

Public Sector Employment in an Equilibrium Search and Matching Model

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Abstract: In this paper, we extend the standard Pissarides model of equilibrium unemployment to incorporate public-sector employment. We modify the Pissarides model in three ways. First, we assume that workers are heterogeneous in terms of human capital. Second, we assume that productivity is match specific and that the distribution of match-specific productivity is more favorable in the sense of first-order stochastic dominance the higher is a worker's human capital. Third, we allow for both private- and public-sector employment. We prove existence of equilibrium and numerically analyze the effects of public-sector employment policy on the distributions of wages, productivities and human capital levels in the two sectors and on overall employment and welfare using data from Colombia.

1 Introduction

The public sector accounts for a substantial fraction of employment in both developed and developing economies. Algan et al. (2002) estimates that the public sector accounted for slightly less than 19% of total employment across 17 OECD countries in 2000, and Mizala et al. (2011) estimates that 13% of total urban employment over the period 1996-2007 across eleven Latin American countries was in the public sector. The basic ordering between public- and private sector wages is similar in most developed and developing countries. On average, there is a public-sector wage premium (see, e.g., Gregory and Borland 1999 for a survey), both in the raw data and after controlling for observable worker characteristics and endogenous sector choice.

Wages in the public sector tend to be more compressed than in the private sector. The public-sector premium is higher at lower quantiles and there is a negative public-sector premium at higher quantiles in some countries. See, e.g., Melly (2005) for Germany, Lucifora and Meurs (2006) for France, Britain and Italy and Mizala et al. (2011) for Latin America.

These stylized facts raise a number of obvious questions. In general, how do the private- and public-sector labor markets interact? What types of workers tend to work in the public sector? How does the size of the public sector affect the overall unemployment rate and the distributions of worker productivities and wages? What types tend to work in the private sector? How do the hiring and wage-setting rules used in the public sector affect the distribution of wages in the private sector? A natural approach to these questions is to incorporate public-sector employment into an equilibrium search and matching model. Surprisingly, there are very few papers that do this.

Our paper fills this void. Specifically, we incorporate a public sector into an extended version of the canonical Diamond-Mortensen-Pissarides (Pissarides 2000) model of equilibrium unemployment. We extend this model in three directions. First, we allow for *ex ante* worker heterogeneity; that is, we assume an exogenous distribution, $y \sim F(y)$, $\underline{y} \leq y \leq \bar{y}$, of human capital across workers. This makes it possible to address questions about which types of workers tend to work in the two sectors. This feature of our model is based on Albrecht, Navarro and Vroman (2009). Second, we allow for *ex post* idiosyncratic match productivity. When a worker of type y meets a prospective employer with a vacancy, she draws a match-specific productivity, $x \sim G_s(x|y)$, $\underline{x} \leq x \leq \bar{x}$, where the subscript $s \in \{p, g\}$ indicates whether the job in question is in the private or public (government) sector. To give content to our notion of human capital, we assume first-order stochastic dominance, i.e., $y' > y \Rightarrow G_s(x|y') < G_s(x|y)$. The higher is a worker's level of human capital, the more favorable is that worker's distribution of match-specific productivity, and this is the case in both sectors. The combination of idiosyncratic match productivity with a first-order stochastic dominance assumption is related to Dolado, Jansen and Jimeno (2009), who assume first-order stochastic dominance in conjunction with a two-point distribution for y – “low-skilled workers” and “high-skilled workers.”¹ Finally, we take into account the fact that the rules governing public-sector employ-

¹ A related idea can also be found in Coşar (2010). In his model, a worker's productivity on a particular job is the product of his or her human capital and a match-specific draw. The two components of the product are independently distributed.

ment and wage determination are in general not the same as those used in the private sector. We assume that the public sector posts an exogenous measure of vacancies, v_g , and that a worker of type y who meets a public-sector vacancy and draws match-specific productivity x is offered the job iff an index that depends on both x and y is large enough, i.e., iff $h_g(x, y) \geq 0$. We assume the index is nondecreasing in both of its arguments, which implies that a worker of type y receives a public-sector offer iff $x \geq \xi_g(y)$. We also assume that a worker's wage in a public-sector job is determined by an exogenous rule, $w_g(x, y)$, and without loss of generality, we set $w_g(x, y) = 0$ for $x < \xi_g(y)$. We assume that $w_g(x, y)$ is nondecreasing in its arguments so long as $x \geq \xi_g(y)$. We experiment with a variety of functional forms for $\xi_g(y)$ and $w_g(x, y)$; for example, we allow for the possibility that the government may pay relatively more attention to formal qualifications, i.e., y , than a private-sector employer would in the same circumstances. As best we know, our combination of these three elements – *ex ante* worker heterogeneity, match-specific productivity with a first-order stochastic dominance assumption, and both private- and public-sector employment – is unique in the search/matching literature.

