Endogenous Fiscal Policy and Capital Market Transmissions In the Presence of Demographic Shocks

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Previous analyses of population aging mainly focused on the social security implications of the aging trend. This paper addresses aging in an open economy framework with two regions that have politically responsive fiscal policy regarding their education finance. The paper shows that capital flows from the aging region to the late-aging region, alleviating the negative growth and welfare effects of this region's demographic shift. While demographic shocks start a growth process that lead to income and consumption improvements in both regions, these results are sensitive to a critical parameter in the model that indicates the return to education spending. Low values of this parameter are associated with less favorable economic outcomes particularly in education spending and human capital. Hence, a policy implication emerges that investing in education system enhancement might pay off in terms of easing the negative growth and welfare consequences of expected demographic shocks. The paper also shows empirical evidence on the negative relationship between population aging and government spending on education based on panel data on 61 countries for the years 1965 through 1995.

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1. Introduction

The global economy is experiencing major changes as a consequence of the aging of developed nations' populations. An important and widely ignored characteristic of the recent aging trend is that it influences fiscal policy decision making concurrently with rapid global capital market integration. In a world connected by an international capital market, population aging in one region of the world leads to capital flows by altering saving and consumption behavior and, by extension, changes fiscal policy. An endogenously changing fiscal policy, in turn, affects human capital accumulation by changing government spending for public goods such as education that serve as inputs to human capital. Through these channels, population aging may have unexpected strong growth and welfare implications.

Recent discussions of aging have noted the potential generational conflict generated by the need to share society's resources between non-working elderly and the younger working population. While the literature focused on social security as the sole source of conflict between generations, an increasing number of studies, particularly from the U.S., also drew attention to public education as another key government spending program that may bring the young and the elderly into conflict amid an aging population.[1] Table 1 shows the size of education expenditures in OECD countries. Total public education expenditure gets as high as 5.5% of GDP (Iceland) and 16.2% of total public expenditure (Mexico) with OECD averages at 4% and 9%, respectively. OECD average for annual educational expenditures per student relative to GDP per capita is also about 23%, which shows the significance of these expenditures in the economies of developed countries. In the U.S., education is the largest public expenditure in the state government budgets. James Poterba (1997) started an interesting literature on aging and

[1] For a selection of general studies on aging and social security, see Börsch-Supan, Ludwig and Winter (2005), Feldstein (2001), Elmendorf and Sheiner (2000) and Bohn (1999).

education spending by providing empirical evidence from the U.S. using state-level data that older citizens prefer lower levels of public spending for education. Ladd and Murray (2001) did not find a strong evidence of generational conflict in a similar study that used county-level data. Another study by Harris, Evans, and Schwab (2001) confirmed Poterba's finding using school-district-level data, however with a smaller estimated impact than Poterba's estimates. A recent study by Grob and Wolter (2005) used panel data on Swiss Cantons to show that elderly population has indeed had a negative effect on public education spending during the period from 1990 to 2002. A branch of the literature on aging and economic growth used this evidence on the relationship between increasing political power of the elderly and government spending on education to examine economic growth effects. Recent examples to such studies are Holtz-Eakin, Lovely and Tosun (2004), Razin and Sadka (2004), Gradstein and Kaganovich (2004) and Razin, Sadka and Swagel (2002). While these studies found strong fiscal policy and/or growth effects of aging through its impact on education spending and human capital, they did not consider how this impact could be transmitted to other regions of the world. Hence they lack an open economy perspective. On the other hand, other recent studies examined open economy aspects of population growth differences between regions without a political economy perspective (see Sayan, forthcoming; Kenc and Sayan, 2001, and Deardorff, 1994).

This paper examines the dynamic transition effects of population aging considering the roles of international capital mobility and a politically responsive fiscal policy. Demographic changes that occur at different regions of the world affect the political process of determining fiscal policy and lead to capital flows by creating interest rate differentials between these regions. The open economy framework enables an examination of the interactions between the domestic and global effects of demographic changes. The paper compares demographic changes in economically similar regions. The goal is to highlight the significance of differential aging trends in regions like the United States, Europe and Japan.

The paper is structured as follows. The next section describes in detail the model used in this paper. Section 3 presents an analytical examination of dynamic capital stock transitions between pre-aging and post-aging steady states. Section 4 describes the transition analysis that requires numerical simulations and presents simulation results. The last section presents the concluding remarks.

2. The Model

2.1. Modeling Strategy

This section describes the three distinct features of the modeling approach. First, the paper employs an extended version of the two-period overlapping generations model. By explicitly tracing the life-cycle of savers, overlapping generations models enable a generational or cohort welfare analysis of population aging. This implies that demographic transitions, which take the form of changing relative cohort size, can be important determinants of national saving and the current account.

Secondly, a political process for fiscal policy making is explicitly modeled. Auerbach and Kotlikoff (1992) assert, “one of the greatest unknowns is how the political process will change as an increasing fraction of the voting population becomes elderly.” To make the political process of fiscal policy determination rich, interesting, yet tractable, a median-voter framework[2] with voter heterogeneity is used. Voter heterogeneity is introduced by assuming a distribution of genetic ability levels for the working generation. The ability level of the individual will, in turn, determine the value she receives from public education.

Finally, the third aspect of the modeling strategy focuses on transitions between pre-aging and post-aging steady states. For simplicity, demographic change is introduced as an

[2] The political process can be modeled using a median voter framework because the conditions for the median voter theorem are satisfied. The choice of voters is over a single dimension since the preferred tax rate is the only choice variable, and the voter preferences are single peaked. The property of single-peakedness has been demonstrated to ensure existence of a voting equilibrium (Black, 1948).

exogenous decrease in the population growth rate in different regions at different time periods. Population growth rates of the two regions are equal in both the initial and the final steady states[3]. Therefore, the analysis focuses on the transition between these two steady states.

The model extends the basic Diamond overlapping generations model[4] in two ways. First, it is a two-region world economy model similar to Buiter (1981), in which regions are perfectly symmetric in the initial steady state equilibrium. Second, it models political decision making for fiscal policy in order to examine political economy related effects of aging. With an increase in the dependency ratio,[5] fiscal policy shifts towards the preferences of the older generation. This combines with the effect of international capital mobility when demographic asymmetries (different population growth rates) between regions are present. Internationally mobile capital is the intermediary in spreading the effects of aging globally. For clarity, the model is presented for one region only. The two-region world equilibrium is described at the end of this section.

2.2. Household Behavior

Individuals live for two periods and seek to maximize a lifetime utility function,

$$U = \ln C_{jt} + \left(\frac{1}{1+\delta}\right)\ln C_{jt+1}, \qquad (1)$$

here j indexes individuals, C_{jt} is consumption when young, C_{jt+1} is consumption when old, and δ is the pure rate of time preference. The period-specific budget constraints in the first and the second periods are:

[3] In a world with divergent population growth, the region with the higher growth rate continuously gets bigger than the other region. In that case, the post-demographic shock steady state is not defined. Deardorff (1994) tackles this problem in a modified Solow growth model by including earnings from investment abroad as income of the region.
[4] See Diamond (1965). However, the earliest formalization of an overlapping generations model is due to Allais (1947) and Samuelson (1958).
[5] The dependency ratio is defined as the ratio of elderly to non-elderly persons, $\left(N_{t-1}/N_t\right) = \left(1/1+\eta_t\right)$ where η_t is the population growth rate. An increase in the dependency ratio is simulated by an exogenous decrease in the population growth rate η_t.

First period: $C_{jt}\left(a_j\right)+S_{jt}\left(a_j\right)=\theta_t w_t l_t\left(a_j\right)$

Second Period: $C_{jt+1}\left(a_j\right)=\left(1+\theta_{t+1}r_{t+1}\right)S_{jt}\left(a_j\right),$ (2)

where $\theta_t=\left(1-\tau_t\right)$, $S_{jt}\left(a_j\right)$ is first period saving, w_t is the wage rate individual *j* faces, $l_t\left(a_j\right)$ is effective labor[6], where a_j is the ability level of individual *j*, r_{t+1} is the rate of return to capital, and τ_t is the rate of income taxation.

Tax policy is a flat tax on the labor income of the young and the capital income of the old. I also assume there is a continuous distribution of abilities that is replicated in each new generation. The ability level of individual j is indexed by a_j, which ranges from 0 to 1. The density function of abilities is denoted by *f(a)* where by definition:

$$\int_0^1 f\left(a\right)da=1\,. \tag{3}$$

Human capital is accumulated from the interaction of ability level (a_j) of the individual and government spending per worker (g_t):

$$l_t\left(a_j\right)=\Phi\left[a_j g_t+1\right]^{\psi}, \tag{4}$$

here, Φ denotes an index on human capital efficiency and ψ is a parameter indicating the return to human capital from the inputs (a_j and g_t). The form of the human capital function is chosen so that even individuals with the lowest ability $(a_j=0)$ will contribute to the economy in terms of human capital (see Holtz-Eakin, Lovely, and Tosun 2004). From the maximization of (1) subject to (2) and (4); we get the familiar first order condition:

$$C_{jt}\left(a_j\right)=\frac{1+\delta}{\left(1+r_{t+1}\theta_{t+1}\right)}C_{jt+1}\left(a_j\right). \tag{5}$$

[6] Here, young supplies one unit of time to the economy. Note that, making the allocation of time between "schooling" and supplying labor endogenous does not change this analysis.

