CrimeNet Explorer: A framework for criminal network knowledge discovery," *ACM Transactions on Information Systems* 23, 201-226.

[98] Xu, J.J. and H. Chen (2003), "Untangling criminal networks: A case study," *ISI*, pp. 232-248.

[99] Zhou, J. and Y.-J. Chen (2012), "The benefit of sequentiality in social networks," Unpublished paper, University of California at Berkeley, Berkeley.

Appendices (For Online Publication)

A Finding the key player

A.1 Intercentrality measure in the invariant case

Let $\boldsymbol{M}(g,\phi) = (\boldsymbol{I} - \phi \boldsymbol{G})^{-1}$. Its entries $m_{ij}(g,\phi)$ count the number of walks in g starting from i and ending at j, where walks of length k are weighted by ϕ^k. The Katz-Bonacich centrality of node i is $b_i(g,\phi) = \sum_{j=1}^n \alpha_j m_{ij}(g,\phi)$, and counts the *total* number of paths in g starting from i weighted by the α_j of each linked node j. Let $b_i(g,\phi)$ be the centrality of i in network g, $B(g,\phi)$ the *total* centrality in network g (i.e. $B(g,\phi) = \boldsymbol{l}'_n \boldsymbol{M}\boldsymbol{\alpha}$) and $B(g^{[-i]},\phi) = \boldsymbol{l}'_{n-1}\boldsymbol{M}^{[-i]}\boldsymbol{\alpha}^{[-i]}$ the *total* centrality in $g^{[-i]}$, where $\boldsymbol{\alpha}^{[-i]}$ is a $(n-1)\times 1$ column vector in which α_i have been removed and $\boldsymbol{M}^{[-i]} = (\boldsymbol{I} - \phi\boldsymbol{G}^{[-i]})^{-1}$ is a $(n-1)\times(n-1)$ matrix in which the ith row and ith column corresponding to i has been removed from $\boldsymbol{M}$. Finally, let $\boldsymbol{\alpha}^{[i]}$ be a $(n\times 1)$ column vector where all entries but i are defined as $\boldsymbol{\alpha}^{[-i]}$, while entry i contains the initial α_i, and let $\boldsymbol{M}^{[i]}$ be the $n\times n$ matrix such that each element is $m^{[i]}_{jk} = \frac{m_{ji}m_{ik}}{m_{ii}}$ so that $B(g^{[i]},\phi) = \boldsymbol{l}'_n\boldsymbol{M}\boldsymbol{\alpha}^{[i]}$ and $\boldsymbol{l}'_n\boldsymbol{M}^{[i]}\boldsymbol{\alpha}^{[i]} = b_{\boldsymbol{\alpha}^{[i]},i}(g,\phi)\sum_{j=1}^n m_{ji}(g,\phi)/m_{ii}(g,\phi)$. Ballester and Zenou (2012) have proposed the following definition:

Definition 3 *For all networks g and for all i, the intercentrality measure of delinquent i is:*

$$\begin{aligned} d_i(g,\phi) &= B(g,\phi) - B(g^{[-i]},\phi) \\ &= \boldsymbol{l}'_n\boldsymbol{M}\boldsymbol{\alpha} - \boldsymbol{l}'_n\boldsymbol{M}\boldsymbol{\alpha}^{[i]} + \boldsymbol{l}'_n\boldsymbol{M}^{[i]}\boldsymbol{\alpha}^{[i]} \\ &= B(g,\phi) - B(g^{[i]},\phi) + \frac{b_{\boldsymbol{\alpha}^{[i]},i}(g,\phi)\sum_{j=1}^n m_{ji}(g,\phi)}{m_{ii}(g,\phi)}. \end{aligned} \tag{19}$$

In this context, we have the following result due to Ballester et al. (2006, 2010) and Ballester and Zenou (2012).

Proposition 4 *A player i^* is the key player if and only if i^* is a delinquent with the highest intercentrality in g, that is, $d_{i^*}(g,\phi) \geq d_i(g,\phi)$, for all $i = 1,...,n$.*

A.2 Dynamic network formation: Theory

The dynamic model Let us formally solve the game described in Section 3.2.1. For each period in time t, we solve the model backwards, as usual. As in section 3.1.1, the unique Nash equilibrium of the "afternoon game" (assuming that $\phi\mu_1(\boldsymbol{G}_t(i,j)) < 1$) is such that

$$y^*_{i,t} = \phi\sum_{k=1}^n g_{ik,t}y^*_{j,t} + \pi_i + \eta + \epsilon_i \tag{20}$$

or in vector form $\boldsymbol{y}^*_t = (\boldsymbol{I} - \phi\boldsymbol{G}_t)^{-1}(\boldsymbol{\pi} + \eta\boldsymbol{l}_n + \boldsymbol{\epsilon})$. Let $\boldsymbol{M}_t \equiv \boldsymbol{M}^{(i,j)}_t = (\boldsymbol{I} - \phi\boldsymbol{G}^{(i,j)}_t)^{-1}$, with $\boldsymbol{m}_{i,t} \equiv \boldsymbol{m}^{(i,j)}_{i,t}$ being $\boldsymbol{M}_t$'s ith row and $m_{ij,t} \equiv m^{(i,j)}_{ij,t}$ being its (i,j)th entry. Then $y^*_{i,t} = \boldsymbol{m}_{i,t}(\boldsymbol{\pi} + \eta\boldsymbol{l}_n + \boldsymbol{\epsilon})$.

For the "morning game", as $\boldsymbol{\epsilon}$ is unobservable in the morning, the chosen agent i makes his/her link formation decision by maximizing his/her expected utility

$$\begin{aligned}\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})] &= (\pi_i + \eta)\,\mathrm{E}(y_{i,t}^*) + \mathrm{E}(\epsilon_i y_{i,t}^*) - \frac{1}{2}\mathrm{E}[(y_{i,t}^*)^2] \\ &\quad + \phi \sum_{k=1}^n g_{ik,t}\mathrm{E}(y_{i,t}^* y_{k,t}^*) - c_i(\bar{g}_{i,t} - \bar{g}_{i,t-1}),\end{aligned} \tag{21}$$

where $\mathrm{E}(y_{i,t}^*) = \boldsymbol{m}_{i,t}(\boldsymbol{\pi}+\eta \boldsymbol{l}_n)$, $\mathrm{E}(\epsilon_i y_{i,t}^*) = \sigma_\epsilon^2 m_{ii,t}$, $\mathrm{E}[(y_{i,t}^*)^2] = [\mathrm{E}(y_{i,t}^*)]^2 + \sigma_\epsilon^2 \boldsymbol{m}_{i,t}\boldsymbol{m}_{i,t}'$ and $\mathrm{E}(y_{i,t}^* y_{k,t}^*) = \mathrm{E}(y_{i,t}^*)\mathrm{E}(y_{k,t}^*) + \sigma_\epsilon^2 \boldsymbol{m}_{i,t}\boldsymbol{m}_{k,t}'$. It is easily seen, as in Section 3.1.1 (see (8)), that

$$\begin{aligned}\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})] &= \frac{1}{2}[\mathrm{E}(y_{i,t}^*)]^2 + \sigma_\epsilon^2 m_{ii,t} - \frac{1}{2}\sigma_\epsilon^2 \boldsymbol{m}_{i,t}\boldsymbol{m}_{i,t}' \\ &\quad + \phi\sigma_\epsilon^2 \sum_{k=1}^n g_{ik,t}\boldsymbol{m}_{i,t}\boldsymbol{m}_{k,t}' - c_i(\bar{g}_{i,t} - \bar{g}_{i,t-1}).\end{aligned} \tag{22}$$

If chosen, an agent i will *not* create a link with any agent j if and only if

$$\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})] \geq \max_{j\neq i} \mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})], \tag{23}$$

that is, i's expected utility of not creating a link is higher than that of creating a link with any agent j. Let

$$\kappa_{i,t}(g_t^{(i,j)}) = \frac{1}{2}\{\mathrm{E}[y_{i,t}^*(g_t^{(i,j)})]\}^2 + \sigma_\epsilon^2 m_{ii,t}^{(i,j)} - \frac{1}{2}\sigma_\epsilon^2 \boldsymbol{m}_{i,t}^{(i,j)}\boldsymbol{m}_{i,t}^{(i,j)\prime} + \phi\sigma_\epsilon^2 \sum_{k=1}^n g_{ik,t}^{(i,j)}\boldsymbol{m}_{i,t}^{(i,j)}\boldsymbol{m}_{k,t}^{(i,j)\prime}. \tag{24}$$

Using (22), the inequality (23) gives the lower bound of c_i:

$$\underline{c}_{i,t} = \max_j \kappa_{i,t}(g_t^{(i,j)}) - \kappa_{i,t}(g_t^{(i,i)}), \tag{25}$$

such that if $c_i \geq \underline{c}_{i,t}$ then agent i will have no incentive to create a new link at period t.

Convergence of the link formation process Let us now determine the convergence and the equilibrium of this dynamic formation model. Let time be measured at countable dates $t = 0, 1, 2, ...$ and consider a *discrete time Markov chain* for the network formation process $(g_t)_{t=0}^\infty$ with $g_t = (N, L_t)$ comprising the set of delinquents $N = \{1, ..., n\}$ together with a set of links L_t at time t between them. $(g_t)_{t=0}^\infty$ is a collection of random variables g_t, indexed by time t on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$, where Ω is the countable state space of all networks with n nodes, $\mathcal{F}$ is the σ-algebra $\sigma(\{g_t : t = 0, 1, 2, ...\})$ generated by the collection of g_t, and $\mathbb{P} : \mathcal{F} \to [0, 1]$ is a countably additive, non-negative measure on $(\Omega, \mathcal{F})$ with total mass $\sum_{g\in\Omega} \mathbb{P}(g) = 1$. At every time $t \geq 0$, links can be created, or not, according to the game described above.