As noted above, there are almost no papers that incorporate public-sector employment into an equilibrium search and matching model. Three exceptions are Burdett (2012), Quadrini and Trigari (2007), and Gomes (2011). Burdett (2012) adds a public sector to the Burdett and Mortensen (1998) model of on-the-job search, extended to allow for free entry of private-sector vacancies along the lines of Mortensen (2000). In Burdett (2012), the public sector is characterized by a measure of job slots (filled jobs plus vacancies), O_g , and a wage, w_g . Given (O_g, w_g) the measure of private-sector vacancies is determined by the free-entry condition, and the distribution of private-sector wage offers is determined by the usual Burdett and Mortensen (1998) equal-profit condition. Burdett (2012) thus captures the idea that public-sector employment policy, i.e., the choice of (O_g, w_g) , has spillover effects on the size of private-sector employment and the distribution of wages in the private sector. On the other hand, the equilibrium wage distribution in this paper has some decidedly unrealistic features – an upward-sloping density of private-sector wage offers (as in the standard homogeneous-firm version of Burdett and Mortensen 1998) and no wage dispersion in the public sector.

The paper by Quadrini and Trigari (2007) is closer to ours in the sense of taking the basic Pissarides (2000) model as its starting point. Their model is designed to analyze the effect of public-sector employment policy on the private-sector labor market over the business cycle. To do this, they consider

a discrete-time version of Pissarides (2000) in which private-sector productivity varies stochastically over time. They assume sector-specific search in the sense that in each period each unemployed worker chooses whether to search for a private- or a public-sector job. In equilibrium, since workers are homogeneous, each worker has to be indifferent between searching in one sector versus the other. Quadrini and Trigari (2007) assume that the level of public-sector employment (or, equivalently, the measure of public-sector vacancies) depends on (i) a target steady-state level for public-sector employment and (ii) the difference between current private-sector employment and its steady-state value, and they make an analogous assumption for the public-sector wage. The model developed in Gomes (2011) is similar to that of Quadrini and Trigari (2007). However, rather than assuming exogenous public-sector employment and wage-setting rules, Gomes (2011) characterizes the optimal public-sector policies. He shows, in particular, that public-sector vacancy posting should be countercyclical but that public-sector wages should vary procyclically.

Finally, there are, of course, papers that model the interaction between the private- and public-sector labor markets without taking an explicitly search-theoretic approach. For example, Algan et al. (2002) present a static model in which workers choose whether to look for work in the private or public sector. The private-sector wage and level of employment are then determined by union bargaining in a right-to-manage framework while the corresponding variables are set exogenously in the public sector. Worker sector choice is determined by an arbitrage condition.

Relative to these papers (and the few others that we know about), our paper offers the following. First, so far as we know, ours is the only paper that allows for *ex ante* worker heterogeneity in an equilibrium search and matching model with public-sector employment. That is, ours is the only paper that can address the question of which types of workers tend to work in the private sector and which types tend to work in the public sector. Second, we allow for match-specific productivity coupled with our first-order stochastic dominance assumption. This implies – as we see in reality – that there is not a perfect sorting of worker types between the two sectors. Finally, relative to earlier papers, we allow for a rich specification of public-sector employment policy.

In the next section, we set out the model and prove existence of equilibrium. In Section 3, we make specific functional form assumptions so that we can solve the model and analyze it numerically. We first present a simple numerical example to show how the model works and then we calibrate the model using data from Colombia. We are able to look at the effects of

changes in public-sector employment policy. We look at the effect of changing the productivity cutoff for public employment and changing the public wage policy. We are able to see how these policy changes effect the distributions of wages, productivities and human capital levels in the two sectors and overall welfare. In Section 4, we conclude.