Using (5) and (2), we derive the optimal saving of an individual j:

$$S_{jt}\left(a_j\right) = \frac{1}{2+\delta}\theta_t w_t l_t\left(a_j\right). \tag{6}$$

Saving of an individual depends on net labor earnings but it is independent of the interest rate. This is due to the Cobb-Douglas form of the utility function. Given (5) and (6), it is easy to derive consumption functions in each period:

$$\begin{aligned} C_{jt}\left(a_j\right) &= \frac{1+\delta}{2+\delta}\theta_t w_t l_t\left(a_j\right) \\ C_{jt+1}\left(a_j\right) &= \frac{\left(1+r_{t+1}\theta_{t+1}\right)\left(\theta_t w_t l_t\left(a_j\right)\right)}{2+\delta}. \end{aligned} \tag{7}$$

2.3. Political Process

The consumption and saving decisions, as seen above, depend on human capital, which is in turn determined by government spending (see equation 4). By plugging these into (1), we get the indirect utility function, which each voter maximizes in determining his or her preferred tax rate, subject to the government budget constraint $(\tau_t y_t = g_t)$[7]. Note the assumption that individual voters are too small to affect w_t, r_{t+1}, and θ_{t+1} in (7) so each voter takes these as given. Then the preferred tax rate of individual *j* when young is:

$$\tau_{jt}\left(a_j\right) = \frac{a_j \psi y_t - 1}{\left(1+\psi\right) a_j y_t}. \tag{8}$$

Equation (8) is the tax rate each individual prefers based on her ability level. This preferred tax rate is increasing in both ability level a_j and income per young y_t. Because the old do not derive any benefit from publicly provided education and there are no bequests in the model, they incur a cost without enjoying any benefits. Therefore, their preferred tax rate will always be zero, regardless of their ability.

At each period of the model, a cohort of size N_t is born. Then total population in each period is $N_{t-1}+N_t$ where $N_t=\left(1+\eta_t\right)N_{t-1}$ and η_t is the population growth rate at period *t*. Given this, the median voter is defined by

$$N_{t-1}+N_t\int_0^m f\left(a\right)da=\frac{N_{t-1}+N_t}{2}, \tag{9}$$

here *m* is the ability level of the median voter.

With population aging, the median voter becomes a person with lower ability (see Appendix I) and the preferred tax rate of the median voter is lower. The intuition behind this is as follows: as the population ages (dependency ratio rises), older people will need fewer young voters to form a majority. These young voters are the ones at the lower end of the ability distribution. They prefer lower taxes than higher ability people because their return from public education is lower.

2.4. Producers' Behavior

Each region produces a single good using a Cobb-Douglas production technology.

$$Y_t=\Lambda K_t^{\alpha}H_t^{1-\alpha}\ , \tag{10}$$

here Λ is the productivity index, K is capital stock and H is aggregate supply of human capital. The aggregate supply of human capital is:

$$H_t=N_t\int_0^1 l\left(a\right)f\left(a\right)da. \tag{11}$$

Human capital per worker, using (4) and (11), is

$$h_t=\Phi\int_0^1\left(ag_t+1\right)^{\psi}f\left(a\right)da. \tag{12}$$

[7] Government uses the entire tax revenue to finance public education for all young equally, regardless of their ability level (Bearse, Glomm and Ravikumar, 2000).

Competitive factor markets require that real wage and interest rates are equal to the marginal products of labor and capital respectively. Therefore, factor demand equations are:

$$w_t = (1-\alpha)\Lambda\left(\frac{k_t}{h_t}\right)^{\alpha} \tag{13}$$

$$r_t = \alpha\Lambda\left(\frac{k_t}{h_t}\right)^{\alpha-1} . \tag{14}$$

Here, $k_t = K_t / N_t$ and $h_t = H_t / N_t$ are capital stock per worker and human capital per worker, respectively.

Using (6) and (12), saving per worker can be expressed as

$$s_t = \left(\frac{1}{2+\delta}\right)\theta_t w_t \Phi \int_0^1 (ag_t + 1)^{\psi} \ f(a)\, da. \tag{15}$$

2.5. World Equilibrium

To close the dynamic model, an international goods market condition and an international capital flow constraint must be specified. It is assumed that there is perfect international financial capital mobility[8]. In each region, claims to domestic and foreign capital are perfect substitutes. International goods market equilibrium requires that world saving is equal to world investment. The domestic capital stock per worker may be higher or lower than the domestic saving per worker, implying international capital flows. Writing this in per worker terms, international goods market equilibrium implies:

$$k_{t+1}^A + \left[\prod_{i=0}^{t}\frac{\left(1+\eta_{i+1}^B\right)}{\left(1+\eta_{i+1}^A\right)}\right]k_{t+1}^B = \frac{s_t^A}{1+\eta_{t+1}^A} + \left[\frac{\prod_{i=1}^{t\neq 0}\left(1+\eta_i^B\right)}{\prod_{i=0}^{t}\left(1+\eta_{i+1}^A\right)}\right]s_t^B, \tag{16}$$

[8] Feldstein and Horioka (1980) challenge this assumption by providing evidence that net capital flows are actually much smaller. However, Taylor (1996) shows that capital became increasingly mobile since 1980s, thanks to rapid integration of international capital markets.

here, superscripts A and B denote regions. Initial population sizes are assumed to be the same in both regions (i.e., $N_0^A = N_0^B$)[9]. In this model, capital income is taxed where income is earned. Thus, a territorial system of capital income taxation is used for both regions[10]. This implies that net-of-tax interest rates are equalized in equilibrium. Therefore, the international capital flow constraint is:

$$r_{t+1}^A\left(1-\tau_{t+1}^A\right) = r_{t+1}^B\left(1-\tau_{t+1}^{\mathrm{B}}\right) \tag{17}$$

Capital will move between regions until both (16) and (17) hold.

As sketched above, the model incorporates the interaction of household behavior, firm behavior, political process, and international capital flows. Given the aforementioned structure, we expect that the model will behave in the following way. Because the population growth rate changes, the median voter also changes. This leads to a change in government spending for public education, human capital accumulation, capital accumulation, and income per worker. As shown in Appendix I, when the population growth rate falls (increased dependency ratio), the ability level of the "new" median voter will be lower. All else equal, this political transition would reduce provision of the public good. However, if the tax rate decreases, national saving, and thus, physical capital accumulation will be enhanced. This will create a positive effect on income per worker, government spending per worker and human capital per worker, *ceteris paribus*. When the feedback effect through human capital accumulation dominates the negative

[9] When population growth rates of regions are equal in all periods, the expression in (16) is reduced to $k_{t+1}^A + k_{t+1}^B = \dfrac{s_t^A + s_t^B}{1+\eta_{t+1}}$.

[10] Under a territorial system, capital income is taxed where income is earned. The model tax treaties of the OECD and the UN both give source countries the first rights to tax income accrued within their borders. Countries claim this first right of taxation to any investment, domestic or foreign. However, in practice, only few countries have pure territorial taxation (e.g., France, Netherlands). Other countries such as the United States, England, Canada, and Japan practice a credit imputation system which is a mixed system of territorial and residence taxation. See Gravelle (1994, pp. 229-231) for a detailed comparison of international tax systems. Under a residence system, however, the interest rates will be equalized between countries, independent of the tax rates. Therefore, there will not be any direct effect of aging on international capital flows through changing tax rates.

effect of a lower preferred tax rate, the demographic shift may induce higher income per worker, higher capital stock per worker, and higher human capital and government spending per worker. However, in an international setting, changes in the interest rate in the aging region due to changes in income per worker and capital per worker may lead to capital flows between regions.

3. Transition Dynamics in Response to Population Aging

3.1. The Experiment

To focus on the role of demographic transition, the regions are assumed to be initially identical. As a benchmark, region A is the region that undergoes demographic transition first and it has a lower population growth rate than region B in the first two periods. region B's population growth rate decreases to region A's growth rate in the third period and the two growth rates are equalized thereafter.