Definition 5 *Consider a discrete time Markov chain* $(g_t)_{t=0}^\infty$ *on the probability space* $(\Omega, \mathcal{F}, \mathbb{P})$. *Consider a network* $g_t = (N, L_t)$ *at time* t *with delinquents* $N = \{1, ..., n\}$ *and links* L_t. *Let* $g_t^{(i,j)}$ *be the network obtained from* g_{t-1} *by the addition of the edge* $ij \notin L_{t-1}$ *between agents* $i, j \in N$. *Let* $\boldsymbol{u}_t(\boldsymbol{y}_t^*, g_t) = (u_{1,t}(\boldsymbol{y}_t^*, g_t), ..., u_{n,t}(\boldsymbol{y}_t^*, g_t))$ *denote the profile of Nash equilibrium payoffs of the* *delinquents in g_t following from the payoff function (9) with parameter $\phi < 1/\mu_1(\boldsymbol{G}_t)$. Then the delinquent j is a best response of delinquent i if $u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)}) \geq u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,k)})$ for all $k \in N \backslash \mathcal{N}_{i,t-1}$, where $\mathcal{N}_{i,t} = \{j \in N : ij \in L_t\}$ is the neighborhood of individual $i \in N$ at t. The set of delinquent i's best responses is denoted by $BR_{i,t}$.*

The key question is how individuals choose among their potential linking partners. This definition shows that, at every t, an agent i, selected uniformly at random from the set N, enjoys an opportunity of updating his/her current links. If an agent i receives such an opportunity, then he/she initiates a link to agent j which increases his/her equilibrium payoff the most. Agent j is said to be the best response of agent i given the network g_t. In our framework, agent j always accepts the link proposal because he/she does not pay the cost of the link and his/her utility always increases following the creation of the link because of local complementarities. If the utility of not creating a link is higher than that of creating a link with individual i's best response, then a link is not created and the network is unchanged. In the previous section, we have shown that this is the case if $c_i > \underline{c}_i$, where $\underline{c}_i$ is defined by (25).

Definition 6 *We define the network formation process $(g_t)_{t=0}^{\infty}$ with $g_t = (N, L_t)$, as a sequence of networks $g_0, g_1, ...$ in which, at every time $t = 0, 1, 2, ...$, a delinquent $i \in N$ is uniformly selected at random. This delinquent i initiates a link to a best response delinquent $j \in BR_{i,t}$. The link is created if $BR_{i,t} \neq \emptyset$ and $u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)}) \geq u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})$. No link will be created otherwise. If $BR_{i,t}$ is not unique, then i randomly selects one delinquent in $BR_{i,t}$.*

In this framework, the newly established link affects the overall network structure and thus the centralities and payoffs of all other delinquents in the network. The formation of links can thus introduce large, unintended and uncompensated externalities.

We now analyze the network formation process $(g_t)_{t=0}^{\infty}$ defined above, in more detail, where g_t is the random variable realized at time $t \geq 0$. Let us show that the network formation process $(g_t)_{t=0}^{\infty}$ introduced in Definition 6 induces a Markov chain on a finite state space Ω. Ω contains all unlabeled graphs with n nodes. Therefore, the number of states is finite and the transition between states can be represented with a transition matrix $\boldsymbol{T}$. Let us show that $(g_t)_{t=0}^{\infty}$ is a *Markov chain*. The network g_{t+1} is obtained from g_t by adding a link to L_t or doing nothing. Thus, the probability of obtaining g_{t+1} depends only on g_t and not on the previous networks $g_{t'}$ for $t' < t$, that is

$$\mathbb{P}\left(g_{t+1} | g_0, g_1, ..., g_t\right) = \mathbb{P}\left(g_{t+1} | g_t\right).$$

The number of possible networks g_t is finite for any time t and the transition probability from a network g_t to g_{t+1} does not depend on t. Therefore, $(g_t)_{t=0}^{\infty}$ is a finite state, discrete time, homogeneous Markov chain. Moreover, the transition matrix $\boldsymbol{T} = [T_{ij}]$ is defined by

$$T_{ij} = \mathbb{P}\left(g_{t+1} = g_j \mid g_t = g_i\right) \text{ for any } g_i, g_j \in \Omega.$$

Let us explain what kind of equilibrium concept we are using for this dynamic network formation model. As above, define $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,j)})]$ is the expected utility at time t for i to create a link with

j, for all j such that $j \neq i$ and the (i,j)th element of the adjacency matrix $\boldsymbol{G}_{t-1}$ is zero and $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})]$ as the expected utility at time t for i not to create a link (see 22).

Definition 7 *Consider the network formation process* $(g_t)_{t=0}^{\infty}$ *with* $g_t = (N, L_t)$ *described in Definition 6, where, at each period in time t, the morning-afternoon game described in Section 3.2.1 is played. We say that the network* g_0 *at time* $t = 0$ *converges to an equilibrium network* g_T *at time* $t = T$ *when each of the n delinquents in the network* g_T *has no incentive to create a new link at time* $t = T$*, that is,* $\mathrm{E}[u_{i,t}(\boldsymbol{y}_t^*, g_t^{(i,i)})] > \max_j \mathrm{E}[u_{i,T}(\boldsymbol{y}_t^*, g_t^{(i,j)})]$ *for all* $i = 1, ..., n$.

This definition says that the first time period for which each delinquent has no incentive to create a new link is T. This means that at times $T+1$, $T+2$, etc., there are also no incentives for them to create a new link. In that case, the network $g_T = (N, L_T)$ is an equilibrium network. In terms of Markov chain, this means that the equilibrium network g_T is an *absorbing state*. From now on, when use the word "equilibrium" or "equilibrium network" in the dynamic network formation model, we refer to the equilibrium concept defined in Definition 7.

A.3 Dynamic network formation and key player: Econometric issues

To simplify notation, we drop the subscript r when there is no ambiguity. To determine the key player, we first need to estimate the total expected crime in equilibrium when a delinquent is removed, i.e. $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})]$. Remember that the expected equilibrium crime effort when i has been removed is given by $\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})] = (\boldsymbol{I}_{n-1} - \phi \boldsymbol{G}_T^{[-i]})^{-1}(\bar{\boldsymbol{X}}^{[-i]}\boldsymbol{\beta} + \eta \boldsymbol{l}_{n-1})$ and $\sum_{j=1, j\neq i}^{n} \mathrm{E}[y_{j,T}^*(g_T^{[-i]})]$ is the sum of each element of the vector $\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]$. We estimate $\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]$ as

$$\widehat{\mathrm{E}[\boldsymbol{y}_T^*(g_T^{[-i]})]} = (\boldsymbol{I}_{n-1} - \hat{\phi} \boldsymbol{G}_T^{[-i]})^{-1}(\bar{\boldsymbol{X}}^{[-i]}\hat{\boldsymbol{\beta}} + \hat{\eta} \boldsymbol{l}_{n-1}), \tag{26}$$

where $\hat{\phi}$ and $\hat{\boldsymbol{\beta}}$ are the estimates obtained from the bias-corrected many-IV GMM estimation procedure in Section 5, and $\boldsymbol{\eta}$ can be estimated by the average of the estimation residuals. The only parameter that is not estimated from the bias-corrected many-IV GMM estimation is c_i, the marginal cost of forming links. Let us assume that the network observed in the AddHealth data is in *equilibrium* ($t = 0$), as per Definition 7, that is, no one has an incentive to create a new link at day $t = 1$, and denote this network by $\boldsymbol{G}_0$ (i.e. the adjacency matrix at day $t = 0$). For each i, $\underline{c}_i = \underline{c}_{i,1}$ is defined by (25).

Let $\hat{\boldsymbol{M}}_t^{(i,j)} = (\boldsymbol{I} - \hat{\phi} \boldsymbol{G}_t^{(i,j)})^{-1}$, with $\hat{\boldsymbol{m}}_{i,t}^{(i,j)}$ being $\hat{\boldsymbol{M}}_t^{(i,j)}$'s ith row and $\hat{m}_{ij,t}^{(i,j)}$ being its (i,j)th entry. As $\mathrm{E}(y_{i,t}^*) = \boldsymbol{m}_{i,t}^{(i,j)}(\bar{\boldsymbol{X}}\boldsymbol{\beta} + \eta \boldsymbol{l}_n)$, from (24) $\kappa_{i,t}(g_t^{(i,j)})$ can be estimated by:

$$\hat{\kappa}_{i,t}(g_t^{(i,j)}) = \frac{1}{2}[\hat{\boldsymbol{m}}_{i,t}^{(i,j)}(\bar{\boldsymbol{X}}\hat{\boldsymbol{\beta}} + \hat{\eta}\boldsymbol{l}_n)]^2 + \hat{\sigma}_\epsilon^2 \hat{m}_{ii,t}^{(i,j)} - \frac{1}{2}\hat{\sigma}_\epsilon^2 \hat{\boldsymbol{m}}_{i,t}^{(i,j)} \hat{\boldsymbol{m}}_{i,t}^{(i,j)\prime} + \hat{\phi}\hat{\sigma}_\epsilon^2 \sum_{k=1}^{n} g_{ik,t}^{(i,j)} \hat{\boldsymbol{m}}_{i,t}^{(i,j)} \hat{\boldsymbol{m}}_{k,t}^{(i,j)\prime}.$$

Hence, in the simulations, we estimate $\underline{c}_i$ by

$$\widehat{\underline{c}}_i = \max_j \hat{\kappa}_{i,1}(g_1^{(i,j)}) - \hat{\kappa}_{i,1}(g_1^{(i,i)}). \tag{27}$$

Observe that even if $c_i = \underline{c}_i$ for all i so that nobody will want to form a link, it will be possible that links will be formed after the removal of the key player since $\underline{c}_i(g) \neq \underline{c}_i(g^{[-i]})$.

We will compare the determination of the key player that solves (10), i.e.

$$\min_i \left\{ \sum_{j=1, j \neq i}^{n} \mathrm{E}[y^*_{j,T}(g_T^{[-i]})] \mid i = 1, ..., n \right\}$$

with the key player calculated in the invariant case, i.e. the network is given and there is no link formation. To be consistent with (10), in the invariant network case, we will calculate the key player as the one whose removal leads to the largest *expected* total crime reduction[51], or equivalently, the key player i^* will be the one that solves:

$$\min_i \left\{ \sum_{j=1, j \neq i}^{n} \mathrm{E}[y^*_{j,0}(g_0^{[-i]})] \mid i = 1, ..., n \right\} \tag{28}$$

B Data appendix

[Insert Tables A1 and A2 here]

C Identification of the local-aggregate model

Let

$$\boldsymbol{\Lambda}_1 = \begin{bmatrix} \boldsymbol{\beta}_2 c_1 & 1 & -\phi_1 c_1 \\ \boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 c_2 & -\phi_1 & -\phi_1 c_2 \\ \boldsymbol{\beta}_2 c_3 & 0 & 1 - \phi_1 c_3 \\ \eta_r & 0 & 0 \end{bmatrix} \tag{29}$$

and

$$\boldsymbol{\Lambda}_2 = \begin{bmatrix} -\boldsymbol{\beta}_2 c_1 & 1 & \phi_1 c_1 \\ \boldsymbol{\beta}_1 - \boldsymbol{\beta}_2 c_2 & -\phi_1 & \phi_1 c_2 \\ -\boldsymbol{\beta}_2 c_3 & -1 & \phi_1 c_3 + 1 \\ \boldsymbol{\beta}_2 - \boldsymbol{\beta}_1 - \boldsymbol{\beta}_2 c_4 & \phi_1 & \phi_1 c_4 - \phi_1 \\ -\boldsymbol{\beta}_2 c_5 & 0 & \phi_1 c_5 - 1 \end{bmatrix}. \tag{30}$$