2 Model

We consider a model with search and matching frictions. Only the unemployed search, and their prospects depend on overall labor market tightness, $\theta = (v_p + v_g)/u$, where v_p and v_g are the measures of private- and public-sector vacancies posted at any instant, and u is the fraction of the workforce that is unemployed. Search is random, so conditional on meeting a prospective employer, the probability that the job is in the private sector is $\phi = v_p/(v_p + v_g)$.² Specifically, job seekers meet prospective employers at Poisson rate $m(\theta)$, and employers meet job seekers at rate $m(\theta)/\theta$. Not all meetings lead to matches. In the private sector, a match forms if and only if the realized value of x is high enough so that the match is jointly worthwhile for the worker and firm. The threshold value of x depends in general on the worker's type. That is, a private-sector match forms if and only if $x \geq R_p(y)$, where $R_p(y)$ is a type-specific reservation productivity. In the public sector, a match forms if and only if $x \geq \xi_g(y)$. The key equilibrium objects are the reservation productivity schedule, $R_p(y)$, overall labor market tightness, θ , and the fraction, ϕ , of vacancy postings that are accounted for by the private sector. These objects are determined in equilibrium by (i) the condition that private-sector matches form only when it is in the joint interest of the worker and firm, (ii) a free-entry condition for private-sector vacancies, and (iii) steady-state conditions for worker flows into and out of unemployment, private-sector employment and public-sector employment.

2.1 Value Functions, Wages, Reservation Values

We start with the optimization problem for a worker of type y . Let $U(y)$, $N_p(x, y)$, and $N_g(x, y)$ be the values (expected discounted lifetime utilities) associated with unemployment and employment in, respectively, a private-sector job and a public-sector job with match-specific productivity x . The

² As will be seen when we lay out our model, an assumption of sector-specific search, as in Gomes (2011) and Quadrini and Trigari (2007), would give the unrealistic prediction of perfect sorting. That is, all workers above some type y^* would search exclusively in one sector while all workers of type below y^* would direct their search to the other sector.

value of unemployment for worker y is defined by

$$rU(y) = z + \phi m(\theta) E \max[N_p(x, y) - U(y), 0] + (1 - \phi) m(\theta) E \max[N_g(x, y) - U(y), 0]. \quad (1)$$

This expression reflects the following assumptions. Time is continuous, and the worker lives forever, discounting the future at rate r . The worker receives a flow value z while unemployed. Private-sector vacancies are met at rate $\phi m(\theta)$, and public-sector vacancies are met at rate $(1 - \phi) m(\theta)$. When the worker meets a vacancy, a match-specific productivity is realized, and the worker realizes a capital gain, either $N_p(x, y) - U(y)$ or $N_g(x, y) - U(y)$ if the relevant difference is positive; zero otherwise.

The two employment values are defined by

$$rN_p(x, y) = w_p(x, y) + \delta_p(U(y) - N_p(x, y)) \quad (2)$$

$$rN_g(x, y) = w_g(x, y) + \delta_g(U(y) - N_g(x, y)). \quad (3)$$

The private-sector wage is determined by Nash bargaining with an exogenous worker share parameter, as described below, while the public-sector wage schedule is exogenous. Job destruction is assumed to occur at exogenous Poisson rate δ_s , and we allow for the possibility that $\delta_p \neq \delta_g$.

On the private-sector firm side, let $J(x, y)$ be the value (expected discounted profit) associated with a job filled by a worker of type y whose match-specific productivity is x , and let V be the value associated with posting a private-sector vacancy. These values are defined by

$$rJ(x, y) = x - w_p(x, y) + \delta_p(V - J(x, y)) \quad (4)$$

$$rV = -c + \frac{m(\theta)}{\theta} E \max[J(x, y) - V, 0]. \quad (5)$$

The expectation in equation (5) is taken with respect to the joint distribution of (x, y) across the population of unemployed job seekers. A private-sector firm with a vacancy doesn't know what worker type it will meet next nor does it know what match-specific productivity this worker will draw. The firm does know, however, the distribution of worker types among the unemployed and the conditional distribution function $G_p(x|y)$.