3.2. Capital Stock Transitions

A decrease in the population growth rate can affect capital per worker in two ways. First, it can affect "directly" by causing fewer workers in the economy, which eventually leads to lower marginal product of capital, and thereby inducing capital outflows, *ceteris paribus*. Second, it can also affect "indirectly" through endogenous fiscal policy by changing the identity of the median voter. As a result of population aging, median voter becomes a lower ability person that votes for a lower preferred tax rate. This is important for an aging region since a lower tax rate implies lower government spending on the productivity enhancing public good, which in turn may produce negative growth effects. These direct and indirect effects can be analyzed by examining the dynamic solution of capital per worker caused by a change in the population growth rate at any period during transition. This is given by (see Appendix 2 for the derivation of this result)

$$\frac{dk_{t+1}^A}{d\eta_{t+1}^A} = \frac{dk_{t+1}^B}{d\eta_{t+1}^B} = \left(Z_2 - \frac{Z_3}{Z_1} \right) \quad \frac{dk_{t+1}^A}{d\eta_{t+1}^B} = \frac{dk_{t+1}^B}{d\eta_{t+1}^A} = \left(Z_2 + \frac{Z_3}{Z_1} \right), \tag{18}$$

$$\frac{dk_{t+1}^A}{d\eta_t^A}=\frac{dk_{t+1}^A}{d\eta_t^B}=\frac{dk_{t+1}^B}{d\eta_t^B}=\frac{dk_{t+1}^B}{d\eta_t^A}=\left(2MZ_2+Z_4\right), \tag{19}$$

where $M=\frac{\alpha\left(1-\frac{\tau^*}{1-\tau^*}\varepsilon_{\tau y}\right)}{2P}$ with the property *2M<1* by the intertemporal stability condition (see Appendix III). $Z_2=\frac{-k^*}{2\left(1+\eta^*\right)}$<0 shows the "direct saving effect" of a change in the population growth rate on capital per worker for a given ability of the median voter.[11] Holding the ability of the median voter fixed, a decrease in the population growth rate increases capital per worker by spreading the same saving over fewer workers. $Z_1=\frac{2\left[\alpha^2 y^*\left[\left(1-\tau^*\right)-\varepsilon_{\tau y}\tau^*\right]-Py^*\left(1-\tau^*\right)\right]}{k^{*2}P}$ shows the "direct net interest rate effect" (through capital changes) of a change in the population growth rate in one region and it is negative under the stability conditions explained in Appendix III. For a given ability of the median voter, an increase in capital per worker in one region depresses the net interest rate by putting a downward pressure on the marginal product of capital. This in turn has a negative effect on capital accumulation in that region through capital outflows, *ceteris paribus*.

There are also indirect saving and net interest rate effects. First, capital per worker in period *t+1* is affected by saving in period *t*. This is the "indirect saving effect" of a change in the ability of the median voter on capital per worker, which is depicted as

$Z_4=\frac{\left(1-\alpha\right)s^*\varepsilon_{hg}\varepsilon_{\tau a}\left(1-\frac{\varepsilon_{\tau y}\tau^*}{1-\tau^*}\right)-s^*\varepsilon_{\tau a}\frac{\tau^*}{1-\tau^*}P}{2P\eta^*\left(1+\eta^*\right)^2}$ in equation (19). For a given level of capital per

[11] In all the Z terms, $P=1-\left[\left(1-\alpha\right)\varepsilon_{hg}\left(1+\varepsilon_{\tau y}\right)\right]$, which is greater than zero by the stability of the political equilibrium, superscript "*" indicates steady state values, and $\varepsilon_{\tau y}$, $\varepsilon_{\tau a}$, and ε_{hg} are the elasticity of the tax rate with respect to income, elasticity of the tax rate with respect to ability level and the elasticity of human capital with respect to government spending, respectively.

worker, a decrease in median voter's ability decreases the tax rate, thereby increasing the net-of-tax labor income ($w_t h_t(1-\tau_t)$) and saving per worker. At the same time, the lower tax rate also lowers government spending, human capital per worker, labor income, and consequently saving per worker, *ceteris paribus*. The sign of this effect depends on the relative magnitudes of these two counteracting impacts.

Finally, there is an "indirect net interest rate effect" through a change in median voter's ability, which is shown as $Z_3 = \frac{(1-P)\alpha y^* \varepsilon_{\tau a}\left[(1-\tau^*)+\varepsilon_{\tau y}\tau^*\right] - \alpha y^* \varepsilon_{\tau a}\tau^* P(1+\varepsilon_{\tau y})}{(1+\varepsilon_{\tau y})k^* P(1+\eta^*)\eta^*}$ in the last term on the right hand side of equation (18). The sign of Z_3 is indeterminate, which makes the signs of the derivatives in (18) indeterminate as well. For a given level of capital per worker, a decrease in median voter's ability decreases the tax rate and thereby government spending and human capital per worker. This in turn would put a downward pressure on income per worker (y) and through this on the marginal product of capital, thus creating a negative effect on the net interest rate. However, since it decreases the tax rate, it creates a counter positive effect on the net interest rate. Z_3/Z_1 can be interpreted as the net interest rate effect of a change in median voter's ability on capital per worker after controlling for the effect of capital stock changes on the interest rate (Z_1). As shown by the Z_3/Z_1 terms in equation (18), a change in one region's population growth rate has opposite effects on capital transitions in two regions. For example, a decrease in the population growth in only region A at any point in time will decrease the net interest rate in region A as long as the decrease in tax rate is outweighed by the decrease in the interest rate. This, in turn, will cause region A capital to flow to region B until the net interest rates are equalized.

Notice that when the elasticity of the preferred tax rate with respect to median voter's ability, $\varepsilon_{\tau a} = \frac{1}{m\psi y - 1} > 0$, approaches zero, Z_3 is also equal to zero, which drives the net interest rate effect (Z_3/Z_1) to zero. This is because zero elasticity eliminates completely the feedback effect from an endogenously changing tax rate and government spending. Also, Z_3 is increasing (in absolute value) in $\varepsilon_{\tau a}$. Regardless of the sign of Z_3/Z_1, higher $\varepsilon_{\tau a}$ indicates a greater impact of the net interest rate effect on capital per worker in both regions. The expression for $\varepsilon_{\tau a}$ also shows that for given values of median voter's ability (*m*), and income per worker (*y*), a low value of ψ would imply a high value for $\varepsilon_{\tau a}$. Thus, the smaller is ψ, the greater is the decrease in the preferred tax rate to a decrease in the ability level and the greater is the magnitude of the net interest rate effect. Since, ψ indicates the return to ability and government spending on education in terms of human capital, demographic transition (in one region) will impose a greater degree of asymmetry between regions with low human capital return.

In order to explore the significance of open economy and endogenous fiscal policy effects, the effects in (18) and (19) are compared to analytic results from two alternative versions of this model. In the exogenous policy case, government spending education (*g*) is fixed[12]. Solutions for these models are:

Closed economy model: $\frac{dk_{t+1}^{A^c}}{d\eta_{t+1}^{A^c}} = 2Z_2$ and $\frac{dk_{t+1}^{A^c}}{d\eta_t^{A^C}} = 2\left(2MZ_2 + Z_4\right).$ (20)

Exogenous fiscal policy model: $\frac{dk_{t+1}^{A^f}}{d\eta_{t+1}^{A^f}} = \frac{dk_{t+1}^{A^f}}{d\eta_{t+1}^{B^f}} = \frac{dk_{t+1}^{B^f}}{d\eta_{t+1}^{A^f}} = \frac{dk_{t+1}^{B^f}}{d\eta_{t+1}^{B^f}} = Z_2$ (21)

[12] Government spending, instead of the tax rate, is fixed because this enables us to do a comparison in the presence of inefficiencies stemming from choosing a suboptimal value of *g*. Fixing the tax rate, however, allows for changes in government spending with changes in the tax base.

$$\frac{dk_{t+1}^{A^f}}{d\eta_t^{A^f}} = \frac{dk_{t+1}^{A^f}}{d\eta_t^{B^f}} = \frac{dk_{t+1}^{B^f}}{d\eta_t^{A^f}} = \frac{dk_{t+1}^{B^f}}{d\eta_t^{B^f}} = 2M^f Z_2. \qquad (22)$$

Comparing (18) and (19) with (20), both the direct saving effect (Z_2), and the indirect saving effect (Z_4) are shared equally between the two regions when these economies are open. Internationally mobile capital partially offsets the direct and indirect effects of population aging on saving by spreading them between the two regions. Comparing (18) and (19) with (20)-(22), it is clear that the net interest rate effect (Z_3/Z_1) is unique to the combined model. Since this effect changes capital per worker in A and B in opposite directions, it is a major factor in creating an asymmetry between the two regions throughout the demographic transition.

This analytical section identified the different effects that are in force in the impact of a demographic shock on capital accumulation. It also showed the significance of open economy and endogenous fiscal policy as two determining aspects of the model. However, theoretical ambiguity prevailed in how a demographic shock impacts on capital accumulation. Hence, the direction and magnitude of the transition effects are examined next in a simulation exercise.

4. Simulation of Transition Effects

The simulation study consists of transition analysis of population aging in both regions at different time periods. It is assumed that both regions are at an identical steady state in period 0. Population aging is simulated by a decrease in the population growth rate (or an increase in the dependency ratio) from an annual 2 percent rate to an annual 1 percent rate.[13] In the benchmark simulation, region A ages starting at period 1 while region B retains its demographic structure until period 3.[14] The population growth rates between the two region are equalized in period 3 and thereafter.

[13] This may seem like a dramatic decrease but Williamson (2001) argued that it's this type of sharp changes in the fertility rate that characterizes a "big demographic shock" such as the OECD baby boom.
[14] Moving the timing of the shock in region B to period 1 or period 3 does not make any qualitative difference in simulation results.