Proof of Proposition 2. We follow the identification strategy as in Bramoullé et al. (2009) by investigating identification conditions which can be revealed via networks as functions on a vector of regressors to outcomes. Thus, we assume $\boldsymbol{X}_r$ is a vector in this proof. First, we consider the case that, for some network r, $\boldsymbol{G}_r$ has non-constant row sums. In this case, $\mathrm{E}(\boldsymbol{J}\boldsymbol{Z}) = \boldsymbol{J}[\mathrm{E}(\boldsymbol{G}\boldsymbol{y}), \boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X}]$ has full column rank if

$$\boldsymbol{J}_r[\mathrm{E}(\boldsymbol{G}_r\boldsymbol{y}_r)d_1 + \boldsymbol{X}_r d_2 + \boldsymbol{G}_r^*\boldsymbol{X}_r d_3] = 0 \tag{31}$$

implies $d_1 = d_2 = d_3 = 0$. As $\boldsymbol{J}_r = \boldsymbol{I}_{n_r} - \frac{1}{n_r}\boldsymbol{l}_{n_r}\boldsymbol{l}'_{n_r}$, (31) can be rewritten as

$$\mathrm{E}(\boldsymbol{G}_r\boldsymbol{y}_r)d_1 + \boldsymbol{X}_r d_2 + \boldsymbol{G}_r^*\boldsymbol{X}_r d_3 + \boldsymbol{l}_{n_r}\mu = 0, \tag{32}$$

[51] Note this is different from the definition of the key player in Section 3.1.2.

where $\mu = -\frac{1}{n_r}\boldsymbol{l}'_{n_r}[\mathrm{E}(\boldsymbol{G}_r\boldsymbol{y}_r)d_1 + \boldsymbol{X}_r d_2 + \boldsymbol{G}_r^*\boldsymbol{X}_r d_3]$. As

$$(\boldsymbol{I}_{n_r} - \phi_1\boldsymbol{G}_r)\mathrm{E}(\boldsymbol{G}_r\boldsymbol{y}_r) = \boldsymbol{G}_r\boldsymbol{X}_r\boldsymbol{\beta}_1 + \boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r\boldsymbol{\beta}_2 + \boldsymbol{G}_r\boldsymbol{l}_{n_r}\eta_r$$

from the reduced form equation, premultiplying (32) by $(\boldsymbol{I}_{n_r} - \phi_1\boldsymbol{G}_r)$ gives

$$\boldsymbol{X}_r d_2 + \boldsymbol{G}_r\boldsymbol{X}_r(\boldsymbol{\beta}_1 d_1 - \phi_1 d_2) + \boldsymbol{G}_r^*\boldsymbol{X}_r d_3 + \boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r(\boldsymbol{\beta}_2 d_1 - \phi_1 d_3) + \boldsymbol{l}_{n_r}\mu + \boldsymbol{G}_r\boldsymbol{l}_{n_r}(\eta_r d_1 - \phi_1\mu) = 0.$$

Note, as $\boldsymbol{G}_r = \boldsymbol{R}_r\boldsymbol{G}_r^*$, where $\boldsymbol{R}_r$ is a diagonal matrix with the ith diagonal element being $\bar{g}_{i,r} = \sum_j g_{ij,r}$, $\boldsymbol{G}_r, \boldsymbol{G}_r^*$ are linearly independent if and only if rows sums of $\boldsymbol{G}_r$ are not constant. For $\boldsymbol{G}_r$ with non-constant row sums, we consider two cases. (i) $\boldsymbol{I}_{n_r}, \boldsymbol{G}_r, \boldsymbol{G}_r^*, \boldsymbol{G}_r\boldsymbol{G}_r^*$ are linearly independent. In this case, $[\boldsymbol{X}_r, \boldsymbol{G}_r\boldsymbol{X}_r, \boldsymbol{G}_r^*\boldsymbol{X}_r, \boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r]$ has full column rank. Thus, for a general $\boldsymbol{X}_r$, $[\boldsymbol{X}_r, \boldsymbol{G}_r\boldsymbol{X}_r, \boldsymbol{G}_r^*\boldsymbol{X}_r, \boldsymbol{G}_r\boldsymbol{G}_r^*\boldsymbol{X}_r, \boldsymbol{l}_{n_r}, \boldsymbol{G}_r\boldsymbol{l}_{n_r}]$ has full column rank, which implies $d_2 = \boldsymbol{\beta}_1 d_1 - \phi_1 d_2 = d_3 = \boldsymbol{\beta}_2 d_1 - \phi_1 d_3 = \mu = \eta_r d_1 - \phi_1\mu = 0$. Therefore, $d_1 = d_2 = d_3 = 0$ if $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \eta_r$ are not all zeros. (ii) $\boldsymbol{G}_r\boldsymbol{G}_r^* = c_1\boldsymbol{I}_{n_r} + c_2\boldsymbol{G}_r + c_3\boldsymbol{G}_r^*$ for some constant scalars c_1, c_2, c_3. In this case,

$$\begin{aligned} 0 \;=\;& \boldsymbol{X}_r(\boldsymbol{\beta}_2 c_1 d_1 + d_2 - \phi_1 c_1 d_3) + \boldsymbol{G}_r\boldsymbol{X}_r[(\boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 c_2)d_1 - \phi_1 d_2 - \phi_1 c_2 d_3] \\ &+ \boldsymbol{G}_r^*\boldsymbol{X}_r[\boldsymbol{\beta}_2 c_3 d_1 + (1 - \phi_1 c_3)d_3] + \boldsymbol{l}_{n_r}\mu + \boldsymbol{G}_r\boldsymbol{l}_{n_r}(\eta_r d_1 - \phi_1\mu), \end{aligned}$$

which implies $d_1 = d_2 = d_3 = 0$ if $\boldsymbol{\Lambda}_1$ given by (29) has full rank. When $\eta_r \neq 0$, a sufficient condition for $\boldsymbol{\Lambda}_1$ to have full rank is $|\phi_1 c_1 + c_2| + |1 - \phi_1 c_3| \neq 0$.

Next, consider the case that $\boldsymbol{G}_r$ has constant row sums such that $\bar{g}_{i,r} = \bar{g}_r$ for all r. In this case, $\boldsymbol{G}_r = g_r\boldsymbol{G}_r^*$. $\mathrm{E}(\boldsymbol{J}\boldsymbol{Z}) = \boldsymbol{J}[\mathrm{E}(\boldsymbol{G}\boldsymbol{y}), \boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X}]$ has full column rank if

$$\boldsymbol{J}[\mathrm{E}(\boldsymbol{G}\boldsymbol{y})d_1 + \boldsymbol{X}d_2 + \boldsymbol{G}^*\boldsymbol{X}d_3] = 0 \tag{33}$$

implies $d_1 = d_2 = d_3 = 0$. As $\boldsymbol{J}(\boldsymbol{I} - \phi_1\boldsymbol{G})^{-1}\boldsymbol{G}\boldsymbol{L} = 0$, substitution of $\mathrm{E}(\boldsymbol{G}\boldsymbol{y}) = (\boldsymbol{I} - \phi_1\boldsymbol{G})^{-1}(\boldsymbol{G}\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{G}\boldsymbol{L}\eta)$ into (33) gives

$$\boldsymbol{J}(\boldsymbol{I} - \phi_1\boldsymbol{G})^{-1}[\boldsymbol{X}d_2 + \boldsymbol{G}\boldsymbol{X}(\boldsymbol{\beta}_1 d_1 - \phi_1 d_2) + \boldsymbol{G}^*\boldsymbol{X}d_3 + \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}(\boldsymbol{\beta}_2 d_1 - \phi_1 d_3)] = 0,$$

which implies

$$\boldsymbol{X}d_2 + \boldsymbol{G}\boldsymbol{X}(\boldsymbol{\beta}_1 d_1 - \phi_1 d_2) + \boldsymbol{G}^*\boldsymbol{X}d_3 + \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}(\boldsymbol{\beta}_2 d_1 - \phi_1 d_3) = c\boldsymbol{L} \tag{34}$$

because $\boldsymbol{J}\boldsymbol{L} = 0$. As $\boldsymbol{G}^*\boldsymbol{L} = \boldsymbol{L}$, premultiplying (34) by $\boldsymbol{G}^*$ gives

$$\boldsymbol{G}^*\boldsymbol{X}d_2 + \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}(\boldsymbol{\beta}_1 d_1 - \phi_1 d_2) + \boldsymbol{G}^{*2}\boldsymbol{X}d_3 + \boldsymbol{G}\boldsymbol{G}^{*2}\boldsymbol{X}(\boldsymbol{\beta}_2 d_1 - \phi_1 d_3) = c\boldsymbol{L}. \tag{35}$$

From (34) and (35), when $\boldsymbol{I}, \boldsymbol{G}, \boldsymbol{G}^*, \boldsymbol{G}\boldsymbol{G}^*, \boldsymbol{G}^{*2}, \boldsymbol{G}\boldsymbol{G}^{*2}$ are linearly independent, $d_1 = d_2 = d_3 = 0$ if $\boldsymbol{\beta}_1, \boldsymbol{\beta}_2$ are not both zeros. When $\boldsymbol{G}\boldsymbol{G}^{*2} = c_1\boldsymbol{I} + c_2\boldsymbol{G} + c_3\boldsymbol{G}^* + c_4\boldsymbol{G}\boldsymbol{G}^* + c_5\boldsymbol{G}^{*2}$ and $\boldsymbol{I}, \boldsymbol{G}, \boldsymbol{G}^*, \boldsymbol{G}\boldsymbol{G}^*, \boldsymbol{G}^{*2}$ are linearly independent, $d_1 = d_2 = d_3 = 0$ if $\boldsymbol{\Lambda}_2$ given by (30) has full rank. On the other hand, if $\bar{g}_r = \bar{g}$ for all r, then $\boldsymbol{G} = g\boldsymbol{G}^*$. When $\boldsymbol{I}, \boldsymbol{G}^*, \boldsymbol{G}^{*2}, \boldsymbol{G}^{*3}$ are linearly independent, (34) and (35) imply $d_1 = d_2 = d_3 = 0$ if $\phi_1 \bar{g} \boldsymbol{\beta}_1 + \boldsymbol{\beta}_2 \neq 0$. ■

D Estimation of the local-aggregate model

D.1 The 2SLS estimator

In this paper, we estimate (13) using the 2SLS and generalized method of moments (GMM) approaches proposed by Liu and Lee (2010). The conventional instruments for the estimation of (13) are $\boldsymbol{Q}_1 = \boldsymbol{J}[\boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X}, \boldsymbol{G}\boldsymbol{X}, \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}]$ (*finite-IV 2SLS*). For the local-aggregate model, Liu and Lee (2010) have proposed to include additional instruments $\boldsymbol{J}\boldsymbol{G}\boldsymbol{L}$ so that $\boldsymbol{Q}_2 = [\boldsymbol{Q}_1, \boldsymbol{J}\boldsymbol{G}\boldsymbol{L}]$ (*many-IV 2SLS*). The additional instruments $\boldsymbol{J}\boldsymbol{G}\boldsymbol{L}$ are based on the leading-order term of the Katz-Bonacich centrality. Those additional IVs could help model identification when the conventional IVs are weak and improve upon the estimation efficiency of the conventional 2SLS estimator based on $\boldsymbol{Q}_1$. As $\boldsymbol{J}\boldsymbol{G}\boldsymbol{L}$ has $\bar{r}$ columns, the number of instruments will increases with the sample size n if the number of networks $\bar{r}$ increases with n. Therefore, The 2SLS using $\boldsymbol{Q}_2$ could be asymptotic biased when $\bar{r}$ increases too fast relative to n (see, e.g., Bekker, 1994; Bekker and van der Ploeg, 2005; Hansen et al., 2008). Liu and Lee (2010) have shown that the many-IV 2SLS estimator has a properly-centered asymptotic normal distribution when the average group size is large relative to the number of networks in the sample. Liu and Lee (2010) have proposed a bias-correction procedure based on the estimated leading-order many-instrument bias. The *bias-corrected 2SLS* is properly centered, asymptotically normally distributed, and efficient when the average group size is sufficiently large.