We assume that the private-sector wage for a worker of type y with match-specific productivity x is determined via Nash bargaining with exogenous worker share parameter β . Imposing the free-entry condition for private-sector vacancy creation in advance, i.e., $V = 0$, the Nash bargaining solution implies

$$w_p(x, y) = \beta x + (1 - \beta)rU(y); \quad (6)$$

that is, the private-sector wage is a weighted average of the flow productivity of the match, x , and the flow value of the worker's outside option, $rU(y)$.

Substituting equation (6) into equation (2) and recalling our assumption that $w_g(x, y)$ is increasing in x for $x \geq \xi_g(y)$, it is clear that $N_p(x, y)$ and $N_g(x, y)$ are nondecreasing in x for any value of y . Accordingly, reservation productivities can be defined for the type- y worker. The private-sector reservation productivity for a type- y worker, $R_p(y)$, is defined by $N_p(R_p(y), y) = U(y)$. Using equations (2) and (6), $N_p(R_p(y), y) = U(y)$ implies $R_p(y) = rU(y)$. That is, at $x = R_p(y)$ the net surplus associated with the match equals zero. The public-sector reservation productivity for a type- y worker is simply $R_g(y) = \xi_g(y)$. This is equivalent to assuming that, given the public-sector wage schedule, $N_g(\xi_g(y), y) \geq U(y)$. If $N_g(\xi_g(y), y) > U(y)$, there is rationing of public-sector jobs for type- y workers as shown in Figure 1. If $N_g(\xi_g(y), y) = U(y)$, then $\xi_g(y) = rU(y) = R_p(y)$; that is, the public- and private-sector reservation productivities are equal for the type- y worker. Finally, we could in principle consider the case of $N_g(\xi_g(y), y) < U(y)$. In this case, however, matches would not form for $x \in [\xi_g(y), R_p(y))$ because workers would reject them. In this sense, it is without loss of generality to assume $N_g(\xi_g(y), y) \geq U(y)$.

To further characterize the private-sector reservation productivity, it is useful to rewrite our expression for $rU(y)$. Using equations (2) and (6) and integrating by parts gives

$$E \max[N_p(x, y) - U(y), 0] = \frac{\beta}{r + \delta_p} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx.$$

Similarly, using equation (3) together with $rU(y) = R_p(y)$ gives

$$E \max[N_g(x, y) - U(y), 0] = \frac{1}{r + \delta_g} \int_{\xi_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y).$$

Substituting into equation (1) then gives

$$\begin{aligned} R_p(y) &= z + \phi m(\theta) \frac{\beta}{r + \delta_p} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx \\ &\quad + (1 - \phi) m(\theta) \frac{1}{r + \delta_g} \int_{\xi_g(y)}^{\bar{x}} (w_g(x, y) - R_p(y)) dG_g(x|y). \end{aligned} \tag{7}$$

Given overall labor market conditions, i.e., θ and ϕ , and the government's employment and wage-setting policy, equation (7) gives a unique solution for $R_p(y)$ since the RHS of equation (7) is positive at $R_p(y) = 0$, goes to z as $R_p(y) \rightarrow \infty$, and the derivative of the RHS with respect to $R_p(y)$ is negative.

2.2 Steady State and Free-Entry Conditions

The next step is to characterize optimal entry by private-sector firms. Imposing $V = 0$ in advance and using equation (4), we have

$$J(x, y) = \frac{x - w_p(x, y)}{r + \delta_p} = (1 - \beta) \frac{x - rU(y)}{r + \delta_p} = (1 - \beta) \frac{x - R_p(y)}{r + \delta_p}.$$

Letting $f_u(y)$ denote the density of y among the unemployed, the free-entry condition, i.e., equation (5) with $V = 0$, can be written as

$$\begin{aligned} c &= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (x - R_p(y)) dG_p(x|y) f_u(y) dy \\ &= \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_{\underline{y}}^{\bar{y}} \int_{R_p(y)}^{\bar{x}} (1 - G_p(x|y)) dx f_u(y) dy, \end{aligned} \quad (8)$$

where the final equality uses integration by parts.