The elasticity of output with respect to capital input is set equal to one-third ($\alpha = 0.33$).[15] The annual rate of time preference is chosen to be 4%[16]. Notice that two parameters, the rate of time preference in the utility specification and the population growth rate, are adjusted to the length of the model period (30 years). In these simulations, the ability level, a, is distributed uniformly on the interval [0,1]. These parameter values and the timing of population aging mentioned above describe the benchmark simulation. However, the critical parameter in the model is the elasticity of human capital with respect to government spending on education and ability level (ψ). Since there is no known consensus on a possible value of this parameter, the literature on returns to education is reviewed in the next section.

4.1. Search for a Human Capital Elasticity Parameter

Laitner (2001b) uses a human capital function that is similar to (4) and sets his human capital elasticity with respect to education equal to 0.1967. Based on an initial value of the ability of the median voter, Laitner's estimate corresponds approximately to $\psi = 0.37$ in this model. Studies on the estimates of returns to education show that the rates of return estimated for the United States generally fall in the range of 5-15%.

Card (1995) gives a good survey of fairly recent rate of return estimates. While the OLS estimates are very low (highest estimate is 9%), the IV estimates range between 7-19%. Psacharopoulos (1985) gives estimates at international level as well. The author makes two key conclusions. First, the rate of return is considerably higher for low income and developing countries than for developed countries. Secondly, the rate of return is much higher for primary education than for secondary and high education. If we take all countries together, the upper

[15] This elasticity estimate is consistent with the data from the United States. See Laitner (2000a) for an argument.
[16] Caldwell, Favreault, Gantman, Gokhale, Johnson and Kotlikoff (1999) argue that a premium of riskiness should be added to the widely used 2% rate. They use 3.5% as the discount rate which is the real safe return on indexed Treasury bonds. See Coronado, Fullerton and Glass (2000) for a recent argument on the variety of discount rates used in studies of social security. They assert that the selection of discount rates ranges between 2%-5%.

bound will be close to 20%. Finally, Cawley, Heckman and Vytlacil (2001) look at wages and cognitive ability. They assert that without controlling for human capital measures such as education, measured ability explains 14-19.9% of wage variance. While, the above literature does not point to a specific value for ψ, ψ between 0.35 and 0.5 seems to be a plausible range. The next section starts with the optimistic value of ψ =0.5 and then use ψ = 0.35 at the low end to show the sensitivity of results. Finally, simulations for ψ = 0.7 are also run to show the importance of having high returns to education in the midst of population aging.

4.2. Simulation Results

With a decrease in the population growth rate (increase in the old-age dependency ratio) in region A, older people will form a majority with fewer young people at the lower end of the ability distribution. Therefore, the ability of the median voter is lower in region A, but unchanged in region B until period 3, after which it is the same in both regions. Figure 1 shows the implications of this on the preferred tax rate. In both regions, the tax rate decreases in the first period of their respective demographic shocks. Then the tax rate increases in both regions following this first period shock. This is due to income growth (see below) in both regions, and the fact that the preferred tax rate exhibits positive income elasticity. In the model, the lower tax rate in the aging region, *ceteris paribus*, causes lower government spending and this causes deterioration in human capital accumulation. In Figure 2, human capital per worker decreases in region A in period 1 and in region B in period 3. However, again due to income growth, human capital increases in both regions following their first period of aging, and both regions end up with a higher steady state human capital per worker.

[Figure 1 and Figure 2 about here.]

It is shown in the analytical section that the effect of a decrease in population growth rate on capital per worker is indeterminate due to the political economy of a changing median voter. Figure 3 shows that with the parameter values chosen for simulation, the capital stock per worker increases (capital deepening) in both regions throughout the entire transition. After the first demographic shock in region A, the abundance of capital per worker compared to a decreasing human capital per worker depresses the domestic interest rate in region A. At a lower interest rate, demand for capital increases in region B, causing capital to flow to region B. Capital per worker is higher in region B than in region A during the first two periods because the decrease in the ability of median voter in A depresses the net interest rate (Figure 4) in A even further causing more capital to flow to region B. The second demographic shock, which takes place in region B in period 3, boosts capital accumulation in both regions. The increase in capital per worker in the second shock is larger because both economies are growing prior to the second shock.

[Figure 3 and Figure 4 about here.]

These results for capital per worker and human capital per worker tell us how income responds to the demographic transition. For the first demographic shock, due to the capital inflow to region B until period 2, both capital and human capital per worker increase in region B leading to an increase in income per worker in the first two periods. On the other hand, even though the capital per worker increases in A, human capital per worker decreases in period 1. The income transitions in Figure 5 show that the decrease in human capital dominates, leading to a decrease in the income per worker in region A in the first period. In the second period however, due to both capital and human capital per worker increases, income per worker increases in region A. When the second demographic shock hits region B, unlike the first shock, there is income growth in both regions. Thus, region B does not experience the same income deterioration as region A did in the first shock even though the shock itself is identical in both cases. This interesting result, which is a clear indication of asymmetric economic transitions of regions aging at different times, can be explained by the observation that economies are growing prior to the second demographic shock.[17] Therefore, this "growth effect" partially offsets the adverse effects of population aging for region B.

[Figure 5 about here.]

Welfare effects of aging can be examined by defining welfare in period t as the lifetime welfare of a person of generation t.[18] Welfare of this person is composed of her consumption when young (in period t) and consumption when old (in period $t+1$). In order to examine welfare effects, an indirect utility function is derived by substituting consumption solutions in (7) into (1), and integrating this over the ability distribution. To understand welfare changes, consumption transitions are examined first. In Figure 6, consumption of old persons of generations 0 and 2 decreases in both regions immediately after the two shocks. On the other hand, consumption of young persons of all generations increases in both regions throughout the entire demographic transition. In periods 0 and 2, consumers internalize the steep decrease in the net of tax interest rate (see Figure 4) in periods 1 and 3. Thus, young persons who save in periods 0 and 2 will consume less from a given level of savings in their old age in periods 1 and 3. This result explains welfare changes in periods 0 and 2.

[Figure 6 about here.]

Figure 7 shows that welfare level of generations 0 and 2 drop in both regions. However, the decrease in welfare in period 2 is less pronounced than the decrease in period 0. The answer to this puzzle lies at the increase in consumption of young in period 2, which is again the consequence of an already growing economy prior to the second shock. As hinted by the simulation results in this section, international capital mobility and the political economy of a

[17] In contrast, I assume that both economies are at a steady state prior to the first demographic shock.
[18] Generation t refers to persons who are young in period t and old in period $t+1$.

changing median voter are the key factors in understanding the effects of population aging on the economic transitions of regions.

[Figure 7 about here.]

In summary, short term impacts of demographic shocks are to decrease education spending and human capital by lowering the tax rate earmarked for education. However, these shocks also start a growth process by increasing the saving per worker, capital per worker and income per worker in the longer term, leading to favorable consumption and welfare transitions. Results also show that region B that starts aging later is expected to experience more favorable transition in capital stock, income, consumption and welfare than region A that starts aging first. This is mainly due to capital flows from region A to region B following the demographic shock in region A. Region B benefits from economic growth triggered by these capital flows which also help alleviate the negative impacts of its own demographic shock in the later periods. This is an important result for a country like the United States which lags behind regions like Europe and Japan in the aging of its population. In the following section, these results are compared to results from alternative versions of the model.

4.3. Comparison with Alternative Versions

Table 2 compares the effects of demographic transitions in an open economy to those without international capital mobility. The open economy results in columns (3), (4), and (5) are considerably different from the closed economy results in columns (1) and (2). The open economy results with exogenous fiscal policy[19] in column 3 are identical for both regions. By affecting capital accumulation in the two regions in opposite directions, a politically determined fiscal policy single-handedly creates asymmetric results for regions. On the other hand,

[19] Here exogenous fiscal policy means a fixed government spending regime. Alternatively, tax rate could be fixed. However, simulation results with a fixed tax rate do not differ qualitatively from results with a fixed government spending. These simulation results are available from the author upon request.

comparing the closed economy results with the open economy results, capital mobility acts as a shock absorber, spreading the effects of aging to both economies. While capital flows smooth consumption and welfare in region A for the first demographic shock, they smooth consumption and welfare in region B for the second shock. These results suggest that the interaction of open economy and endogenous fiscal policy changes the magnitude of the effects of aging in a considerable way.

[Table 2 about here.]

An even more striking result is that it is possible to change the pattern as well as the magnitude of these effects. Aging increases region A's income per worker in the closed economy version, but it decreases first period income per worker in region A in the open economy model with endogenous fiscal policy. The explanation lies in the relatively smaller increase in the first period capital stock and greater decrease in first period human capital and government spending per worker in region A. Another change is seen in welfare transitions. Unlike the closed economy, the open economy results in columns (3) and (4) show that the welfare of generation 0 in region B and welfare of generation 2 in region A both decrease because of the decreases in old age consumption.