To summarize, the 2SLS estimators of $\boldsymbol{\theta} = (\phi, \boldsymbol{\beta}_1', \boldsymbol{\beta}_2')'$ considered in the empirical studies of this paper are:

(i) *Finite-IV 2SLS*: $\hat{\boldsymbol{\theta}}_{2sls-1} = (\boldsymbol{Z}'\boldsymbol{P}_1\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_1\boldsymbol{y}$, where $\boldsymbol{P}_1 = \boldsymbol{Q}_1(\boldsymbol{Q}_1'\boldsymbol{Q}_1)^{-1}\boldsymbol{Q}_1'$ and $\boldsymbol{Q}_1$ contains the linearly independent columns of $\boldsymbol{J}[\boldsymbol{X}, \boldsymbol{G}^*\boldsymbol{X}, \boldsymbol{G}\boldsymbol{X}, \boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X}]$.

(ii) *Many-IV 2SLS*: $\hat{\boldsymbol{\theta}}_{2sls-2} = (\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{y}$, where $\boldsymbol{P}_2 = \boldsymbol{Q}_2(\boldsymbol{Q}_2'\boldsymbol{Q}_2)^{-1}\boldsymbol{Q}_2'$ and $\boldsymbol{Q}_2$ contains the linearly independent columns of $[\boldsymbol{Q}_1, \boldsymbol{J}\boldsymbol{G}\boldsymbol{L}]$.

(iii) *Bias-corrected 2SLS*: $\hat{\boldsymbol{\theta}}_{c2sls-2} = (\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{Z})^{-1}[\boldsymbol{Z}'\boldsymbol{P}_2\boldsymbol{y} - \tilde{\sigma}^2\mathrm{tr}(\boldsymbol{P}_2\boldsymbol{G}\tilde{\boldsymbol{M}})\boldsymbol{e}_1]$, where $\tilde{\boldsymbol{M}} = (\boldsymbol{I} - \tilde{\phi}\boldsymbol{G})^{-1}$, and $\tilde{\sigma}^2, \tilde{\phi}$ are $\sqrt{n}$-consistent initial estimators of σ^2, ϕ.

D.2 The GMM estimator

The 2SLS approach can be generalized to the GMM with additional quadratic moment equations. While the IV moments use the information of the mean of the reduced-form equation for estimation, the quadratic moments explore the correlation structure of the reduced-form disturbances. Liu and Lee (2010) have shown that the many-IV GMM estimators can be consistent, asymptotically normal, and efficient when the sample size grows fast enough relative to the number of networks. Liu and Lee (2010) have also suggested a bias-correction procedure for the many-IV GMM estimator based on the estimated leading order many-instrument bias. The *bias-corrected GMM* is shown to be more efficient than the corresponding 2SLS estimator.

Let $\boldsymbol{M} = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}$, $\boldsymbol{U} = \boldsymbol{J}\boldsymbol{G}\boldsymbol{M}\boldsymbol{J} - \mathrm{tr}(\boldsymbol{J}\boldsymbol{G}\boldsymbol{M})\boldsymbol{J}/\mathrm{tr}(\boldsymbol{J})$ and $\boldsymbol{\epsilon}(\theta) = \boldsymbol{J}(\boldsymbol{y} - \boldsymbol{Z}\theta)$. To summarize, the GMM estimators of $\boldsymbol{\theta} = (\phi, \boldsymbol{\beta}_1', \boldsymbol{\beta}_2')'$ considered in the empirical studies of this paper are:

(i) *Finite-IV GMM*: $\hat{\boldsymbol{\theta}}_{gmm-1}$ is the (feasible) optimal GMM estimator with moment conditions $\boldsymbol{g}_1(\theta) = [\boldsymbol{Q}_1, \boldsymbol{U}\boldsymbol{\epsilon}(\boldsymbol{\theta})]'\boldsymbol{\epsilon}(\boldsymbol{\theta})$, and $\boldsymbol{Q}_1$ is constructed as in the finite-IV 2SLS.

(ii) *Many-IV GMM*: $\hat{\boldsymbol{\theta}}_{gmm-2}$ is the (feasible) optimal GMM estimator with moment conditions $\boldsymbol{g}_2(\theta) = [\boldsymbol{Q}_2, \boldsymbol{U}\boldsymbol{\epsilon}(\boldsymbol{\theta})]'\boldsymbol{\epsilon}(\boldsymbol{\theta})$, and $\boldsymbol{Q}_2$ is constructed as in the many-IV 2SLS.

(iii) *Bias-corrected GMM*: $\hat{\boldsymbol{\theta}}_{cgmm-2} = \hat{\boldsymbol{\theta}}_{gmm-2} - \tilde{\boldsymbol{b}}_{gmm}$, where $\tilde{\boldsymbol{b}}_{gmm}$ is the estimated leading-order many-instrument bias given in Liu and Lee (2010).

D.3 The OIR test

Our identification strategy and the validity of the moment conditions employed by the 2SLS and GMM estimators rest on the exogeneity of the adjacency matrix $\boldsymbol{G}$ (conditional on covariates and network fixed effects). We test the exogeneity of $\boldsymbol{G}$ using the over-identifying restrictions (OIR) test. If the OIR test cannot reject the null hypothesis that the moment conditions are correctly specified, then it provides evidence that $\boldsymbol{G}$ is exogenous. If the number of moment restrictions is fixed, the OIR test statistic given by the 2SLS (or GMM) objective function evaluated at the 2SLS (or GMM) estimator follows a chi-squared distribution with degrees of freedom equal to the number of over-identifying restrictions (Lee, 2007b, Proposition 2). However, the OIR test might not be robust in the presence of a large number of moment restrictions.[52] Hence, we only consider the OIR test for the 2SLS and GMM estimator based on IV matrix $\boldsymbol{Q}_1$.

E Robustness checks

E.1 Estimation of the local-aggregate model with endogenous crime participation

Without loss of generality, we assume the first n (of the $\bar{n}$) individuals will choose to become criminals. For those who become criminals,

$$\boldsymbol{y}_1^* = \boldsymbol{X}_1\boldsymbol{\gamma} + \boldsymbol{\epsilon}_1,$$

where $\boldsymbol{y}_1^* = (y_{1,1}^*, \cdots, y_{n,1}^*)' > 0$, $\boldsymbol{X}_1 = (\boldsymbol{x}_{1,1}, \cdots, \boldsymbol{x}_{n,1})'$ and $\boldsymbol{\epsilon}_1 = (\epsilon_{1,1}, \cdots, \epsilon_{n,1})'$, and their crime effort levels in vector form are given by (12), i.e.,

$$\boldsymbol{y} = \phi\boldsymbol{G}\boldsymbol{y} + \boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta} + \boldsymbol{\epsilon}_2, \tag{36}$$

where $\boldsymbol{y} = (y_1, \cdots, y_n)'$, $\boldsymbol{X} = (\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n)'$ and $\boldsymbol{\epsilon}_2 = (\epsilon_{1,2}, \cdots, \epsilon_{n,2})'$. Let $\boldsymbol{M} = (\boldsymbol{I} - \phi\boldsymbol{G})^{-1}$ and the ith row of $\boldsymbol{M}$ be $\boldsymbol{m}_i$. Then, from the reduced form equation of (36), we have $y_i = \boldsymbol{m}_i(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta}) + \boldsymbol{m}_i\boldsymbol{\epsilon}_2$. Given the participation decisions of the $\bar{n}$ individuals in the first stage, the expected crime effort of criminal i is equal to:

$$\mathrm{E}[y_i|\boldsymbol{y}_1^* > 0] = \boldsymbol{m}_i(\boldsymbol{X}\boldsymbol{\beta}_1 + \boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2 + \boldsymbol{L}\boldsymbol{\eta}) + \boldsymbol{m}_i\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{\epsilon}_1 > -\boldsymbol{X}\boldsymbol{\gamma}\right] \tag{37}$$

The term $\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{\epsilon}_1 > -\boldsymbol{X}\boldsymbol{\gamma}\right]$ is the *correction term* for the sample-selection bias.

Suppose, for $i, j = 1, \cdots, n$,

$$\begin{bmatrix} \epsilon_{i,1} \\ \epsilon_{i,2} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}\right),$$

[52] For the case with independent observations, Chao et al. (2010) have proposed an OIR test that is robust to many IVs. However, no robust OIR test with many IVs is available when observations are spatially correlated.

and $\mathrm{E}\left(\epsilon_{i,1}|\epsilon_{j,2}\right)=0$ for $i\neq j$. Then, $\mathrm{E}\left[\boldsymbol{\epsilon}_2|\boldsymbol{\epsilon}_1>-\boldsymbol{X}_1\gamma\right]=\sigma_{12}\boldsymbol{\lambda}$, where $\boldsymbol{\lambda}=\left[\lambda\left(\boldsymbol{x}_{1,1}'\gamma\right),...,\lambda\left(\boldsymbol{x}_{n,1}'\gamma\right)\right]'$ and $\lambda\left(\boldsymbol{x}_{i,1}'\gamma\right)=\phi\left(\boldsymbol{x}_{i,1}'\gamma\right)/\Phi\left(\boldsymbol{x}_{i,1}'\gamma\right)$. Substitution of $\sigma_{12}\boldsymbol{\lambda}$ into (37) gives, in matrix notation,

$$\mathrm{E}[\boldsymbol{y}|\boldsymbol{y}_1^*>0]=\boldsymbol{M}(\boldsymbol{X}\boldsymbol{\beta}_1+\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2+\boldsymbol{L}\boldsymbol{\eta})+\sigma_{12}\boldsymbol{M}\boldsymbol{\lambda}. \tag{38}$$

From (38), the infeasible selection-bias-corrected outcome equation is given by

$$\boldsymbol{y}=\phi\boldsymbol{G}\boldsymbol{y}+\boldsymbol{X}\boldsymbol{\beta}_1+\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2+\boldsymbol{L}\boldsymbol{\eta}+\sigma_{12}\boldsymbol{\lambda}+\boldsymbol{\epsilon}^*,$$

where $\boldsymbol{\epsilon}^*=\boldsymbol{\epsilon}_2-\sigma_{12}\boldsymbol{\lambda}$ is the bias-corrected error term such that $\mathrm{E}[\boldsymbol{\epsilon}^*|\boldsymbol{y}_1^*>0]=0$. The conditional variance of $\boldsymbol{\epsilon}^*$ is $\mathrm{Var}[\boldsymbol{\epsilon}^*|\boldsymbol{y}_1^*>0]=\sigma_2^2\boldsymbol{I}-\sigma_{12}^2\boldsymbol{A}$, where $\boldsymbol{A}=\mathrm{diag}\{\lambda(\boldsymbol{x}_{i,1}'\gamma)\boldsymbol{x}_{i,1}'\gamma+\lambda^2(\boldsymbol{x}_{i,1}'\gamma)\}_{i=1}^n$.