The only unknown function in equation (8) is the contaminated density, $f_u(y)$. Assuming that the exogenous distribution function, $F(y)$, is continuous with corresponding density, $f(y)$, one can use Bayes Law (as in Albrecht, Navarro and Vroman 2009) to write

$$f_u(y) = \frac{u(y)f(y)}{u};$$

that is, the density of types among the unemployed, $f_u(y)$, can be written as the type-specific unemployment rate, $u(y)$, times the population density, $f(y)$, normalized by the overall unemployment rate,

$$u = \int_{\underline{y}}^{\bar{y}} u(y)f(y)dy.$$

To derive the type-specific unemployment rates, $u(y)$, let $n_p(y)$ and $n_g(y)$ be the fractions of time that a type- y worker spends in private-sector and

public-sector employment, respectively. In steady state, the following two equations must hold:

$$\delta_p n_p(y) = \phi m(\theta)(1 - G_p(R_p(y)|y))u(y) \quad (9)$$

$$\delta_g n_g(y) = (1 - \phi)m(\theta)(1 - G_g(\xi_g(y)|y))u(y). \quad (10)$$

The first condition equates the flow from private-sector employment to unemployment and vice versa, and the second condition equates the flow from public-sector employment to unemployment and vice versa. Using

$$u(y) + n_p(y) + n_g(y) = 1,$$

equations (9) and (10) imply

$$\begin{aligned} u(y) &= \frac{\delta_g \delta_p}{\delta_g \delta_p + \delta_g \phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p (1 - \phi)m(\theta)(1 - G_g(\xi_g(y)|y))} \\ n_p(y) &= \frac{\delta_g \phi m(\theta)(1 - G_p(R_p(y)|y))}{\delta_g \delta_p + \delta_g \phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p (1 - \phi)m(\theta)(1 - G_g(\xi_g(y)|y))} \\ n_g(y) &= \frac{\delta_p (1 - \phi)m(\theta)(1 - G_g(\xi_g(y)|y))}{\delta_g \delta_p + \delta_g \phi m(\theta)(1 - G_p(R_p(y)|y)) + \delta_p (1 - \phi)m(\theta)(1 - G_g(\xi_g(y)|y))} \end{aligned} \quad (11)$$

Substituting the expression for $u(y)$ into equation (8) completes the characterization of the private-sector free-entry condition.

The final unknown that needs to be characterized is ϕ , the fraction of vacancies that are posted by private-sector firms. To do this, note that since

$$v_p + v_g = \theta u,$$

$$\phi = v_p / (v_p + v_g)$$

implies

$$\phi = \frac{\theta u - v_g}{\theta u}. \quad (12)$$

This closes the model.

2.3 Equilibrium

Definition: A steady-state equilibrium is a function, $R_p(y)$, that satisfies equation (7) for all $y \in [\underline{y}, \bar{y}]$ together with scalars θ and ϕ that satisfy equations (8), (11) and (12).

An equilibrium always exists. First, as noted above, for given values of θ and ϕ , the reservation productivity, $R_p(y)$, is uniquely determined.

Second, given any value of ϕ , equation (8) has at least one solution for θ . The argument is standard. The RHS of equation (8) is continuous in θ , it converges to infinity as $\theta \rightarrow 0$, and it goes to zero as $\theta \rightarrow \infty$. Finally, once $R_p(y)$ and θ are determined as functions of ϕ , equation (12) has at least one solution in ϕ . (The complication, of course, is that u depends on ϕ .) Note that we do not claim uniqueness. In equation (8), $f_u(y)$ need not be monotonically decreasing in θ nor is it obvious that equation (12) has a unique solution. Uniqueness depends on the form of $F(y)$, $G_p(x|y)$, $G_g(x|y)$ and public-sector employment policy and needs to be investigated numerically.³

2.4 Wage Distributions

Let $H_s(w)$ denote the distribution function of wages paid in sector s . We can develop expressions for $H_p(w)$ and $H_g(w)$ as follows. Consider first the distribution of private-sector wages across workers of type y , say $H_p(w|y)$. Of course,

$$H_p(w|y) = 0 \text{ for } w < w_p(R_p(y), y).$$

That is, no wages are paid to a type- y worker below the wage that makes that worker indifferent between accepting and rejecting a private-sector match. The conditional distribution function of w in private-sector jobs given y for $w \geq R_p(y)$ then follows from

$$H_p(w|y) = P[w_p(R_p(y), y) \leq w_p(X, y) \leq w|y].$$

Since

$$w_p(R_p(y), y) = \beta R_p(y) + (1 - \beta)R_p(y) = R_p(y),$$

we have

$$\begin{aligned} H_p(w|y) &= P[R_p(y) \leq \beta X + (1 - \beta)R_p(y) \leq w|y] \\ &= P[R_p(y) \leq X \leq \frac{w - (1 - \beta)R_p(y)}{\beta}|y] \\ &= G_p\left(\left(\frac{w - (1 - \beta)R_p(y)}{\beta}\right)|y\right) - G_p(R_p(y)|y). \end{aligned}$$