Finally, results in columns (4) and (5) show that there are transition asymmetries between the first and the second demographic shocks. In terms of welfare, both regions are better off in the second shock compared to the first one. This can be explained by the economic growth that is taking place prior to the second demographic shock. Therefore, even though the demographic shocks are symmetric (a decrease in population growth from 2 percent to 1 percent), the economic transitions are not.

4.4. Sensitivity to Returns to Education Spending

The simulations, described in the previous sections, are based on parameter values explained in the beginning of section 5. Among those, the parameter of the human capital function (ψ), which can be interpreted as the return, in terms of human capital, from government spending on education, is an important aspect of the model and the welfare results. Accordingly, in Table 3, the results from a sensitivity analysis for ψ are presented. Higher values of ψ are associated with higher values of the elasticity of human capital with respect to government spending on education (ε_{hg}) and lower values of the elasticity of the preferred tax rate with respect to median voter's ability $(\varepsilon_{\tau a})$. Thus, for a high value of ψ, there will be a relatively smaller decline in the tax rate and government spending from a decrease in median voter's ability, while there will be a relatively greater human capital feedback from government spending on education for a given ability level. For $\psi = 0.7$ in columns (5) and (6), regions overall have more favorable transitions, indicated by capital, income and welfare growth throughout the transition periods. However, for $\psi = 0.35$, the decrease in government spending and human capital due to a decrease in median voter's ability is so large that there is even a decrease in capital per worker in period 2 in both regions. This leads to substantial welfare deterioration in periods 1 and 2 for region A and in period 2 for region B.

[Table 3 about here.]

The above analysis shows that changes in economic variables are highly sensitive to the returns to education spending parameter ψ. For ψ =0.35, which is the more likely parameter value given the empirical estimates of return to education discussed earlier, both government spending on education and human capital per worker deteriorate in the period following the demographic shock and between pre-shock and post-shock steady states despite the growth in capital per worker and income per worker. This finding is in line with empirical evidence from the U.S. on aging and education spending presented by Poterba (1997) and Harris, Evans and Schwab (2001). Results for higher ψ suggest that regions can overcome this negative effect from population aging by improving their returns to education spending. One robust result that was first mentioned in section 4.2 is that late-aging region B experiences a more favorable transition following its own demographic shock compared to region A's transition following its demographic shock two periods earlier.

4.5. Empirical Evidence on Population Aging and Education Spending

In a final exercise, empirical evidence is provided on how well the model, examined in previous sections, explains the historical relationship between population aging and government spending on education. Regression analysis is used to estimate the determinants of the natural logarithm of real per worker education spending. Before proceeding to the empirical analysis, several econometric issues warrant discussion. The data are a panel of 427 observations that include 61 countries and five-year data for the years 1965 through 1995. Table 4 provides the descriptive statistics for the variables used in the regressions. Two conventional approaches for estimating panel data are the fixed-effects and random-effects procedures. However, if the individual country fixed-effects are correlated with other exogenous variables, the random-effects estimation procedure yields inconsistent estimates. A Hausman test (with chi-square test statistic 11.23) shows that the fixed country-effects are correlated with the other exogenous variables, which suggests that the fixed-effects estimation procedure is more appropriate for this analysis.

The model suggests that the tax rate for education in equation (8) and government spending on education are mainly a function of income per worker and the demographic change, which in turn changes the identity of the median voter and his preferred spending. It is also demonstrated in the model and the simulation exercise that open economy has a significant effect on how aging could affect the economic variables including government spending on education. Thus, the following general specification is used to run regressions with government spending on education as the dependent variable:

$$Education_{it} = \alpha + \beta_1 GDP_{it} + \beta_2 Openness_{it} + \beta_3 Elderly_{it} + f_i + \phi_t + \varepsilon_{it}, \qquad (23)$$

where "$Education_{it}$" is the real government spending on primary, secondary and tertiary education in country *i* at time *t*. GDP_{it} is real GDP per worker, which controls for income changes. $Openness_{it}$ is trade openness, which proxies for capital mobility between countries and is measured as the ratio of the sum of exports and imports of goods and services to the gross domestic product.[20] $Elderly_{it}$ is the share of 65 and older population in total population, which controls for population aging.[21] f_i represents the unobservable country specific, time-invariant effects, ϕ_t represents unobservable time specific effects, and ε_{it} represents time-variant unsystematic effects and is i.i.d. The inclusion of year dummies controls for the time specific effects, ϕ_t in equation (23).[22] All variables in the regression except country and year dummies are in natural logs. The results of this regression are shown in column (1) of Table 5. GDP per worker has a positive sign and is statistically significant as predicted by the model, exhibiting a slightly elastic relationship. Openness also has a positive sign and is statistically significant. Population 65 and Older has a negative sign, which is in line with the model, but it is not statistically significant. While this simple regression provides some evidence on the relationship between aging and education spending, it draws on data from a variety of countries that have quite different economic structures and different degrees of openness. Hence, in the next regression, a dummy variable for high income developed countries, *OECD*, is used and interacted with Openness and Population 65 and Older to derive the effects for the relatively

[20] A direct measure for capital mobility such as capital flows was not available for the group of countries and the time period used in the empirical analysis.
[21] Other indicators of population aging such as old-age dependency ratio can also be used. Results presented here are found to be insensitive to the use of this alternative measure. Share of 65 and older population is used more widely in empirical studies on aging.
[22] The year 1965 is excluded to avoid the dummy variable trap.

open developed countries. Column (2) in Table 5 reports the results of this new regression.[23] The sign of Population 65 and Older is still negative but it is now statistically significant as well. The sign of OECD * Openness * 65 and Older is also negative and this interaction is statistically significant, which means that population aging (rising share of 65 and older population) in relatively more open OECD countries have had a negative effect on education spending per worker in those countries. This result is in line with the model and the simulation results discussed in section 4.3. As these results suggest, population aging may still have a positive effect on government spending on education since aging induced income growth may dominate the negative effect from rising political power (in terms of lower support for education spending) of the elderly population.

5. Summary and Concluding Remarks

Population aging affects education spending policy by changing the political balance in favor of the preferences of older generations. This triggers changes in the economy of both the aging region and other regions through the medium of internationally mobile capital. The numerical simulation exercises show that capital flows from the aging region to the late-aging region, alleviating the negative growth and welfare effects of this region's demographic shift. The paper distinguishes between the direct effects of having fewer workers in the economy and indirect effects arising from fiscal policy dynamics caused by a changing identity of the median voter. The paper shows the significance of combining open economy with endogenous fiscal policy in the same model by comparing this to closed economy and exogenous fiscal policy models. While an effect that is unique to the combined model is identified, it is not possible to determine the direction and exact magnitude of this effect. A numerical simulation exercise shows that despite the negative effects of the demographic shocks on education spending and human capital per worker, these shocks still trigger a growth process that lead to consumption

[23] Note that OECD, Openness and OECD * Openness are dropped from regression to avoid perfect multicollinearity.

and welfare improvements in both regions. Separate simulations also reveal that these results are sensitive to a critical parameter in the model that indicates the return to education spending. Indeed, the low values of this parameter are associated with less favorable economic outcomes particularly in education spending and human capital. Hence, a policy implication emerges that investing in education system enhancement might pay off in terms of easing the negative growth and welfare consequences of expected demographic shocks. The paper also shows empirical evidence on the negative relationship between population aging and government spending on education based on panel data on 61 countries for the years 1965 through 1995. This result was particularly significant for OECD countries that have been relatively more open. At the same time, these results showed a strong positive relationship between GDP per worker and government spending on education. Hence, to the extent that the aging trend causes income growth, population aging may still have a net positive impact on government spending on education.

Appendix

I. The Effect of Increasing Dependency Ratio on the Ability Level of the Median Voter

Recall that median voter is defined by $N_{t-1} + N_t \int_0^m f(a)\,da = \frac{N_{t-1}+N_t}{2}$. Rewriting this:

$N_{t-1} + N_t F(m) - N_t F(0) = \frac{N_{t-1}+N_t}{2}$, dividing both sides by N_{t-1}:

$1 + (1+\eta_t)\{F(m) - F(0)\} = \frac{2+\eta_t}{2}$ which can be rearranged as $F(m) - F(0) = \frac{\eta_t}{2(1+\eta_t)}$.

Differentiating both sides we get, $F'(m)\,dm = \frac{2(1+\eta_t)\,d\eta_t - 2\eta_t\,d\eta_t}{4(1+\eta_t)^2}$. Finally this can be rearranged as $\frac{dm}{d\eta_t} = \frac{2}{\underset{(+)}{F'(m)}\,4(1+\eta_t)^2}$ which is positive. Therefore, with a decrease in the population growth rate the ability level of the median voter will be lower.