The selection-bias-corrected outcome equation can be estimated following Heckman's two-step approach. In the first step, we estimate $\boldsymbol{\gamma}$ in the participation equation (17) using the probit maximum likelihood estimator $\hat{\boldsymbol{\gamma}}$. With estimated inverse Mills ratios $\hat{\boldsymbol{\lambda}}=\left[\lambda\left(\boldsymbol{x}_{1,1}'\hat{\boldsymbol{\gamma}}\right),...,\lambda\left(\boldsymbol{x}_{n,1}'\hat{\boldsymbol{\gamma}}\right)\right]'$, the (feasible) selection-bias-corrected outcome equation is

$$\boldsymbol{y}=\phi\boldsymbol{G}\boldsymbol{y}+\boldsymbol{X}\boldsymbol{\beta}_1+\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2+\boldsymbol{L}\boldsymbol{\eta}+\sigma_{12}\hat{\boldsymbol{\lambda}}+\boldsymbol{\epsilon}, \tag{39}$$

where $\boldsymbol{\epsilon}=\boldsymbol{\epsilon}_2-\sigma_{12}\hat{\boldsymbol{\lambda}}=\boldsymbol{\epsilon}^*-\sigma_{12}(\hat{\boldsymbol{\lambda}}-\boldsymbol{\lambda})$. After we eliminate the fixed group effect by the projector $\boldsymbol{J}$, (39) becomes

$$\boldsymbol{J}\boldsymbol{y}=\phi\boldsymbol{J}\boldsymbol{G}\boldsymbol{y}+\boldsymbol{J}\boldsymbol{X}\boldsymbol{\beta}_1+\boldsymbol{J}\boldsymbol{G}^*\boldsymbol{X}\boldsymbol{\beta}_2+\sigma_{12}\boldsymbol{J}\hat{\boldsymbol{\lambda}}+\boldsymbol{J}\boldsymbol{\epsilon}.$$

Let $\boldsymbol{Q}=\boldsymbol{J}[\boldsymbol{X},\boldsymbol{G}^*\boldsymbol{X},\boldsymbol{G}\boldsymbol{X},\boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X},\boldsymbol{\lambda}]$ be an (infeasible) IV matrix and $\hat{\boldsymbol{Q}}'=\boldsymbol{J}[\boldsymbol{X},\boldsymbol{G}^*\boldsymbol{X},\boldsymbol{G}\boldsymbol{X},\boldsymbol{G}\boldsymbol{G}^*\boldsymbol{X},\hat{\boldsymbol{\lambda}}]$.[53] Let $\boldsymbol{P}=\boldsymbol{Q}(\boldsymbol{Q}'\boldsymbol{Q})^{-1}\boldsymbol{Q}'$, $\hat{\boldsymbol{P}}=\hat{\boldsymbol{Q}}(\hat{\boldsymbol{Q}}'\hat{\boldsymbol{Q}})^{-1}\hat{\boldsymbol{Q}}'$, $\boldsymbol{Z}=[\boldsymbol{G}\boldsymbol{y},\boldsymbol{X},\boldsymbol{G}^*\boldsymbol{X},\boldsymbol{\lambda}]$, and $\hat{\boldsymbol{Z}}=[\boldsymbol{G}\boldsymbol{y},\boldsymbol{X},\boldsymbol{G}^*\boldsymbol{X},\hat{\lambda}_n]$. Then the 2SLS estimator of $\boldsymbol{\theta}=(\phi,\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\sigma_{12})'$ is given by $\hat{\boldsymbol{\theta}}=(\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\hat{\boldsymbol{Z}})^{-1}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{y}$. As, by the mean value theorem,

$$\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon}=\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}[\boldsymbol{\epsilon}^*-\sigma_{12}(\hat{\boldsymbol{\lambda}}-\boldsymbol{\lambda})]=\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon}^*-\sigma_{12}\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{A}(\bar{\gamma})\boldsymbol{X}_1(\hat{\gamma}-\gamma),$$

the asymptotic variance of $\hat{\boldsymbol{\theta}}$ is

$$\mathrm{Avar}\sqrt{n}(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})=\mathrm{Avar}(\frac{1}{n}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\hat{\boldsymbol{Z}})^{-1}\frac{1}{\sqrt{n}}\hat{\boldsymbol{Z}}'\hat{\boldsymbol{P}}\boldsymbol{\epsilon}=\mathrm{plim}(\frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z})^{-1}\boldsymbol{\Sigma}(\frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}\boldsymbol{Z})^{-1},$$

where $\boldsymbol{\Sigma}=\frac{1}{n}\boldsymbol{Z}'\boldsymbol{P}[\sigma_2^2\boldsymbol{I}-\sigma_{12}^2\boldsymbol{A}+\sigma_{12}^2\boldsymbol{A}\boldsymbol{X}_1(\boldsymbol{X}_{1\bar{n}}'\boldsymbol{\Lambda}_{\bar{n}}\boldsymbol{X}_{1\bar{n}})^{-1}\boldsymbol{X}_1'\boldsymbol{A}']\boldsymbol{P}\boldsymbol{Z}$ and $\boldsymbol{\Lambda}_{\bar{n}}=\mathrm{diag}\{\frac{\phi^2(\boldsymbol{x}_{i,1}'\boldsymbol{\gamma})}{\Phi(\boldsymbol{x}_{i,1}'\boldsymbol{\gamma})(1-\Phi(\boldsymbol{x}_{i,1}'\boldsymbol{\gamma}))}\}_{i=1}^{\bar{n}}$.

To estimate σ_2^2, we consider $\mathrm{E}[\boldsymbol{\epsilon}^*\boldsymbol{\epsilon}^{*\prime}|\boldsymbol{y}_1^*>0]=\sigma_2^2\boldsymbol{I}-\sigma_{12}^2\boldsymbol{A}$. Let $\hat{\boldsymbol{\epsilon}}^*=\boldsymbol{J}\boldsymbol{y}-\boldsymbol{J}\hat{\boldsymbol{Z}}\hat{\boldsymbol{\theta}}$. Then σ_2^2 can be estimated by $\hat{\sigma}_2^2=\hat{\boldsymbol{\epsilon}}^{*\prime}\hat{\boldsymbol{\epsilon}}^*/\mathrm{tr}(\boldsymbol{J})+\hat{\sigma}_{12}^2\frac{1}{n}\sum_{i=1}^n[\lambda(\boldsymbol{x}_{i,1}'\gamma)\boldsymbol{x}_{i,1}'\gamma+\lambda^2(\boldsymbol{x}_{i,1}'\gamma)]$.

E.2 Network mispecification issues

[Insert Table A3 here]

[53] As the selection-bias correction introduces heteroskedasticity to the error term, the asymptotic properties of the many-instrument 2SLS and GMM become very difficult to derive. Hence, we only use the "conventional" instruments, which are very informative as indicated by the first-stage partial F-statistic, to estimate the selection-bias-corrected model.

Figure 1: Example networks where the local-aggregate model can be identified

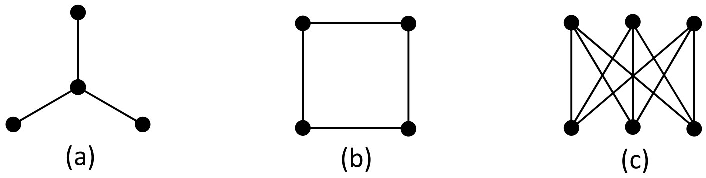


Figure 2: Distribution of networks by size

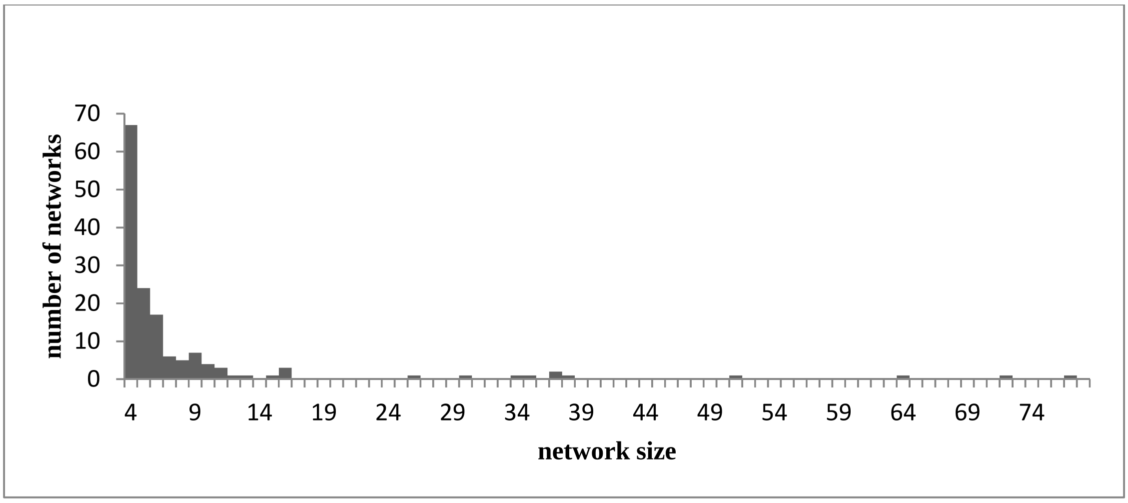


Figure 3: Who is the key player?