³The possibility of non-uniqueness of equilibrium is a common feature of models with worker heterogeneity. See, e.g., Albrecht, Navarro and Vroman (2009) and Chéron, Hairault and Langot (2011).

Summarizing, we have

$$H_p(w|y) = \begin{cases} 0 & \text{for } w < R_p(y) \\ G_p\left(\left(\frac{w-(1-\beta)R_p(y)}{\beta}\right)|y\right) - G_p(R_p(y)|y) & \text{for } w \geq R_p(y) \end{cases} \quad (13)$$

The unconditional distribution of private-sector wages is then found by integrating the conditional distribution function against the density of Y among private-sector employees. That is,

$$H_p(w) = \int_{\underline{y}}^{\bar{y}} H_p(w|y) f_p(y) dy, \quad (14)$$

where

$$f_p(y) = \frac{n_p(y)f(y)}{n_p}.$$

The same approach can be used to find the distribution of public-sector wages, namely,

$$H_g(w|y) = \begin{cases} 0 & \text{for } w < w_g(\xi_g(y), y) \\ P[w_g(\xi_g(y), y) \leq w_g(X, y) \leq w|y] & \text{for } w \geq w_g(\xi_g(y), y) \end{cases} \quad (15)$$

and

$$H_g(w) = \int_{\underline{y}}^{\bar{y}} H_g(w|y) f_g(y) dy. \quad (16)$$

To go further requires specifying the functional form of $w_g(x, y)$ in order to invert the inequality in (15). We do this below.

3 Solving the Model

To solve the model, we make functional form assumptions for $F(y)$, $G_p(x|y)$, $G_g(x|y)$, $\xi_g(y)$ and $w_g(x, y)$. Specifically, we assume

- (i) $F(y) = y$ for $0 \leq y \leq 1$
- (ii) $G_s(x|y) = \Phi\left(\frac{\ln x - y}{\sigma_s}\right)$ for $x > 0$ and $s = p, g$
- (iii) $\xi_g(y) = \alpha + R_p(y)$
- (iv) $w_g(x, y) = \gamma x + (1 - \gamma)rU(y) = \gamma x + (1 - \gamma)R_p(y)$.

How can we understand these assumptions? Assumption (i), namely, that Y follows a standard uniform distribution, can be viewed as a normalization. The effect on model outcomes of changing this assumption, e.g., to assuming that Y follows a more general Beta distribution, can equally well be achieved by changing the assumed distributions of match-specific productivities conditional on y . Assumption (ii) states that, conditional on y , match-specific productivity in sector s follows a log-normal distribution. This functional form clearly satisfies first-order stochastic dominance; i.e., $G_s(x|y') < G_s(x|y)$ for $y' > y$. The particular parameterization we are using also implies that, conditional on y , the expected value of match-specific productivity in sector s is $\exp\{y + \frac{\sigma_s^2}{2}\}$. If $\sigma_g^2 < \sigma_p^2$, then the conditional distribution of private-sector match productivity first-order stochastically dominates the corresponding distribution of public-sector match productivity and, of course, vice versa if $\sigma_g^2 > \sigma_p^2$. Our log normality assumption is parsimonious, but we also expect it to do a good job of matching the data.

Our specifications for the public-sector hiring rule and wage schedule are expressed as “deviations from” the corresponding private-sector rules. Our specification for $\xi_g(y)$ requires $\alpha \geq 0$. At $\alpha = 0$, there is no rationing of public-sector jobs; a worker of type y who draws $x = rU(y)$ ($= R_p(y)$) is just indifferent between accepting the job and continuing to search. When $\alpha > 0$, some public-sector jobs are rationed. A worker of type y who draws $x = \xi_g(y)$ receives a wage above the level needed to get her to accept the job. Specifically, at $x = \xi_g(y)$, the gap between the wage the worker receives and her reservation wages is $\alpha\gamma$. See Figure 2.