II. Derivation of Steady State and Capital Stock Transitions

Transitions can be examined by tracing the behavior of the capital stock over time. For this analytical approach, I totally differentiate the equations given by (8), (10), and (12) through (15) for each region, and (16) and (17), which can be reduced to a vector difference equation:

$$F_{t+1} = \Omega F_t + \hat{\Omega} X_t \qquad \text{(A.1)}$$

where

$$F_{t+1} = \begin{bmatrix} dk_{t+1}^A \\ dk_{t+1}^B \end{bmatrix},$$

$$F_t = \begin{bmatrix} dk_t^A \\ dk_t^B \end{bmatrix}, X_t = \begin{bmatrix} d\eta_t^A \\ d\eta_t^B \\ d\eta_{t+1}^A \\ d\eta_{t+1}^B \end{bmatrix}$$

and

$$\Omega = \begin{bmatrix} M & M \\ M & M \end{bmatrix}, \quad \hat{\Omega} = \begin{bmatrix} Z_4 & Z_4 & Z_2 - \frac{Z_3}{Z_1} & Z_2 + \frac{Z_3}{Z_1} \\ Z_4 & Z_4 & Z_2 + \frac{Z_3}{Z_1} & Z_2 - \frac{Z_3}{Z_1} \end{bmatrix}$$

The vector difference equation (A.1) is used for two purposes. First, for a constant population growth across periods, it can be used to examine the stability of the dynamic system. Ω summarizes the relationship between k_t and k_{t+1}. Secondly, for a given capital stock in period t, I examine the effect of a change in the population growth rate on the capital stock in period $t+1$. I will address each of these below.

In order to analyze the stability of the dynamic system, I examine the matrix Ω in which $M = \frac{\alpha\left(1-\frac{\tau^*}{1-\tau^*}\varepsilon_{\tau y}\right)}{2P}$, and $P = 1-\left[(1-\alpha)\varepsilon_{hg}\left(1+\varepsilon_{\tau y}\right)\right]$ within M. Political equilibrium is locally stable when $P > 0$ and the system is dynamically stable when $2M < 1$ (see Appendix III for derivation of the stability conditions).[24] We can see how the dynamic system will reach a steady state after a demographic transition. As an example, let's assume that region A is the region that undergoes demographic transition first and it has a lower population growth rate than region B in the first two periods. Region B's population growth rate decreases to region A's growth rate in the third period and the two growth rates are equalized thereafter.

Expanding (A.1) for a fixed level of initial capital stock per worker for $t = 0,1,2$:

$$\begin{aligned} F_1 &= \Omega F_0 + \hat{\Omega} X_0 \\ F_2 &= \Omega^2 F_0 + \Omega \hat{\Omega} X_0 + \hat{\Omega} X_1 \\ F_3 &= \Omega^3 F_0 + \Omega^2 \hat{\Omega} X_0 + \Omega \hat{\Omega} X_1 + \hat{\Omega} X_2 . \end{aligned} \quad \text{(A.2)}$$

Then generalizing this for $t > 2$,

$$F_{t+1} = \Omega^{t+1} F_0 + \Omega^t \hat{\Omega} X_0 + \Omega^{t-1} \hat{\Omega} X_1 + \ldots + \Omega \hat{\Omega} X_{t-1} + \hat{\Omega} X_t \quad \text{(A.3)}$$

where $F_0 = \begin{bmatrix} dk_0^A \\ dk_0^B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

24. Due to Cobb-Douglas, $\varepsilon_{\tau y} < \frac{1-\tau}{\tau}$. Then, it can be shown that $0 < 2M < 1$.

Notice that the coefficients of the terms in (A.3) resemble a geometric series. In line with my assumptions regarding the demographic transition I can assume that $X_3 = X_4 = ... = X_{t-1} = X_t$. Using this, (A.3) can be written as:

$$F_{t+1} = \Omega^t \hat{\Omega} X_0 + \Omega^{t-1} \hat{\Omega} X_1 + \Omega^{t-2} \hat{\Omega} X_2 + (\Omega^0 + \Omega^1 + \Omega^2 + ... \Omega^{t-3}) \hat{\Omega} X_t \qquad \text{(A.4)}$$

As $t \to \infty$, the first three terms in the above expression converge to zero and since all the elements of the matrix Ω are less than 1 by dynamic local stability, the geometric series in parentheses converges to $(I - \Omega)^{-1}$. Thus, assuming that $d\eta_t^A = d\eta_t^B = d\eta$ (identical changes in the steady state population growth rates), as $t \to \infty$, (A.4) can be written as:

$$\underset{t \to \infty}{F_{t+1}} = (I - \Omega)^{-1} \hat{\Omega} X \qquad \text{(A.5)}$$

By using Ω, and $\hat{\Omega}$, and converting (A.5) back to the scalar form, we get the change in capital per worker from the initial steady state to the post-demographic transition steady state:

$$\frac{dk^A}{d\eta} = \frac{dk^B}{d\eta} = \frac{2(Z_2 + Z_4)}{1 - 2M} \qquad \text{(A.6)}$$

Using (A.1) and (A.3) for any period of transition, we get

$$\frac{dk_{t+1}^A}{d\eta_{t+1}^A} = \frac{dk_{t+1}^B}{d\eta_{t+1}^B} = \left(Z_2 - \frac{Z_3}{Z_1} \right) \quad \frac{dk_{t+1}^A}{d\eta_{t+1}^B} = \frac{dk_{t+1}^B}{d\eta_{t+1}^A} = \left(Z_2 + \frac{Z_3}{Z_1} \right), \qquad \text{(A.7)}$$

$$\frac{dk_{t+1}^A}{d\eta_t^A} = \frac{dk_{t+1}^A}{d\eta_t^B} = \frac{dk_{t+1}^B}{d\eta_t^B} = \frac{dk_{t+1}^B}{d\eta_t^A} = (2MZ_2 + Z_4), \qquad \text{(A.8)}$$

which are (18) and (19) in the text.

III. Stability

A. Stability of the Political Equilibrium

The political equilibrium is locally stable if starting from a given level of government spending (*g*), the economy moves automatically to an equilibrium (Holtz-Eakin, Lovely, and Tosun 2004). For an analysis of stability, Marshallian-type adjustment rule is assumed:

$$\frac{dg}{dt} = c\left[\frac{\tau_p}{\tau} - 1\right] = \varphi(g) \qquad \text{(A.9)}$$

In this expression c is a positive constant. The variable τ is the actual tax rate, which is a function of government spending per worker and income per worker. The variable τ_p is median voter's preferred tax rate for a given level of government spending and income. Equation (A.9) states that if median voter's preferred tax rate exceeds actual tax rate, government spending will rise. For a political equilibrium, $\varphi(\hat{g}) = 0$ and $\hat{g}$ is locally stable if and only if $\varphi'(\hat{g}) < 0$ (where $\hat{g}$ is the equilibrium level of government spending). Differentiating (8), (10) and (12) in the text, we get:

$$d\tau_{t\,p} = \frac{1}{y^*}(1-\alpha)\varepsilon_{hg}\varepsilon_{\tau y} dg_t \qquad \text{(A.10)}$$

Differentiating the government budget constraint $(\tau_t y_t = g_t)$ and using the differentiated form of (10), we get:

$$d\tau_t = \frac{\left[1-(1-\alpha)\varepsilon_{hg}\right]}{y^*} dg_t \qquad \text{(A.11)}$$

Finally, differentiating (A.9) and using (A.10) and (A.11), we get the stability condition for thr political equilibrium:

$$1 - \left[(1-\alpha)\varepsilon_{hg}\left(1+\varepsilon_{\tau y}\right)\right] > 0 \qquad \text{(A.12)}$$

This makes P in the text positive.

B. Stability of the Intertemporal Equilibrium

B.1 Intertemporal stability requires that the eigen value of the matrix Ω_{2x2} in (A.1) is less than 1 in absolute value. Eigen value is found by solving $|\Omega - \lambda I| = 0$, where λ is the eigen value and I_{2x2} is the identity matrix. The two possible values of λ are:

$$\lambda = 0 \text{ or } \lambda = 2M = \frac{\alpha\left(1-\frac{\tau^*}{1-\tau^*}\varepsilon_{\tau y}\right)}{P} \tag{A.13}$$

Using the nonzero root, intertemporal stability requires:

$$|\lambda| < 1 \text{ or } |2M| = \left|\frac{\alpha\left(1-\frac{\tau^*}{1-\tau^*}\varepsilon_{\tau y}\right)}{P}\right| < 1$$

However, since $\varepsilon_{\tau y} < \frac{1-\tau}{\tau}$ (as shown in the proof below) and $P > 0$ by political stability, $2M$ is positive and the intertemporal stability condition can be written as:

$$\alpha\left(1-\frac{\tau^*}{1-\tau^*}\varepsilon_{\tau y}\right) < P \tag{A.14}$$

B.2 $2M > 0$

Proof by contradiction: Let $\varepsilon_{\tau y} \geq \frac{1-\tau}{\tau}$,

Since $\varepsilon_{\tau y} = \frac{1}{m\psi y - 1}$

and

$$\frac{1}{m\psi y - 1} \geq \frac{1}{\tau} - 1,$$

then

$$\frac{m\psi y}{m\psi y - 1} \geq \frac{(1+\psi)my}{m\psi y - 1}$$

which can not be true since $m\psi y - 1 > 0$ and $\psi > 0$, $m > 0$, $y > 0$.

Then $\varepsilon_{\tau y} < \frac{1-\tau}{\tau}$, and under political stability ($P > 0$) this implies $2M > 0$.