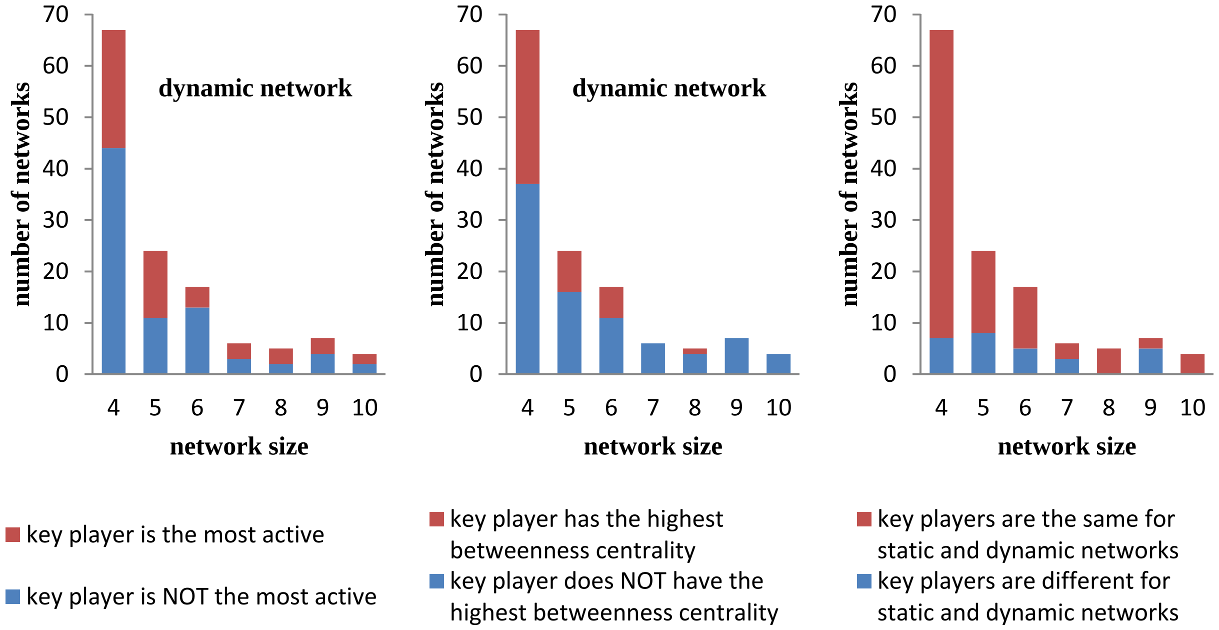
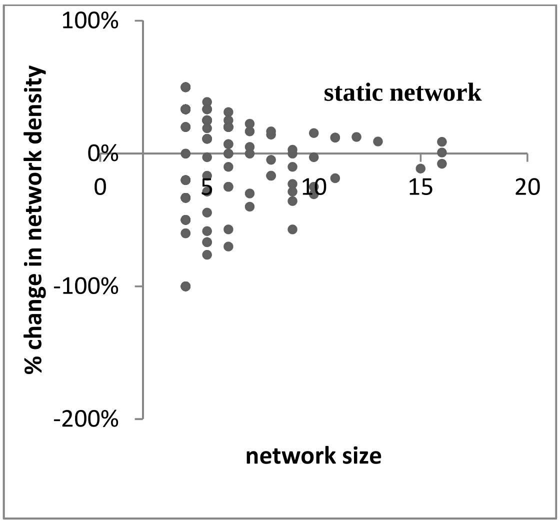


Figure 4a: Change in network topology after removal of the key player



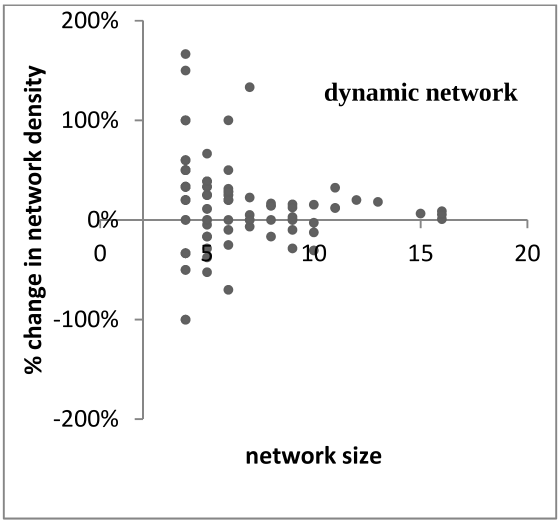


Figure 4b: Change in network topology from the static network to the dynamic network