These functional form assumptions simplify the equations that define equilibrium. Equation (7) becomes

$$\begin{aligned}
 R_p(y) = & z + \phi m(\theta) \frac{\beta}{r + \delta_p} \int_{R_p(y)}^{\infty} (1 - \Phi(\frac{\ln x - y}{\sigma_p})) dx \\
 & + (1 - \phi) m(\theta) \frac{1}{r + \delta_g} \left(\alpha \gamma \left(1 - \Phi(\frac{\ln(\alpha + R_p(y)) - y}{\sigma_g}) \right) + \gamma \int_{\alpha + R_p(y)}^{\infty} (1 - \Phi(\frac{\ln x - y}{\sigma_g})) dx \right)
 \end{aligned} \tag{17}$$

An extreme special case of equation (17) holds when $\alpha = 0$ (no rationing of public-sector jobs) and $\gamma = 0$ (conditional on meeting the type-specific productivity hurdle for the job, workers are paid solely based on their credentials). In this case, the private-sector reservation productivity for a worker

of type y only depends on the distribution of private-sector opportunities, and the public-sector wage schedule is

$$w_g(x, y) = \begin{cases} 0 & \text{for } x < R_p(y) \\ R_p(y) & \text{for } x \geq R_p(y) \end{cases}.$$

Similarly, equation (8) becomes

$$c = \frac{m(\theta)}{\theta} \left(\frac{1 - \beta}{r + \delta_p} \right) \int_0^1 \int_{R_p(y)}^\infty \left(1 - \Phi\left(\frac{\ln x - y}{\sigma_p}\right) \right) dx \frac{u(y)}{u} dy, \quad (18)$$

where

$$u(y) = \frac{\delta_g \delta_p}{\delta_g \delta_p + \delta_g \phi m(\theta) (1 - \Phi(\frac{\ln R_p(y) - y}{\sigma_p})) + \delta_p (1 - \phi) m(\theta) (1 - \Phi(\frac{\ln(\alpha + R_p(y)) - y}{\sigma_g}))}. \quad (19)$$

Finally, we have equation (12),

$$\phi = \frac{\theta u - v_g}{\theta u}.$$

3.1 Numerical Example

In this section, we present a simple numerical simulation of our model to illustrate its properties. We assume a Cobb Douglas matching function $m(\theta) = A\theta^{1-\beta}$ and use an implicit time unit of one quarter. For our baseline simulation we use the following parameter values:

| | |
|------------|-------|
| r | 0.012 |
| z | 0.5 |
| A | 2 |
| β | 0.75 |
| c | 0.5 |
| δ_p | 0.1 |
| δ_g | 0.05 |
| σ_p | 0.2 |
| σ_g | 0.15 |

and take the public policy parameters as:

| | |
|----------|-------|
| v_g | 0.014 |
| α | 0 |
| γ | 0.25 |

The assumption that $\alpha = 0$ implies that the government uses the same productivity hiring cutoff as the private sector, while $\gamma = 0.25$ implies that the government puts much less weight on productivity when setting wages. Below we also consider the case in which $\gamma = \beta$. The results of the baseline simulation for the endogenous variables are:

| | |
|----------|-------|
| u | 0.109 |
| θ | 0.94 |
| ϕ | 0.86 |

Given this solution, we can derive the distributions of productivity in the private and public sectors, which are given in Figures 3 and 4.

If we increase the size of the public sector by increasing government vacancies, we find a decrease in unemployment, an increase in labor market tightness, and a decrease in the fraction of jobs in the private sector. That is, if we now use

| | |
|----------|-------|
| v_g | 0.028 |
| α | 0 |
| γ | 0.25 |

the results change to

| | |
|----------|-------|
| u | 0.092 |
| θ | 1.03 |
| ϕ | 0.7 |

Figures 5 and 6 illustrate the new distributions of productivity and wages.