Table 1. *Education Expenditures in OECD Countries*

COUNTRIES	Annual Expenditure on Educational Institutions per student (2002)*	Annual Expenditure on Educational Institutions per student relative to GDP per capita (2002)	Total Public Expenditure on Education as a percentage of Total Public Expenditure (2002)	Total Public Expenditure on Education as a percentage of GDP (2002)
Australia	19,665	23.65	10.6	3.7
Austria	28,373	31.42	7.6	3.8
Belgium	13,936	24.34	8.3	4.2
Canada	m	m	m	m
Czech Republic	7,328	14.73	6.5	3.0
Denmark	15,730	26.18	8.7	4.8
Finland	12,208	21.95	7.9	4.0
France	20,402	24.76	7.7	4.1
Germany	21,458	26.84	6.4	3.1
Greece	10,819	18.91	5.3	2.5
Hungary	11,583	26.88	6.2	3.3
Iceland	14,400	25.34	12.0	5.5
Ireland	15,883	16.27	9.2	3.1
Italy	14,799	28.08	7.2	3.5
Japan	13,069	24.02	8.0	2.7
Korea, Rep	9,435	25.58	13.2	3.3
Luxembourg	25,807	24.74	9.2	4.0
Mexico	3,235	17.26	16.2	3.6
Netherlands	18,253	20.32	7.2	3.4
New Zealand	10,234	22.96	14.7	4.7
Norway	5,481	24.07	9.4	4.5
Poland	5,481	24.48	m	4.1
Portugal	11,861	31.51	9.2	4.3
Slovak Rep	3,665	14.57	5.5	2.9
Spain	10,601	22.85	7.5	3.0
Sweden	18,495	21.90	8.5	5.0
Switzerland	28,268	28.96	9.1	4.1
Turkey	m	m	m	2.4
United Kingdom	11,655	20.16	9.0	3.7
United States	17,147	23.68	10.3	3.8
OECD Average	**14,260**	**23.44**	**8.91**	**3.72**

*In equivalent US dollars converted using PPPs for GDP, based on full time equivalents
Source: OECD Education at a Glance, 2005.

Table 2. *Comparisons with Alternative Versions*[a]

		Closed Economy		Open Economy		
				Exogenous Policy	Endogenous Policy	
	Time Periods	Region A (1)	Region B (2)	Region A and B (3)	Region A (4)	Region B (5)
Capital stock per worker	1	34.39	0.00	14.67	13.26	15.72
	2	13.59	0.00	4.10	3.68	3.59
	3	6.25	34.39	21.87	23.55	21.02
	4	2.93	13.59	9.05	9.13	9.13
	steady state	71.42	71.42	69.39	71.42	71.42
Human capital per worker	1	-3.09	0.00	0.00	-8.42	3.87
	2	4.19	0.00	0.00	1.22	0.92
	3	1.94	-3.09	0.00	7.17	-5.23
	4	0.91	4.19	0.00	2.82	2.82
	steady state	4.73	4.73	0.00	4.73	4.73
Income per worker	1	7.95	0.00	4.62	-1.77	7.64
	2	7.20	0.00	1.33	2.03	1.79
	3	3.35	7.95	6.74	12.32	2.74
	4	1.57	7.20	2.90	4.86	4.86
	steady state	23.22	23.22	19.00	23.22	23.22
Government spending per worker	1	-7.25	0.00	0.00	-19.22	9.41
	2	10.33	0.00	0.00	3.03	2.17
	3	4.66	-7.25	0.00	18.27	-11.94
	4	2.17	10.33	0.00	6.85	6.85
	steady state	11.56	11.56	0.00	11.56	11.56
Consumption of young	1	13.59	0.00	6.34	4.71	6.98
	2	6.26	0.00	1.80	1.74	1.65
	3	2.93	13.59	9.06	10.59	8.33
	4	1.39	6.26	3.81	4.25	4.25
	steady state	27.55	27.55	26.05	27.55	27.55
Consumption of old	1	-12.24	0.00	-5.75	-5.97	-5.97
	2	8.00	0.00	4.51	3.18	5.42
	3	3.76	-12.24	-6.49	-6.52	-6.61
	4	1.78	8.00	5.10	6.86	4.68
	Steady state	1.73	1.73	0.53	1.73	1.73
Welfare	0	-3.93	0.00	-1.80	-1.90	-1.90
	1	16.25	0.00	7.79	5.60	8.81
	2	7.54	-3.93	-0.29	-0.36	-0.46
	3	3.52	16.25	10.76	13.03	9.81
	4	1.66	7.54	4.49	5.10	5.10
	steady state	28.31	28.31	26.25	28.31	28.31

[a]All numbers are percentage changes from the previous period. Numbers for steady state indicate percentage change between the initial and the final steady state.
Source: Computed by author.

Table 3. *Sensitivity of Simulation Results to Various Values of* ψ [a]

	ψ	0.35		0.5		0.7	
	Time Periods	Region A (1)	Region B (2)	Region A (3)	Region B (4)	Region A (5)	Region B (6)
Capital stock per worker	1	6.70	20.61	13.26	15.72	14.07	15.12
	2	-0.95	-0.60	3.68	3.59	6.24	6.20
	3	30.72	15.23	23.55	21.02	25.02	23.92
	4	5.98	5.98	9.13	9.13	14.08	14.08
	steady state	53.99	53.99	71.42	71.42	113.56	113.56
Human capital per worker	1	-26.91	3.39	-8.42	3.87	3.37	6.35
	2	-0.69	-0.11	1.22	0.92	2.78	2.66
	3	15.31	-18.96	7.17	-5.23	10.59	7.61
	4	2.34	2.34	2.82	2.82	6.09	6.09
	steady state	-12.70	-12.70	4.73	4.73	37.03	37.03
Income per worker	1	-17.19	8.78	-1.77	7.64	6.79	9.17
	2	-0.78	-0.27	2.03	1.79	3.91	3.82
	3	20.19	-8.98	12.32	2.74	15.16	12.74
	4	3.53	3.53	4.86	4.86	8.66	8.66
	steady state	5.28	5.28	23.22	23.22	58.64	58.64
Government spending per worker	1	-91.63	15.98	-19.22	9.41	4.97	9.42
	2	-14.02	-0.46	3.03	2.17	4.08	3.91
	3	419.65	-67.62	18.27	-11.94	15.81	11.29
	4	16.96	16.96	6.85	6.85	8.99	8.99
	steady state	-50.73	-50.72	11.56	11.56	58.24	58.24
Consumption of young	1	-4.83	7.58	4.71	6.98	8.00	9.00
	2	-0.58	-0.23	1.74	1.65	3.79	3.76
	3	15.14	1.50	10.59	8.33	14.73	13.72
	4	2.76	2.76	4.25	4.25	8.45	8.45
	steady state	14.58	14.59	27.55	27.55	58.91	58.91
Consumption of old	1	-8.54	-8.54	-5.97	-5.97	-4.21	-4.21
	2	-4.56	7.89	3.18	5.42	6.05	7.04
	3	-9.73	-9.42	-6.52	-6.61	-2.85	-2.89
	4	12.53	-0.81	6.86	4.68	10.42	9.44
	steady state	-8.61	-8.61	1.73	1.73	26.75	26.75
Welfare	0	-2.74	-2.74	-1.90	-1.90	-1.32	-1.32
	1	-6.54	10.22	5.60	8.81	9.99	11.34
	2	-3.72	-3.25	-0.36	-0.46	2.88	2.83
	3	19.59	0.91	13.03	9.81	18.33	16.95
	4	3.32	3.32	5.10	5.10	10.42	10.42
	steady state	11.21	11.21	28.31	28.31	71.12	71.12

[a] All numbers are percentage changes from the previous period. Numbers for steady state indicate percentage change between the initial and the final steady state.
Source: computed by author.

Table 4. Descriptive Statistics

Variable	Number of Observations	Mean	Standard Error	Minimum	Maximum
Education spending per worker (constant 1995 $)	406	982.00	1,186.79	1.70	5,223.21
Number of workers (labor force)	427	21,800,000	80,600,000	80,300	720,000,000
GDP per worker (constant 1995 $)	425	19,343.68	21,092.28	171.75	102,965.90
Total population	427	46,200,000	154,000,000	194,000	1,200,000,000
Openness (total trade % of GDP)	420	68.19	42..42	3.68	302.78
Population 65 and older (% of total population)	427	7.08	4.43	1.19	17.90
OECD	427	0.34	0.48	0	1
OECD * Openness	420	23.93	40.46	0	233.45
OECD * 65 and Older	427	4.22	6.00	0	17.9
Openness * 65 and Older	420	501.07	511.79	15.88	3,081.50
OECD * Openness * 65 and Older	420	461.14	797.09	0	4,433.82

Source: Author’s calculations.