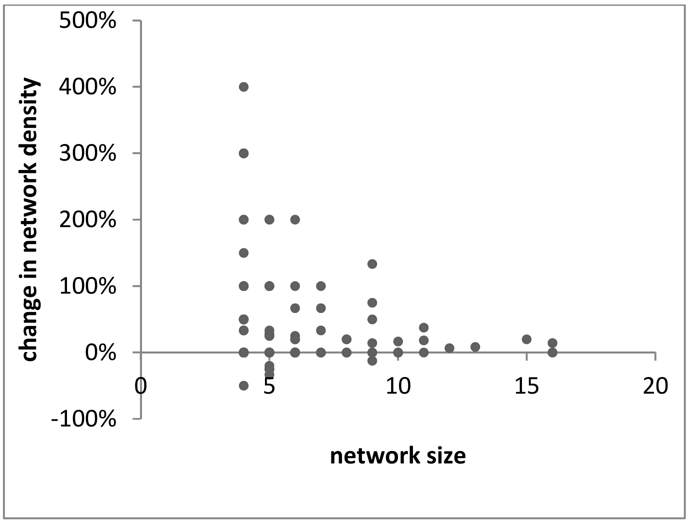


Figure 5: Crime reduction and network size

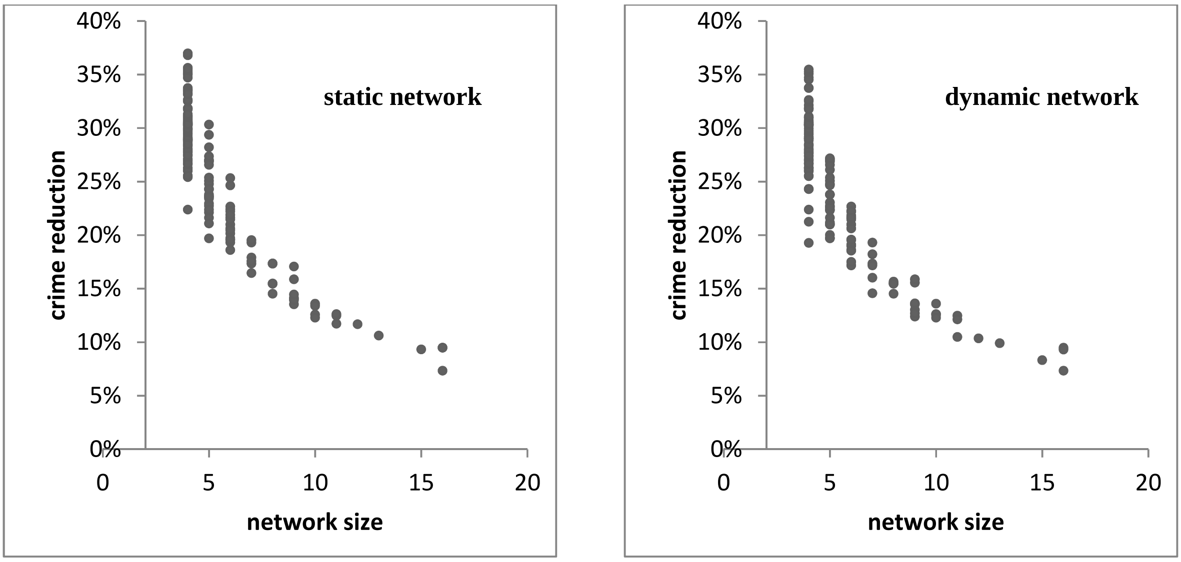


Figure 6: Difference between a key-player and a random-target policy

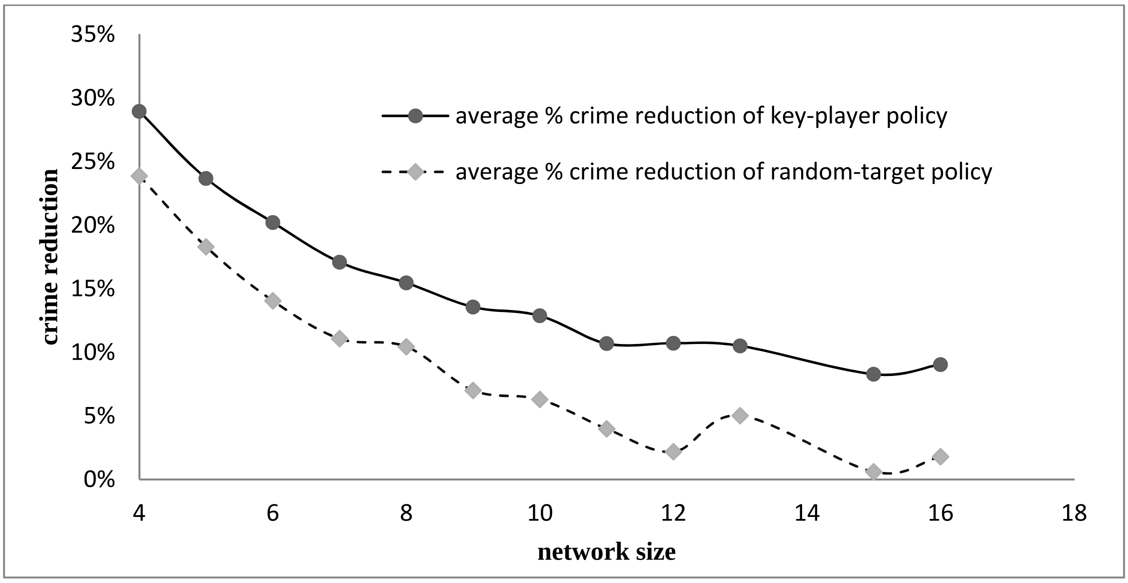


Table A1: Description of Variables

	Variable definition
Individual socio-demographic variables	
Female	Dummy variable taking value one if the respondent is female.
Student grade	Grade of student in the current year.
Black or African American	Race dummies. "White" is the reference group.
Other races	"
Mathematics score	Score in mathematics at the most recent grading period, coded as4= D or lower, 3= C, 2=B, 1=A.
Self esteem	Response to the question: "Compared with other people your age, how intelligent are you", coded as 1= moderately below average, 2= slightly below average, 3= about average, 4= slightly above average, 5= moderately above average, 6= extremely above average.
Physical development	Response to the question: "How advanced is your physical development compared to other boys/girls your age", coded as 1= I look younger than most, 2= I look younger than some, 3= I look about average, 4= I look older than some, 5= I look older than most
Family background variables	
Household size	Number of people living in the household.
Two married parent family	Dummy taking value one if the respondent lives in a household with two parents (both biological and non biological) that are married.
Single parent family	Dummy taking value one if the respondent lives in a household with only one parent (both biological and non biological).
Parent education	Schooling level of the (biological or non-biological) parent who is living with the child, distinguishing between "never went to school", "not graduate from high school", "high school graduate", "graduated from college or a university", "professional training beyond a four-year college", coded as 1 to 5. We consider only the education of the father if both parents are in the household.
Parent occupation dummies	Closest description of the job of (biological or non-biological) parent that is living with the child. If both parents are in the household, the occupation of the father is considered. The categories are: Parent occupation professional/technical , Parent occupation office or sales worker , Parent occupation manual , Parent occupation military or security , Parent occupation farm or fishery , Parent occupation other, "none" is the reference group
Protective factors	
School attachment	Response to the question: "You feel like you are part of your school coded as 5= strongly agree, 4= agree, 3=neither agree nor disagree, 2= disagree, 1= strongly disagree.
Relationship with teachers	Response to the question: "How often have you had trouble getting along with your teachers?" 0= never, 1= just a few times, 2= about once a week, 3= almost everyday, 4=everyday
Social inclusion	Response to the question: "How much do you feel that adults care about you, coded as 5= very much, 4= quite a bit, 3= somewhat, 2= very little, 1= not at all
Parental care	Dummy taking value one if the respondent reports that the (biological or non-biological) parent that is living with her/him or at least one of the parents if both are in the household cares very much about her/him
Residential neighborhood variables	
Residential building quality	Interviewer response to the question "How well kept is the building in which the respondent lives", coded as 4= very poorly kept (needs major repairs), 3= poorly kept (needs minor repairs), 2= fairly well kept (needs cosmetic work), 1= very well kept.
Residential area urban	Interviewer description of the residential immediate area or street (one block, both sides). Dummy taking value 1 if the area is "urban" and 0 otherwise ("suburban", "industrial properties - mostly wholesale" , "rural area", "other type")
Additional selection equation variables	
Religion practice	Response to the question: "In the past 12 months, how often did you attend religious services", coded as 1=no religion, 2= never, 3= less than once a month, 4= once a month or more, but less than once a week, 5= once a week or more.
Outside Options	Composite score of the answers to the following questions: 1. Money related: "How much money do you earn in a typical non-summer week from all your jobs combined"; Coded as 0=none, range $1 to $900 otherwise. "How much is your allowance each week"; Coded as 0=none, range $1 to $95 otherwise. 2.Daily activities: "During the past week, how many times did you do work around the house, such as cleaning, cooking, laundry, yard work, or caring for a pet"; "During the past week, how many times did you do hobbies, such as collecting baseball cards, playing a musical instrument, reading, or doing arts and crafts"; "During the past week, how many times did you go rollerblading, roller-skating, skate-boarding, or bicycling". Coded as 0= not at all, 1= 1 or 2 times, 2= 3 or 4 times, 3= 5 or more times.

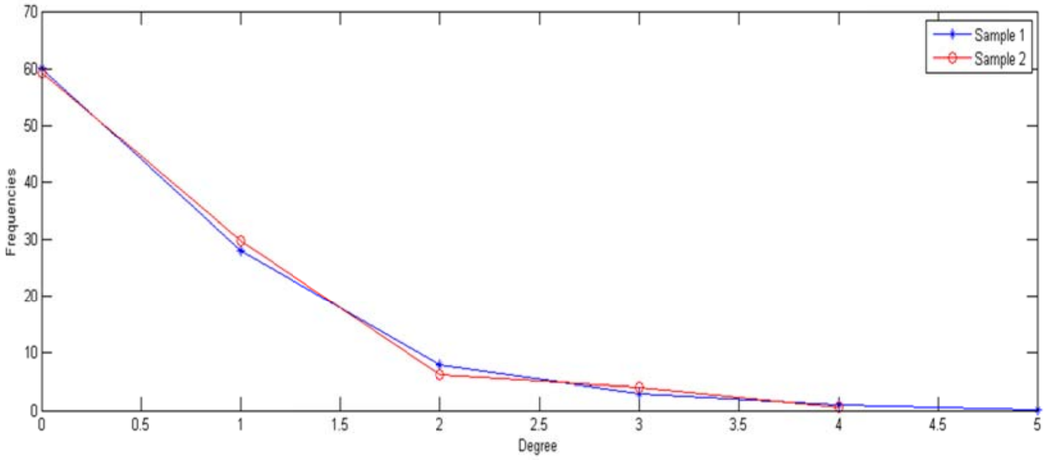
Table A2: Summary statistics

		Non criminals (population) N = 6,993		Criminals (population) N = 4,868		Criminals (selected sample) N = 1,297	
		Network size range 2-1,050				Network size range 4-150	
VARIABLES	range min-max	mean	sd	mean	sd	mean	sd
Female	0-1	0.55	0.50	0.50	0.50	0.49	0.50
Student grade	7-12	9.41	1.66	9.63	1.58	9.01	1.56
Black or African American	0-1	0.19	0.39	0.20	0.40	0.22	0.42
Other races	0-1	0.14	0.35	0.16	0.36	0.05	0.23
Mathematics score	1-4	2.14	0.99	2.36	1.04	2.19	1.01
Self esteem	1-6	4.03	1.10	3.90	1.08	4.00	1.09
Physical development	1-5	3.10	1.09	3.26	1.11	3.34	1.11
Household size	1-11	4.57	1.48	4.61	1.51	4.39	1.33
Two married parent family	0-1	0.73	0.44	0.71	0.45	0.71	0.45
Single parent family	0-1	0.23	0.42	0.25	0.43	0.25	0.43
Parent education	0-5	3.15	1.08	3.10	1.09	3.23	1.10
Parent occupation manager	0-1	0.11	0.31	0.10	0.30	0.12	0.32
Parent occupation professional/technical	0-1	0.18	0.38	0.19	0.39	0.21	0.41
Parent occupation office or sales worker	0-1	0.11	0.31	0.11	0.32	0.11	0.31
Parent occupation manual	0-1	0.34	0.47	0.34	0.47	0.30	0.46
Parent occupation military or security	0-1	0.03	0.18	0.03	0.16	0.02	0.14
Parent occupation farm or fishery	0-1	0.02	0.15	0.01	0.11	0.02	0.13
Parent occupation other	0-1	0.13	0.34	0.14	0.34	0.14	0.35
School attachment	1-5	4.07	0.88	3.88	0.98	4.06	0.92
Relationship with teachers	0-4	0.52	0.77	0.92	0.96	1.08	1.01
Social inclusion	1-5	4.52	0.76	4.40	0.79	4.46	0.76
Parental care	0-1	0.94	0.24	0.91	0.28	0.92	0.28
Residential building quality	1-4	1.53	0.80	1.58	0.80	1.53	0.81
Residential area urban	0-1	0.67	0.47	0.72	0.45	0.61	0.49
Religion practice	1-5	3.85	1.36	3.66	1.39	3.73	1.40
Outside options	1.1-3.9	2.8	0.44	2.78	0.43	2.83	0.42

Variables in italics denote statistically significant differences in means between criminals and non-criminals at least at the 10 percent significance level

Table A3: Sample comparisons

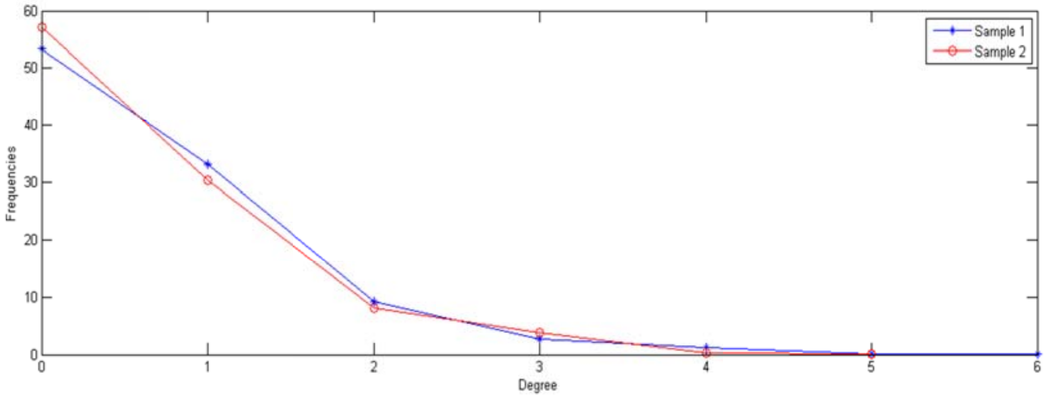
Panel (a) : Degree distribution of in-home criminals by friends' nomination type



Sample 1: criminals of the in-home sample
Sample 2: criminals of the in- home sample who nominate friends who are all in the in-home sample

Kolmogorov Smirnov test: *0.9935*
Whitney-Wolcox-Mann test: *0.5767*

Panel (b): Degree distribution of in-home criminals by friends' activity type



Sample 1: criminals of the in-home sample
Sample 2: criminals of the in-home sample who nominate friends who are all criminals

Kolmogorov Smirnov test: *0.4199*
Whitney-Wolcox-Mann test: *0.1498*

Notes: Degree is measured using out-degree. For each comparison, we even out the sample sizes of the two different samples by randomly removing nodes from the original (larger) one (see, e.g. Lehmann, 1998, Chapter 3). p-values of Kolmogorov Smirnov test and Whitney-Wolcox-Mann test in *Italics.*