If the government is assumed to place the same weight on productivity as the private sector when setting wages ($\gamma = \beta$), then we find more unemployment and less labor market tightness than in the baseline case. Figures 7 and 8 illustrate the new distributions of productivity and wages. In this case, the policy parameters are

| | |
|----------|-------|
| v_g | 0.014 |
| α | 0 |
| γ | 0.75 |

and the results are

| | |
|----------|-------|
| u | 0.117 |
| θ | 0.86 |
| ϕ | 0.86 |

3.2 Calibration Strategy

Now that we have checked that our model “works” for reasonable parameter values, we want to calibrate. Our strategy is as follows. We observe ϕ , θ and u . Given these observed values, given any collection of assumed values for the parameters of the model, and given an assumed functional form for $m(\theta)$, we can solve equation (17) for $R_p(y)$ for, e.g., $y = 0.01, 0.02, \dots, 0.98, 0.99$. Then the solution for $R_p(y)$ can be substituted into equations (18) and (19) to get an updated value of θ . Finally, equation (12) gives an updated value for ϕ . Then we iterate. The essence of this procedure is to choose the free parameters of the problem to minimize the distance between the observed values of ϕ , θ and u and the corresponding values generated by the iterative procedure. Once we have the solution for the free parameters, we can derive the distribution of productivity of workers in the two sectors.

3.3 Calibration: Colombia

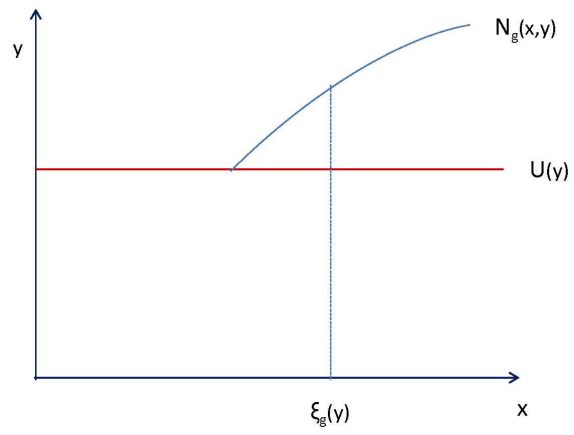
To be added

4 Conclusion

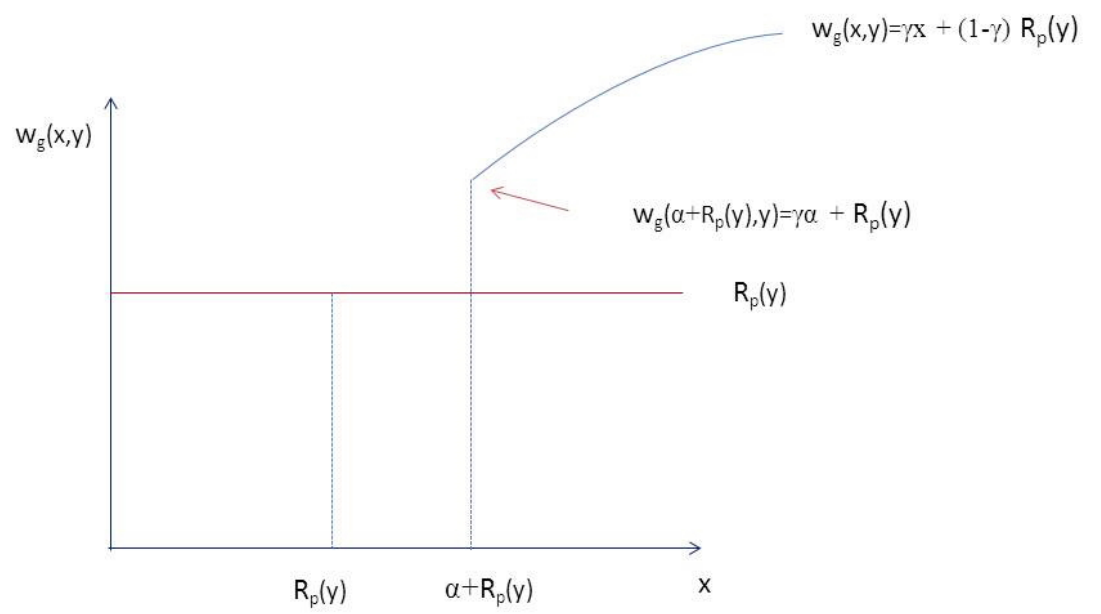
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Public Sector Hiring Rule



Rationing of Public Sector Jobs