Table 5. Fixed Effects Regressions[a]
(Dependent Variable: Logged education spending per worker)

Variable	**(1)**	**(2)**
GDP per capita	1.047***	1.050***
	(0.07)	(0.06)
Openness	0.134**	
	(0.07)	
Population 65 and Older	-0.136	-0.448***
	(0.132)	(0.17)
OECD * 65 and Older		1.075***
		(0.406)
Openness * 65 and Older		0.181***
		(0.07)
OECD * Openness * 65 and Older		-0.375**
		(0.18)
1970	0.106**	0.105**
	(0.05)	(0.05)
1975	0.205***	0.198***
	(0.05)	(0.05)
1980	0.218***	0.209***
	(0.06)	(0.06)
1985	0.205***	0.201***
	(0.06)	(0.06)
1990	0.199***	0.175**
	(0.06)	(0.06)
1995	0.292***	0.264***
	(0.07)	(0.07)
Constant	-4.126***	-4.180***
	(0.57)	(0.58)
Observations	400	400
R-squared (within)	0.70	0.71

[a] Results for country fixed effects are suppressed due to limited space. Standard errors are reported in parentheses.
*Indicate 10 percent significance level.
**Indicate 5 percent significance levels.
***Indicate 1 percent significance levels.

FIGURE 1

Tax Rate Transitions in A and B

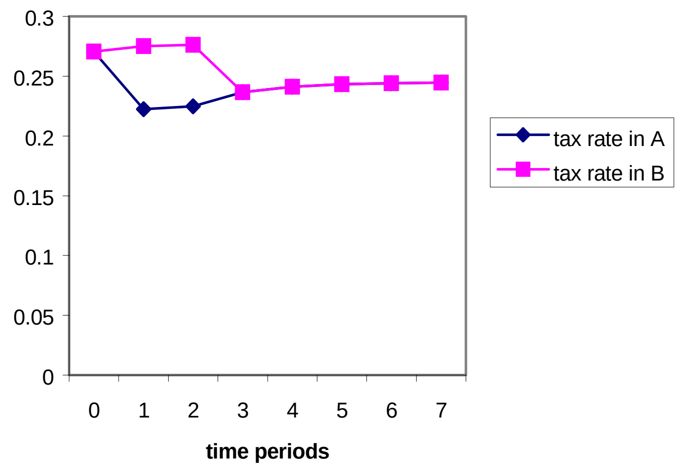


FIGURE 2

Human Capital Transitions in A and B

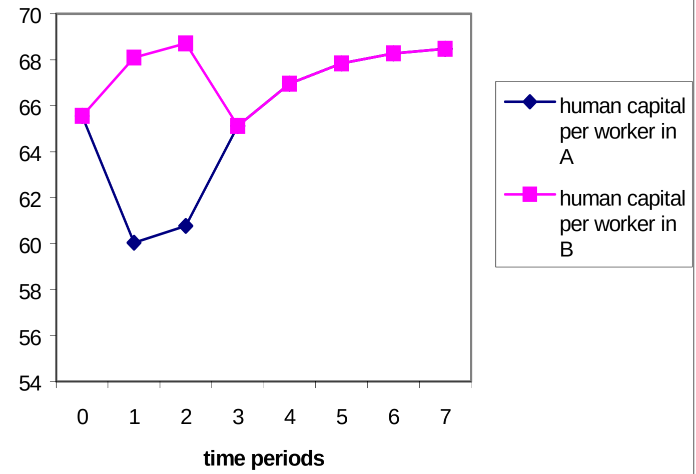


FIGURE 3

Capital Stock Transitions in A and B

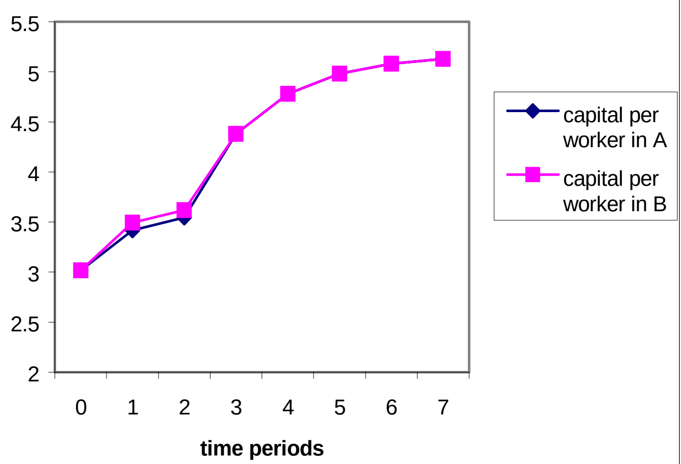


FIGURE 4

World Net Interest Rate Transitions

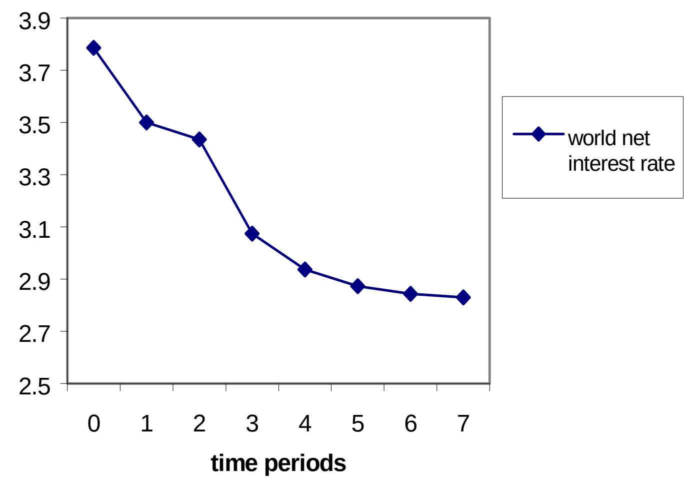


FIGURE 5

Income Transitions in Region A and B

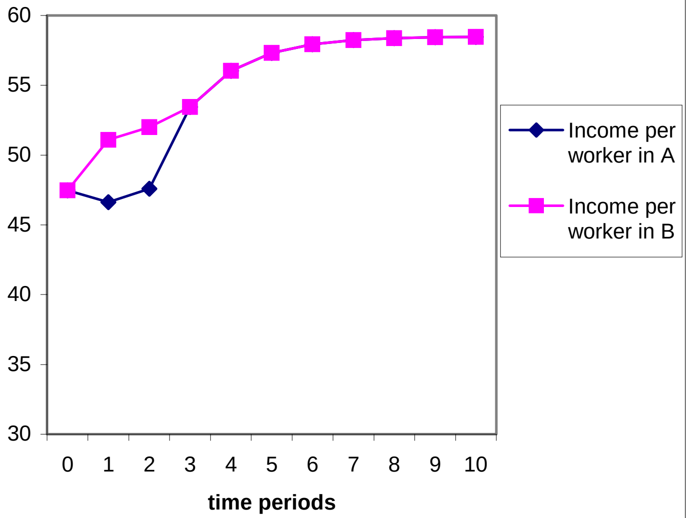


FIGURE 6

Consumption Transitions of Old and Young Generations in A and B

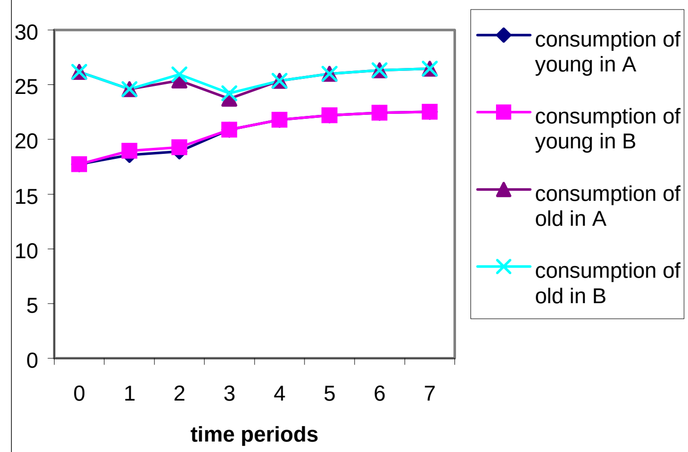
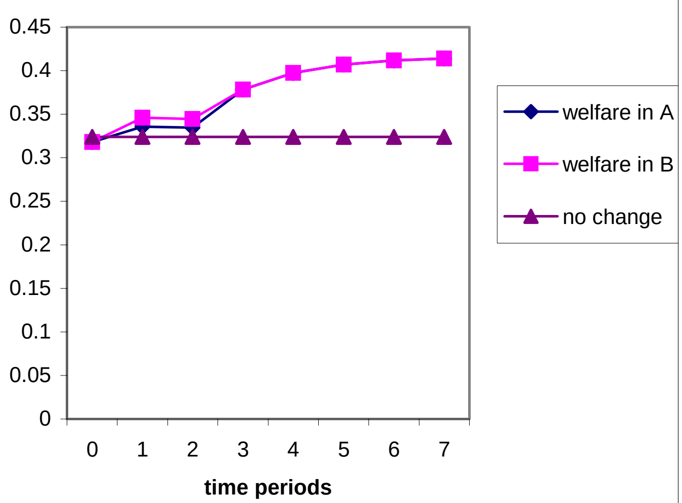


FIGURE 7

Welfare Transitions in A and B



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