Table 1. 2SLS and GMM estimates of the crime model

	Finite-IV 2SLS	Many-IV 2SLS	Bias-Corrected 2SLS	Finite-IV GMM	Many-IV GMM	Bias-Corrected GMM
Peer effects (φ)	0.0425***	0.0284***	0.0359***	0.0517***	0.0391***	0.0457***
	(0.0150)	(0.0119)	(0.0119)	(0.0133)	(0.0111)	(0.0111)
Female	-0.3019***	-0.2983***	-0.3002***	-0.3020***	-0.2973***	-0.2990***
	(0.0605)	(0.0604)	(0.0604)	(0.0606)	(0.0604)	(0.0605)
Student grade	-0.0324	-0.0359	-0.0340	-0.0294	-0.0323	-0.0308
	(0.0329)	(0.0328)	(0.0328)	(0.0329)	(0.0328)	(0.0329)
Black or African American	0.0383	0.0297	0.0343	0.0448	0.0375	0.0413
	(0.1490)	(0.1488)	(0.1488)	(0.1490)	(0.1488)	(0.1489)
Other races	-0.0133	-0.0198	-0.0163	-0.0138	-0.0240	-0.0214
	(0.1541)	(0.1539)	(0.1539)	(0.1542)	(0.1539)	(0.1540)
Parental Education	0.0048	0.0056	0.0051	0.0035	0.0039	0.0035
	(0.0352)	(0.0352)	(0.0352)	(0.0352)	(0.0352)	(0.0352)
Mathematics score	0.0767**	0.0757**	0.0762**	0.0766**	0.0753**	0.0757**
	(0.0341)	(0.0341)	(0.0341)	(0.0341)	(0.0341)	(0.0341)
Self esteem	0.0039	0.0041	0.0040	0.0038	0.0039	0.0039
	(0.0309)	(0.0308)	(0.0308)	(0.0309)	(0.0308)	(0.0309)
Physical development	0.0591**	0.0595**	0.0593**	0.0605**	0.0618**	0.0616**
	(0.0271)	(0.0271)	(0.0271)	(0.0272)	(0.0271)	(0.0271)
Parental care	-0.1810	-0.1865	-0.1836	-0.1795	-0.1862	-0.1836
	(0.1167)	(0.1165)	(0.1165)	(0.1168)	(0.1166)	(0.1166)
School attachment	-0.0436	-0.0428	-0.0432	-0.0433	-0.0420	-0.0423
	(0.0347)	(0.0347)	(0.0347)	(0.0347)	(0.0347)	(0.0347)
Relationship with teachers	0.2554***	0.2540***	0.2547***	0.2568***	0.2560***	0.2566***
	(0.0328)	(0.0327)	(0.0327)	(0.0328)	(0.0327)	(0.0328)
Social inclusion	-0.0986**	-0.0972**	-0.0979**	-0.0997**	-0.0987**	-0.0993**
	(0.0433)	(0.0433)	(0.0433)	(0.0434)	(0.0433)	(0.0433)
Residential building quality	0.0859**	0.0856**	0.0858**	0.0858**	0.0853**	0.0854**
	(0.0416)	(0.0415)	(0.0415)	(0.0416)	(0.0415)	(0.0416)
Residential area urban	-0.1965**	-0.2000***	-0.1981**	-0.1905**	-0.1912**	-0.1896**
	(0.0853)	(0.0852)	(0.0852)	(0.0853)	(0.0852)	(0.0853)
Household size	-0.0229	-0.0232	-0.0230	-0.0228	-0.0231	-0.0229
	(0.0243)	(0.0243)	(0.0243)	(0.0243)	(0.0243)	(0.0243)
Two married parent family	-0.2633	-0.2665*	-0.2648*	-0.2596	-0.2619	-0.2605
	(0.1605)	(0.1604)	(0.1604)	(0.1606)	(0.1604)	(0.1605)
Single parent family	-0.2419	-0.2441	-0.2429	-0.2406	-0.2430	-0.2421
	(0.1620)	(0.1620)	(0.1620)	(0.1622)	(0.1620)	(0.1621)
Parental occupation dummies	yes	yes	yes	yes	yes	yes
Network fixed effects	yes	yes	yes	yes	yes	yes
First stage F statistic	54.920	36.870				
OIR test p-value	0.249			0.245		

Notes: Dependent variable: Index of delinquency. Different Estimators are summarized in Appendix D. The number of observations is 1,297 individuals over 150 networks. Estimated coefficients and Standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10%, 5% and 1% level.

Table 2. Robustness check: Endogenous crime participation Heckman selection model estimation

First step: selection equation	Dep. Var.: Probability to become a delinquent		
		IV1: moral cost	IV2: moral cost & outside options
Religion Practice		-0.0331** (0.0132)	-0.0328** (0.0132)
Outside Options			-0.0891** (0.0436)
Log likelihood		-3394.8303	-3396.6793
Pseudo R2		0.0563	0.0558
Second step: outcome equation	Dep. Var.: Index of delinquency		
	No exclusion restrictions	IV1: moral cost	IV2: moral cost & outside options
Peer effects (φ)	0.0412*** (0.0146)	0.0416*** (0.0148)	0.0416*** (0.0146)
Inverse Mills Ratio	2.2151* (1.2297)	1.4632 (0.9897)	2.1545** (0.9673)
Female	-0.3390*** (0.0732)	-0.3269*** (0.0671)	-0.3399*** (0.0719)
Student grade	0.0182 (0.0479)	0.0007 (0.0417)	0.0150 (0.0430)
Black or African American	0.0870 (0.1670)	0.0735 (0.1579)	0.0802 (0.1654)
Other races	0.0714 (0.1753)	0.0515 (0.1669)	0.0828 (0.1745)
Parental Education	-0.0033 (0.0412)	-0.0008 (0.0380)	-0.0030 (0.0407)
Mathematics score	0.1291*** (0.0511)	0.1111*** (0.0442)	0.1269*** (0.0468)
Self esteem	-0.0285 (0.0401)	-0.0185 (0.0364)	-0.0257 (0.0380)
Physical development	0.1401*** (0.0578)	0.1113*** (0.0469)	0.1350*** (0.0483)
Parental care	-0.2599* (0.1543)	-0.2307* (0.1355)	-0.2531* (0.1496)
School attachment	-0.0792* (0.0477)	-0.0678 (0.0420)	-0.0791* (0.0455)
Relationship with teachers	0.5014*** (0.1517)	0.4169*** (0.1191)	0.4926*** (0.1211)
Social inclusion	-0.1362*** (0.0563)	-0.1215*** (0.0499)	-0.1330*** (0.0540)
Residential building quality	0.1090** (0.0504)	0.0999** (0.0459)	0.1081** (0.0495)
Residential area urban	-0.0973 (0.1147)	-0.1275 (0.1031)	-0.0935 (0.1092)
Household size	-0.0149 (0.0286)	-0.0180 (0.0263)	-0.0162 (0.0281)
Two married parent family	-0.2718 (0.1867)	-0.2699 (0.1725)	-0.2732 (0.1855)
Single parent family	-0.2852 (0.1911)	-0.2721 (0.1758)	-0.2894 (0.1893)
Parental occupation dummies	yes	yes	yes
Network fixed effects	yes	yes	yes

Notes: First stage probit estimation results and second stage finite-IV 2SLS estimation results are reported. *,**,*** denote statistical significance at the 10%, 5% and 1% level.

Table 3. Further robustness checks
Network misspecification issues

	(1)	(2)	(3)
Peer effects (φ)	0.0467**	0.0655***	0.0698*
	(0.0215)	(0.0263)	(0.0371)
Female	-0.3642***	-0.4022***	0.0037
	(0.0931)	(0.1065)	(0.0236)
Student grade	-0.0928**	-0.1134**	-0.0949
	(0.0466)	(0.0536)	(0.091)
Black or African American	0.0452	0.1586	-0.0732
	(0.2146)	(0.2302)	(0.0745)
Other races	-0.3699	-0.2028	0.0058
	(0.2460)	(0.2746)	(0.0162)
Parental Education	0.0379	0.0347	0.0193
	(0.0500)	(0.0588)	(0.0211)
Mathematics score	0.0790	0.1161*	0.0006
	(0.0528)	(0.0599)	(0.0203)
Self esteem	-0.0677	-0.0670	0.0285
	(0.0456)	(0.0531)	(0.0183)
Physical development	0.0514	0.0315	-0.1657**
	(0.0418)	(0.0484)	(0.0743)
Parental care	-0.3772**	-0.4003**	-0.0571**
	(0.1697)	(0.2024)	(0.0218)
School attachment	-0.0267	-0.0566	-0.0395
	(0.0517)	(0.0604)	(0.0265)
Relationship with teachers	0.1595***	0.1390***	0.1399***
	(0.0472)	(0.0536)	(0.0224)
Social inclusion	-0.1321**	-0.1583**	0.0112
	(0.0633)	(0.0704)	(0.0275)
Residential building quality	0.1306**	0.1987***	0.0357
	(0.0636)	(0.0688)	(0.0573)
Residential Area Urban	-0.3337***	-0.3569***	-0.0003
	(0.1300)	(0.1387)	(0.0143)
Household size	-0.0594*	-0.0647*	-0.1144
	(0.0345)	(0.0373)	(0.0991)
Two married parent family	-0.3152	-0.3616	-0.0985
	(0.2353)	(0.2589)	(0.1027)
Single parent family	-0.3287	-0.4663*	0.0311
	(0.2437)	(0.2639)	(0.0986)
Parental occupation dummies	yes	yes	yes
Network fixed effects	yes	yes	yes

Notes: Dependent variable: Index of delinquency. (1) Individuals who only nominate friends in the in-home survey. The number of observations is 648 individuals over 109 networks. (2) Individuals who only nominate friends who are criminals. The number of observations is 503 individuals over 98 networks. (3) Entire sample considering network of criminal and non-criminal friends. Finite-IV 2SLS estimated coefficients and standard errors (in parentheses) are reported. *,**,*** denote statistical significance at the 10%, 5% and 1% level.

Table 4. Who is the key player?
Significant differences

	All Criminals		Key Player		
	Mean	St. dev	Mean	St. dev	t-test
Individual characteristics					
Female	0.53	0.50	0.23	0.42	0.0000
Religion practice	3.65	1.41	3.28	1.57	0.0078
Parent education	3.23	1.06	3.01	1.14	0.0279
Mathematics score	2.18	1.00	2.53	1.05	0.0003
Parental care	0.93	0.26	0.80	0.40	0.0002
School attachment	4.12	0.87	3.71	1.07	0.0000
Relationship with teachers	0.99	0.92	1.79	1.22	0.0000
Social inclusion	4.47	0.74	4.23	0.86	0.0018
Residential building quality	1.51	0.79	1.70	0.96	0.0226
Two married parent families	0.74	0.44	0.61	0.49	0.0020
Single parent family	0.22	0.42	0.30	0.46	0.0706
Parent occupation manager	0.11	0.31	0.17	0.38	0.0704
Parent occupation military or security	0.02	0.14	0.00	0.00	0.0000
Parent occupation other	0.16	0.37	0.11	0.31	0.0673
Friends' characteristics					
Religious practice	2.52	1.98	3.02	1.80	0.0025
Student grade	6.42	4.33	7.64	3.85	0.0006
Parental education	2.30	1.66	2.61	1.54	0.0279
Mathematics score	1.54	1.24	1.87	1.24	0.0033
Self esteem	2.84	1.99	3.28	1.76	0.0066
Physical development	2.44	1.76	2.69	1.52	0.0810
Parental care	0.65	0.46	0.75	0.42	0.0152
School attachment	2.90	1.99	3.35	1.74	0.0055
Social inclusion	3.12	2.09	3.65	1.83	0.0019
Residential building quality	1.05	0.89	1.19	0.83	0.0621
Residential area urban	0.43	0.48	0.55	0.48	0.0033
Household size	3.13	2.22	3.48	1.97	0.0474
Single parent families	0.14	0.31	0.23	0.39	0.0105
Number of obs.	893		145		

Notes: T-test for differences in means with unequal variances had been performed. P-values are reported